1. $r = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

i) $b^2-4ac>0$

$$y = c_1 e^{r_1 t} + c_2 e^{r_2 t} \quad \left[\begin{array}{l} r_1 = \dfrac{-b+\sqrt{b^2-4ac}}{2a} < 0 \\ r_2 = \dfrac{-b-\sqrt{b^2-4ac}}{2a} < 0 \end{array} \right]$$

$$\therefore \lim_{t\to\infty} y = 0 \quad \cdots \quad (1)$$

ii) $b^2-4ac=0$

$$y = c_1 e^{\frac{-b}{2a}t} + c_2 t e^{\frac{-b}{2a}t}$$

$$\lim_{t\to\infty} y = \lim_{t\to\infty} c_1 e^{\frac{-b}{2a}t} + \lim_{t\to\infty} c_2 t e^{\frac{-b}{2a}t}$$

$$\lim_{t\to\infty} c_1 e^{\frac{-b}{2a}t} = 0 \quad \left(\because a, b>0,\ \frac{-b}{2a}<0\right)$$

$$\lim_{t\to\infty} c_2 t e^{\frac{-b}{2a}t} = \lim_{t\to\infty} \frac{c_2 t}{e^{\frac{b}{2a}t}} = \lim_{t\to\infty} \frac{c_2}{\frac{b}{2a}e^{\frac{b}{2a}t}} = 0 \quad \left(\because \frac{b}{2a}>0\right)$$

$$\therefore \lim_{t\to\infty} y = 0 \quad \cdots \quad (2)$$

iii) $b^2-4ac<0$

$$y = c_1 e^{-\frac{b}{2a}t}\cos\left(\frac{|\sqrt{b^2-4ac}|}{2a}t\right) + c_2 e^{-\frac{b}{2a}t}\sin\left(\frac{|\sqrt{b^2-4ac}|}{2a}t\right)$$

$$\therefore \lim_{t\to\infty} y = 0 \quad \cdots \quad (3)$$

by (1), (2), (3)

$$\lim_{t\to\infty} y = 0$$

2. $\frac{d^2y}{dt^2} - 4t\frac{dy}{dt} + (4t^2 - 2)y = 0 \qquad y_1(t) = e^{t^2}$

$$U = \frac{1}{e^{2t^2}} e^{-\int -4t\, dt} = \frac{1}{e^{2t^2}} e^{2t^2} = 1$$

$$\therefore y_2 = e^{t^2} \int 1\, dt = te^{t^2}$$

$$\therefore \underline{y = c_1 e^{t^2} + c_2 t e^{t^2}} \;//$$

3. $t^2 \frac{d^2y}{dt^2} - 2y = t^2$

$t^2 \frac{d^2y}{dt^2} - 2y = 0$, Let's $y = t^r$

$t^2 (r-1) r\, t^{r-2} - 2t^r = 0$

$r^2 - r - 2 = 0$

$\therefore r = -1$ or 2

$\therefore y_1 = t^{-1}, \; y_2 = t^2$

$W = y_1 y_2' - y_2 y_1' = -$

$= t^{-1} 2t - t^2(-1t^{-2}) = 2 + 1 = 3$

$u = -\int \frac{t^2}{3}\, dt \qquad (r = 1) \Leftarrow \frac{d^2y}{dt^2}$의 계수를 1로

$v = \int \frac{t^{-1}}{3}\, dt$

$u = -\frac{1}{9}t^3, \; v = \frac{1}{3}\ln t$

$\therefore y_p = -\frac{1}{9}t^3 t^{-1} + \frac{1}{3}\ln t\; t^2$

$= -\frac{1}{9}t^2 + \frac{1}{3}t^2 \ln t$

$\therefore y = c_1 t^{-1} + c_2 t^2 - \frac{1}{9}t^2 + \frac{1}{3}t^2 \ln t$

$= c_1 t^{-1} + c_3 t^2 + \frac{t^2}{3}\ln t$ //

4. a) $y'' + y' - 6y = \sin t + te^{2t}$

$y'' + y' - 6y = 0$ 에서

$r^2 + r - 6 = 0 \Rightarrow r = -3$ or 2

e^{2t} 는 homogeneous equation 의 solution.

