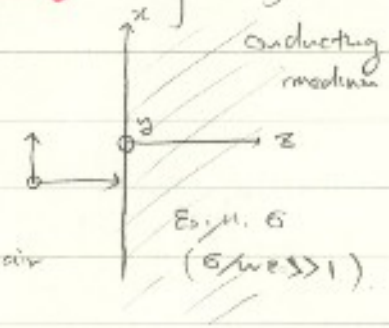
8.28 uniform plane wave in air $\vec{E}_i(z) = \vec{a}_x E_0 \exp(-j\beta_0 z)$



a) reflection coefficient

$$\Gamma = \frac{E_r}{E_i} = \frac{\eta_c - \eta_0}{\eta_c + \eta_0}$$

where $\eta_0 = 120\pi$, $\eta_c = \sqrt{j\omega\mu/\sigma}$ $(\because \sigma/\omega\varepsilon >> 1)$

(b) $$|\Gamma|^2 = \left|\frac{\eta_c - \eta_0}{\eta_c + \eta_0}\right|^2 = \left|\frac{1 - \eta_c/\eta_0}{1 + \eta_c/\eta_0}\right|^2 \approx \left|1 - 2\frac{\eta_c}{\eta_0}\right|^2$$

$$= \left(1 - 2\frac{\eta_c}{\eta_0}\right)\left(1 - 2\frac{\eta_c^*}{\eta_0}\right) \approx 1 - 4\,\mathrm{Re}(\eta_c)/\eta_0$$

the fraction of the incident power absorbed by the conducting medium

$$P = 1 - |\Gamma|^2 = \frac{4}{\eta_0}\,\mathrm{Re}\left(\sqrt{\frac{j\omega\mu}{\sigma}}\right) = \frac{4}{\eta_0}\sqrt{\frac{\omega\mu}{2\sigma}}$$

(c) the fraction of the power absorbed at 1MHz if the medium is iron

$\omega = 2\pi \times 10^6$ (Hz)

for iron: $\mu = 4000 \times (4\pi \times 10^{-7})$ (H/m)

$\sigma = 10^7$ (S/m)

$$P \doteq \frac{4}{\eta_0}\sqrt{\frac{\omega\mu}{2\sigma}} = \frac{4}{120\pi}\sqrt{\frac{(2\pi\times10^6)(4000)(4\pi\times10^{-7})}{2\times10^7}}$$

$$= 4.21 \times 10^{-4}$$

8.30 a transparent dielectric coating → glass ($\varepsilon_r = 4$, $\mu_r = 1$)
to eliminate the reflection of red light ($\lambda_0 = 0.75\,\mu m$)

(a) Determine ε_r and d (thickness) of the coating.

from example 8-12,

$$\eta_2 = \sqrt{\eta_1\eta_3} \longrightarrow \varepsilon_{r2} = \sqrt{\varepsilon_{r1}\varepsilon_{r3}} = 2$$

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wavelength of red light in dielectric coating

$$\lambda_2 = \frac{0.75}{\sqrt{\varepsilon_{r_2}}} = \frac{0.75}{2} = 0.530\,(\mu m)$$

$$\Rightarrow \boxed{d = \frac{\lambda_2}{4} = 0.133\,\mu m}$$

(b) violet light ($\lambda_0 = 0.42\,\mu m$) → coating → glass

what percentage of the incident power will be reflected?

for violet light

wavelength of violet light → $\lambda_2' = \frac{0.42}{\sqrt{2}} = 0.297\,(\mu m)$

$$\frac{d}{\lambda_2'} = 0.447 \Rightarrow \beta_2 d = 0.894\pi$$

from Eq. (8-173)

$$Z_2(0) = \eta_2 \frac{\eta_3 + j\eta_2 \tan\beta_2 d}{\eta_2 + j\eta_3 \tan\beta_2 d} = \frac{1}{\sqrt{2}} \frac{(1/2) + (j/\sqrt{2})\tan\beta_2 d}{(1/\sqrt{2}) + (j/2)\tan\beta_2 d} = \frac{0.5 - j(0.24)}{1.0 - j(0.24)}$$

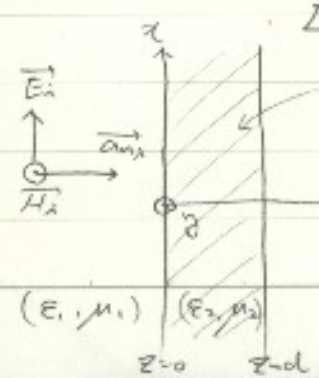
$$\Rightarrow \Gamma = \frac{Z_2(0) - 1}{Z_2(0) + 1} = 0.317\, e^{j198^\circ}$$

