Homework set 4 selected solution

P. 3-17 c) If a 10-(mm) thick plexiglass is inserted between the plates, what is the maximum voltage that can be applied to the plates without a breakdown?

Solution)

Breakdown voltage:

$V_b = E_a da + E_p dp$ (a : air, p: plate)

Since $D_a = D_p \Rightarrow \varepsilon_0 E_a = \varepsilon E_p \Rightarrow E_a = \frac{\varepsilon}{\varepsilon_0} E_p = \varepsilon_r E_p = 3E_p$

$\Rightarrow E_a > E_p$ (Higher field is applied in air)

Since $E_{break,air} = 3kV/mm$, $E_{break,plate} = 20\,kV/mm \Rightarrow$ Breakdown occurs in air first.

Therefore

$$V_b = E_{break,air} da + \frac{E_{break,air}}{\varepsilon_r} dp = 3kV/mm\left(40 + \frac{10}{3}\right) = 130kV$$

P. 3-27 A parallel-plate capacitor of width w, length L, and separation d has a solid dielectric slab of permittivity ε in the space between the plates. The capacitor is charged to a voltage V_0 by a battery, as indicated in Fig. 3-32. Assuming that the dielectric slab is withdrawn to the position shown and the switch is opened, determine the force acting on the slab.

Solution)

Total capacitance C:

$$C = C_d + C_a = \varepsilon \frac{xw}{d} + \varepsilon_0 \frac{(L-x)w}{d} = \frac{w}{d}\{\varepsilon x + \varepsilon_0 (L-x)\}$$

Charge on the plate when witch open:

$Q = CV_0$

Stored electric energy:

$$W_e = \frac{1}{2} CV_0^2 = \frac{V_0^2}{2} \frac{w}{d}\{\varepsilon x + \varepsilon_0 (L-x)\}$$

Force acting on the slab by method of virtual displacement

$$\mathbf{F}_Q = -\nabla W_e = -\hat{x} \frac{\partial}{\partial x}\left[\frac{V_0^2}{2} \frac{w}{d}\{\varepsilon x + \varepsilon_0 (L-x)\}\right] = -\hat{x} \frac{V_0^2 w}{2d}(\varepsilon - \varepsilon_0)$$