Homework set 2 (David K. Cheng, Fundamentals of Engineering Electromagnetics)

P. 2-18 Given a scalar field $V = 2xy - yz + xz$

a) find the vector representing the direction and the magnitude of the maximum rate of increase of V at point $P(2,-1,0)$, and

b) find the rate of increase of V at point $P(2,-1,0)$ in the direction toward the point $Q(0,2,6)$.

P. 2-20 Find the divergence of the following radial fields:

a) $f_1(\mathbf{R}) = \mathbf{a}_R R^n$,

b) $f_2(\mathbf{R}) = \mathbf{a}_R k/R^2$, where k is a constant.

P. 2-21 Given a vector field $\mathbf{F} = \mathbf{a}_x xy - \mathbf{a}_y yz + \mathbf{a}_z zx$,

a) compute the total outward from the surface of a unit cube in the first octant with one corner at the origin, and

b) find $\nabla \cdot \mathbf{F}$ and verify the divergence theorem.

P. 2-23 For a vector function $\mathbf{A} = \mathbf{a}_z Z$,

a) find $\oint \mathbf{A} \cdot d\mathbf{s}$ over the surface of a hemispherical region that is the top half of a sphere of radius 3 centered at the origin with its flat base coinciding with the xy-plane,

b) find $\nabla \cdot \mathbf{A}$, and

c) verify the divergence theorem.

P. 2-26 Assume a vector field $\mathbf{A} = \mathbf{a}_x (2x^2 + y^2) + \mathbf{a}_y (xy - y^2)$,

a) Find $\oint \mathbf{A} \cdot dl$ around the triangular contour shown in Fig. 2-27

b) Find $\oint (\nabla \times \mathbf{A}) \cdot d\mathbf{s}$ over the triangular area.

c) Can $\mathbf{A}$ be expressed as the gradient of a scalar? Explain.

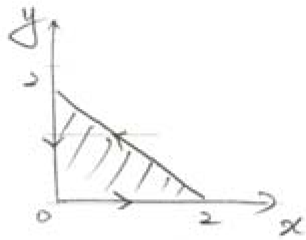

Fig. 2-27 Graph for Problem P. 2-26

P. 2-29 For a scalar function f and a vector function $\mathbf{G}$, prove

$$\nabla \times (f\mathbf{G}) = f(\nabla \times \mathbf{G}) + (\nabla f) \times \mathbf{G}$$

In Cartesian coordinates. In addition, also prove (2-115) by using summation convention and Levi-Civita symbol ε_{ijk}.

P. 2-30 Given a vector function

$$\mathbf{F} = \mathbf{a}_x(x + 3y - c_1 z) + \mathbf{a}_y(c_2 x + 5z) + \mathbf{a}_z(2x - c_3 y + c_4 z)$$

a) determine $c_1, c_2,$ and c_3 if $\mathbf{F}$ is irrotational , and

b) determine c_4 if $\mathbf{F}$ is also solenoidal.