Percentage of power reflected

$$= |\Gamma|^2 \times 100\% = (0.317)^2 \times 100\% \approx \boxed{10\%}$$

8.32 uniform plane wave

$$\vec{E}_i(z,t) = \vec{a}_x E_{i0} \cos\omega\left(t - \frac{z}{u_p}\right)$$



$$\vec{E}_1 = \vec{a}_{x1}\left(E_{i0}\, e^{-j\beta_1 z} + E_{r0}\, e^{j\beta_1 z}\right)$$

$$\vec{H}_1 = \vec{a}_y \frac{1}{\eta_1}\left(E_{i0}\, e^{-j\beta_1 z} - E_{r0}\, e^{j\beta_1 z}\right)$$

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$$\vec{E}_2 = \vec{a}_x \left(E_2^+ e^{-j\beta_2 z} + E_2^- e^{j\beta_2 z} \right)$$

$$\vec{H}_2 = \vec{a}_y \frac{1}{\eta_2} \left(E_2^+ e^{-j\beta_2 z} - E_2^- e^{j\beta_2 z} \right)$$

at $z = d$, $\vec{E}_2 = 0 \Rightarrow E_2^- = -E_2^+ e^{-j2\beta_2 d}$

$$\Rightarrow \vec{E}_2 = \vec{a}_x E_2^+ \left[e^{-j\beta_2 z} - e^{j\beta_2 (z-2d)} \right]$$

$$\vec{H}_2 = \vec{a}_y \frac{E_2^+}{\eta_2} \left[e^{-j\beta_2 z} + e^{j\beta_2 (z-2d)} \right]$$

Boundary conditions at $z = 0$

$$\vec{E}_1(0) = \vec{E}_2(0) \longrightarrow E_{i0} + E_{r0} = E_2^+ \left(1 - e^{-j2\beta_2 d} \right)$$

$$\vec{H}_1(0) = \vec{H}_2(0) \longrightarrow E_{i0} - E_{r0} = E_2^+ \frac{\eta_0}{\eta_2} \left(1 + e^{-j2\beta_2 d} \right)$$

$$\Rightarrow E_2^+ = \frac{2\eta_2 E_{i0}}{(\eta_0 + \eta_2) + (\eta_0 - \eta_2) e^{-j2\beta_2 d}}$$

$$E_{r0} = -\left(\frac{\eta_0 - j\eta_2 \tan\beta_2 d}{\eta_0 + j\eta_2 \tan\beta_2 d} \right) E_{i0}$$

(a) $$\vec{E}_1(z,t) = \vec{a}_x E_{i0} \cos\left(\omega\left(t - \frac{z}{u_p}\right) + \pi - 2\tan^{-1}\left(\frac{\eta_2}{\eta_0} \tan\beta_2 d \right) \right)$$

(b) $$\vec{E}_1(z,t) = \vec{a}_x E_{i0} \left[\cos\left(\omega\left(t - \frac{z}{u_p}\right)\right) + \cos\left(\omega\left(t - \frac{z}{u_p}\right) + \pi - 2\tan^{-1}\left(\frac{\eta_2}{\eta_0} \tan\beta_2 d \right) \right) \right]$$

(c) $$\vec{E}_2(z,t) = \vec{a}_x \cdot \frac{2\eta_2 E_{i0}}{\sqrt{2\left[(\eta_0^2 + \eta_2^2) + (\eta_0^2 - \eta_2^2)\right] \cos 2\beta_2 d}} \cdot \left\{ \cos\left[\omega\left(t - \frac{z}{u_{p2}}\right) + \psi \right] - \cos\left[\omega\left(t + \frac{z}{u_{p2}}\right) - \frac{2\pi d}{u_{p2}} + \psi \right] \right\}$$

where $$\psi = \tan^{-1}\left[\frac{(\eta_0 - \eta_2)\sin 2\beta_2 d}{(\eta_0 + \eta_2) + (\eta_0 - \eta_2)\cos 2\beta_2 d} \right]$$

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(d) $(\vec{P}_{av})_1 = \frac{1}{2} Re(\vec{E}_1 \times \vec{H}_1^*) = \boxed{0}$

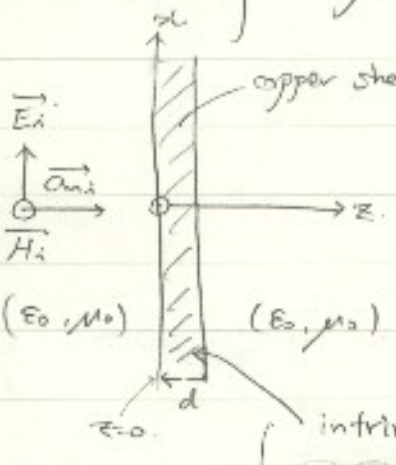
(e) $(\vec{P}_{av})_2 = \frac{1}{2} Re(\vec{E}_2 \times \vec{H}_2^*) = 0$