$\therefore y_p = c_5 \sin t + c_4 \cos t + (c_3 t^2 + c_2 t + c_1) e^{2t}$ 라 가정

$y_p' = c_5 \cos t - c_4 \sin t + (2c_3 t + c_2) e^{2t} + 2(c_3 t^2 + c_2 t + c_1) e^{2t}$

$y_p'' = -c_5 \sin t - c_4 \cos t + (2c_3) e^{2t} + 2(2c_3 t + c_2) e^{2t}$
$\quad + 2(2c_3 t + c_2) e^{2t} + 4(c_3 t^2 + c_2 t + c_1) e^{2t}$

준식에 대입

$-c_5 \sin t - c_4 \cos t + (4c_3 t^2 + (8c_3 + 4c_2)t + 2c_3 + 4c_2 + 4c_1) e^{2t}$

$+ c_5 \cos t - c_4 \sin t + (2c_3 t^2 + (2c_3 + 2c_2)t + c_2 + 2c_1) e^{2t}$

$- 6c_5 \sin t - 6c_4 \cos t - (6c_3 t^2 + 6c_2 t + 6c_1) e^{2t}$

$= \sin t + te^{2t}$

$\therefore \begin{array}{l} -c_4 + c_5 - 6c_4 = 0 \\ -c_5 - c_4 - 6c_5 = 1 \end{array} \Rightarrow \left.\begin{array}{l} c_5 - 7c_4 = 0 \\ -7c_5 - c_4 = 1 \end{array}\right) \Rightarrow \begin{array}{l} c_5 = \frac{-7}{50} \\ c_4 = \frac{-1}{50} \end{array}$

$4c_3 + 2c_3 - 6c_3 = 0$

$8c_3 + 4c_2 + 2c_3 + 2c_2 - 6c_2 = 10c_3 = 1 \quad \therefore c_3 = \frac{1}{10}$

$2c_3 + 4c_2 + 4c_1 + c_2 + 2c_1 - 6c_1 = 2c_3 + 5c_2 = 0 \quad \therefore c_2 = \frac{-1}{25}$

$\therefore y_p = -\frac{7}{50}\sin t - \frac{1}{50}\cos t + \left(\frac{1}{10}t^2 - \frac{1}{25}t + c_1\right) e^{2t}$

여기서 $c_1 e^{2t}$ 는 homogeneous equation 의 solution 이므로 생략

$\therefore y_p = -\frac{7}{50}\sin t - \frac{1}{50}\cos t + \left(\frac{1}{10}t^2 - \frac{1}{25}t\right) e^{2t}$ //

b) $y'' + 4y = t\sin 2t$

$r^2 + 4 = 0 \qquad r = \pm j2$

$\therefore$ $\sin 2t$, $\cos 2t$는 homogeneous equation 의 solution

Let's $y_p = (c_1 t^2 + c_2 t)\sin 2t + (c_3 t^2 + c_4 t)\cos 2t$

($\sin 2t$, $\cos 2t$ term 은 homogeneous equation 의 solution 임으로 생략)

$$y_p' = (2c_1 t + c_2)\sin 2t + 2(c_1 t^2 + c_2 t)\cos 2t + (2c_3 t + c_4)\cos 2t - 2(c_3 t^2 + c_4 t)\sin 2t$$

$$= (2c_1 t + c_2 - 2c_3 t^2 - 2c_4 t)\sin 2t + (2c_1 t^2 + 2c_2 t + 2c_3 t + c_4)\cos 2t$$

$$y_p'' = (2c_1 - 4c_3 t - 2c_4)\sin 2t + 2(2c_1 t + c_2 - 2c_3 t^2 - 2c_4 t)\cos 2t + (4c_1 t + 2c_2 + 2c_3)\cos 2t - 2(2c_1 t^2 + 2c_2 t + 2c_3 t + c_4)\sin 2t$$

준식에 대입

$$(2c_1 - 4c_3 t - 2c_4 - 4c_1 t^2 - 4c_2 t - 4c_3 t - 2c_4)\sin 2t$$
$$+ (4c_1 t + 2c_2 - 4c_3 t^2 - 4c_4 t + 4c_1 t + 2c_2 + 2c_3)\cos 2t$$
$$+ 4(c_1 t^2 + c_2 t)\sin 2t + 4(c_3 t^2 + c_4 t)\cos 2t = t\sin 2t$$

$$-4c_3 t^2 + (4c_1 - 4c_4 + 4c_1)t + 4c_2 + 2c_3 + 4c_3 t^2 + 4c_4 t = 0$$

$$-4c_1 t^2 + (-4c_3 - 4c_2 - 4c_3)t + 2c_1 - 2c_4 - 2c_4 + 4c_1 t^2 + 4c_2 t = t$$

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$$\begin{cases} 4c_1 - 4c_4 + 4c_1 + 4c_4 = 0, & 4c_2 + 2c_3 = 0 \\ -4c_3 - 4c_2 - 4c_3 + 4c_2 = 1, & 2c_1 - 2c_4 - 2c_4 = 0 \end{cases}$$

$$\begin{cases} 8c_1 = 0, & 2c_2 + c_3 = 0 \\ -8c_3 = 1, & c_1 - 2c_4 = 0 \end{cases}$$

$$\therefore c_1 = 0, \quad c_3 = -\frac{1}{8}, \quad c_2 = \frac{1}{16}, \quad c_4 = 0$$

$$\therefore y_p = \frac{1}{16} t\sin 2t - \frac{1}{8} t^2 \cos 2t$$ //