Homework set 5 selected solution

P. 4-3 Lightning strikes a lossy dielectric sphere - $\varepsilon = 12\varepsilon_0$, $\sigma = 10(S/m)$ -of radius 0.1 (m) at time t=0, depositing uniformly in the sphere a total charge 1 (mC). Determine, for all t,

Solution)

Charge density inside the sphere

$$\rho(t) = \rho_0 e^{-(\sigma/\varepsilon)t},\ t \ge 0 \qquad \text{Where } \rho(t=0) = \rho_0 = \frac{Q_0}{4\pi a^3/3} \approx 0.24\ (C/m^3)$$

a) the electric field intensity both inside and outside the sphere,

For $R < b$ (inside the sphere)

$$\mathbf{E}_i(R,t) = \hat{R}\frac{(4\pi R^3/3)f(t)}{4\pi\varepsilon R^2} = \hat{R}\frac{\rho_0 R}{3\varepsilon}e^{-(\sigma/\varepsilon)t} = \hat{R}7.5\times10^9 R\, e^{-9.42\times10^{11}t}\ (V/m)$$

For $R > b$ (outside the sphere)

$$\mathbf{E}_o(R,t) = \hat{R}\frac{Q_0}{4\pi\varepsilon_0 R^2} = \hat{R}\frac{9\times10^9}{R^2}\ (V/m)$$

b) the current density in the sphere.

For $R < b$, $\mathbf{J}_i(R,t) = \sigma\mathbf{E}_i(R,t) = \hat{R}7.5\times10^{10} R\, e^{-9.42\times10^{11}t}\ (V/m)$

For $R > b$, $\mathbf{J}_o(R,t) = 0$

P. 4-11 Refer to the flat conducting quarter-circular washer in Example 4-4 and Fig. 4-4. Find the resistance between the curved sides.

Solution)

Boundary value problem, Laplace equation: $\frac{1}{r}\frac{d}{dr}\left(r\frac{dV}{dr}\right) = 0,\ a \le r \le b$ ①

Boundary conditions: $V(r)\big|_{r=a} = V_0,\ V(r)\big|_{r=b} = 0$ ②

General solution: $V(r) = C_1 \ln r + C_2$ ③

①, ② -> ③ then $V(r) = V_0\frac{\ln(r/b)}{\ln(a/b)}$ ④

$$\mathbf{E}(r) = -\hat{r}\frac{\partial V}{\partial r} = \hat{r}\frac{V_0}{\ln(b/a)}\frac{1}{r}$$

$$I = \int_S \mathbf{J}\cdot d\mathbf{s} = \int_S \sigma\mathbf{E}\cdot d\mathbf{s} = \sigma\int_0^{\pi/2} \mathbf{E}(r) hr d\phi = \frac{\pi}{2}\frac{\sigma h V_0}{\ln(b/a)}$$

$$R = \frac{V_0}{I} = \frac{2\ln(b/a)}{\pi\sigma h}$$