(f) determine the thickness d that makes $\vec{E}_1(z,t)$ the same as if the dielectric slab were absent

$\Rightarrow E_r = -E_{i0} \Rightarrow \tan\beta_2 d = 0 \Rightarrow \boxed{d = \frac{n\lambda_2}{2}, \quad n = 0, 1, 2 \ldots}$

$\boxed{8.33}$ uniform plane wave $\vec{E}_i(z) = \vec{a}_x E_{i0} e^{-j\beta_0 z}$

neglecting multiple reflections within the copper sheets



a) from problem 8.29

$$\boxed{E_2^+} = \boxed{\eta_2 H_2^+} = \boxed{-j\left(\frac{\eta_2}{\eta_0}\right) \frac{e^{\alpha_2 d} e^{j\beta_2 d} E_{i0}}{\sin(\beta_2 - j\alpha_2)d}}$$

intrinsic impedance η_2

α_2, β_2 the attenuation constants of copper sheet

H_2^+, H_2^-, H_{30} 도 쉽게 구할수 있

(b) $$\boxed{E_2^-} = \boxed{-\eta_2 H_2^-} = \boxed{-j\left(\frac{\eta_2}{\eta_0}\right) \frac{e^{-\alpha_2 d} e^{-j\beta_2 d} E_{i0}}{\sin(\beta_2 - j\alpha_2)d}}$$

(c) $$\boxed{E_{30}} = \boxed{\eta_0 H_{30}} = \boxed{-j\left(\frac{\eta_2}{\eta_0}\right) \frac{2e^{j\beta_0 d} E_{i0}}{\sin(\beta_2 - j\alpha_2)d}}$$

(d) $$E_{r0} = \frac{E_{i0}}{1 - j\frac{\eta_2}{\eta_0}\cot(\beta_2 - j\alpha_2)d}$$

$$\simeq -\left(1 + j\frac{\eta_2}{\eta_0} \frac{1 + j\tan\beta_2 d \tanh\alpha_2}{\tan\beta_2 d - j\tanh\alpha_2 d}\right) E_{i0}$$

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$$\Rightarrow (\vec{P}_{av})_1 = \frac{1}{2} R_e \left[\left(\vec{E}_{10} \times \vec{H}_{10}'^{*} \right) - \left(\vec{E}_{r0} \times \vec{H}_{r0}'^{*} \right) \right]$$

$$= \vec{a}_z \frac{\alpha}{\eta_0^2 6} (A + jB)$$

where $$\frac{1 + j \tan\beta_2 d \tanh\alpha_2 d}{\tan\beta_2 d - j \tanh\alpha_2 d} = A + jB.$$

$$\Rightarrow (P_{av})_1 = \frac{\alpha}{\eta_0^2 6} \frac{\sin\beta_2 d \cos\beta_2 d + \sinh\alpha_2 d \cosh\alpha_2 d}{(\sin\beta_2 d \cosh\alpha_2 d)^2 + (\cos\beta_2 d \sinh\alpha_2 d)^2}$$

$$\Rightarrow (P_{av})_2 = \frac{1}{2\eta_0} |E_{20}|^2 = \frac{1}{\eta_0^2} \left(\frac{\alpha}{6} \right)^2 \frac{4E_{10}^2}{(\sin\beta_2 d \cosh\alpha_2 d)^2 + (\cos\beta_2 d \sinh\alpha_2 d)^2}$$

$$\therefore \frac{(P_{av})_2}{(P_{av})_1} = \frac{4}{\eta_0} \left(\frac{\alpha}{6} \right) \frac{1}{\sin\beta_2 d \cos\beta_2 d + \sinh\alpha_2 d \cosh\alpha_2 d}$$

Since $(P_{av})_1 = \frac{1}{2\eta_0} E_{10}^2$

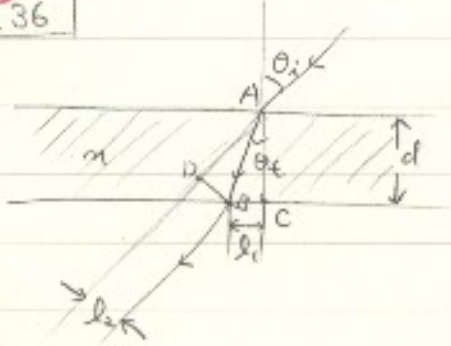
$$\Rightarrow \left| \frac{(P_{av})_2}{(P_{av})_1} \right| = \left| \frac{8}{\eta_0^2} \left(\frac{\alpha}{6} \right)^2 \frac{1}{(\sin\beta_2 d \cosh\alpha_2 d)^2 + (\cos\beta_2 d \sinh\alpha_2 d)^2} \right|$$

Let $f = 10^7 (Hz)$, $6 = 5.80 \times 10^7 (S/m)$,

$\alpha_2 = \beta_2 = \frac{1}{\delta} = \sqrt{\pi f \mu_2 6} = 4.785 \times 10^4$, $d = \delta = \frac{1}{\alpha_2}$

$$\Rightarrow \left| \frac{(P_{av})_2}{(P_{av})_1} \right| = \boxed{1.839 \times 10^{-11}}$$

8.36



(a) By Snell's law

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{1}{n}$$

$$\Rightarrow \boxed{\theta_t = \sin^{-1}\left(\frac{1}{n} \sin\theta_i \right)}$$

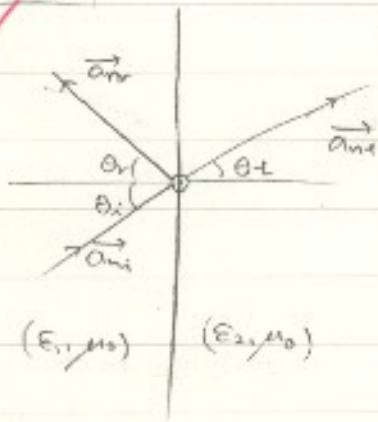
(b) $\cos\theta_t = \sqrt{1-\left(\frac{1}{n}\sin\theta_i\right)^2}$

$$\boxed{l_1 = \overline{BC} = \overline{AC}\tan\theta_t = d\frac{\sin\theta_t}{\cos\theta_t} = \frac{d\sin\theta_i}{\sqrt{n^2-\sin^2\theta_i}}}$$

(c) $$\boxed{l_2 = \overline{BD} = \overline{AC}\sin(\theta_i-\theta_t) = \frac{d}{\cos\theta_t}(\sin\theta_i\cos\theta_t - \cos\theta_i\sin\theta_t)}$$

$$= \boxed{d\sin\theta_i\left[1-\frac{\cos\theta_i}{\sqrt{n^2-\sin^2\theta_i}}\right]}$$

8.42 prove the following.



(a) for perpendicular polarization.

$$\Gamma_\perp = \frac{\sin(\theta_t-\theta_i)}{\sin(\theta_t+\theta_i)} \qquad \tau_\perp = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_t+\theta_i)}$$

from Eq. 8-185. $\frac{n_2}{n_1} = \frac{\sin\theta_t}{\sin\theta_i}$

(a) from Eq. 8-206, $\boxed{\Gamma_\perp} = \frac{(n_2/n_1)\cos\theta_i - \cos\theta_t}{(n_2/n_1)\cos\theta_i + \cos\theta_t}$

$$= \frac{\sin\theta_t\cos\theta_i - \cos\theta_t\sin\theta_i}{\sin\theta_t\cos\theta_i + \cos\theta_t\sin\theta_i}$$

$$= \boxed{\frac{\sin(\theta_t-\theta_i)}{\sin(\theta_t+\theta_i)}}$$

from Eq. 8-207. $\boxed{\tau_\perp} = \frac{2(n_2/n_1)\cos\theta_i}{(n_2/n_1)\cos\theta_i+\cos\theta_t} = \boxed{\frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_t+\theta_i)}}$

(b) for parallel polarization.

$$\Gamma_{\parallel} = \frac{\sin 2\theta_t - \sin 2\theta_i}{\sin 2\theta_t + \sin 2\theta_i} \qquad \tau_{\parallel} = \frac{4\sin\theta_t\cos\theta_i}{\sin 2\theta_t + \sin 2\theta_i}$$

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from Eq. (8-221)

$$\boxed{\Gamma_{11}} = \frac{(\eta_2/\eta_1)\cos\theta_t - \cos\theta_i}{(\eta_2/\eta_1)\cos\theta_t + \cos\theta_i} = \frac{\sin\theta_t\cos\theta_t - \sin\theta_i\cos\theta_i}{\sin\theta_t\cos\theta_t + \sin\theta_i\cos\theta_i}$$

$$= \boxed{\frac{\sin 2\theta_t - \sin 2\theta_i}{\sin 2\theta_t + \sin 2\theta_i}}$$

from Eq. (8-222)

$$\boxed{\tau_{11}} = \frac{2(\eta_2/\eta_1)\cos\theta_i}{(\eta_2/\eta_1)\cos\theta_t + \cos\theta_i} = \boxed{\frac{4\sin\theta_t\cos\theta_i}{\sin 2\theta_t + \sin 2\theta_i}}$$

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