3장

1.

1. $y'''' - 5y''' + 6y'' + 4y' - 8y = 0$

$$r^4 - 5r^3 + 6r^2 + 4r - 8 = 0$$

$$(r-2)^2 (r+1) = 0$$

$$\therefore y = (c_1 + c_2 t + c_3 t^2) e^{2t} + c_4 e^{-t}$$

2. $y'''' + 4y''' + 14y'' - 20y' + 25y = 0$, $y(0) = y'(0) = y''(0) = y'''(0) = 0$

$$r^4 + 4r^3 + 14r^2 - 20r + 25 = 0$$

$r_1 \approx 0.6707 + j0.8977$, $r_2 \approx 0.6707 - j0.8977$ $\Big) \Rightarrow$ 복소수임을 확인

$r_3 = -2.6707 + j3.5745$ $r_4 \approx -2.6707 - j3.5745$

복소수 임으로

$$y = e^{\alpha t}(c_1 \cos\beta t + c_2 \sin\beta t) + e^{\gamma t}(c_3 \cos\gamma t + c_4 \sin\gamma t)$$

$$y(0) = c_1 + c_3 = 0$$

$$y' = \alpha e^{\alpha t}(c_1 \cos\beta t + c_2 \sin\beta t) + e^{\alpha t}(-c_1 \beta \sin\beta t + c_2 \beta \cos\beta t)$$
$$+ \gamma e^{\gamma t}(c_3 \cos\gamma t + c_4 \sin\gamma t) + e^{\gamma t}(-c_3 \gamma \sin\gamma t + c_4 \gamma \cos\gamma t)$$

$$y'(0) = \alpha c_1 + c_2 \beta + c_3 \gamma + c_4 \gamma = 0$$

위와 같은 방법으로 $y''(0)$, $y'''(0)$를 구해보면 $c_1 = c_2 = c_3 = c_4 = 0$

$$\therefore y(t) = 0$$

※ 라플라스를 이용한 풀이

$$y'''' + 4y''' + 14y'' - 20y' + 25y = 0$$

$$s^4 Y + 4s^3 Y + 14s^2 Y - 20sY + 25Y = 0$$

$$\therefore Y = 0$$

$$\therefore y(t) = 0$$

3. $y''' - 4y' = t + \cos t + 2e^{-2t}$

$r^3 - 4r = 0$

$r(r-2)(r+2) = 0 \qquad \therefore r = 0, 2, -2$

상수항, e^{-2t}는 homogeneous equation의 solution

Let's $y_p = C_1 t + C_2 \cos t + C_3 \sin t + C_4 t e^{-2t}$

$y_p' = C_1 - C_2 \sin t + C_3 \cos t + C_4 e^{-2t} - 2C_4 t e^{-2t}$

$y_p'' = -C_2 \cos t - C_3 \sin t - 2C_4 e^{-2t} - 2C_4 e^{-2t} + 4C_4 t e^{-2t}$

$y_p''' = C_2 \sin t - C_3 \cos t + 8C_4 e^{-2t} + 4C_4 e^{-2t} - 8C_4 t e^{-2t}$

준식에 대입

$C_2 \sin t - C_3 \cos t + 12C_4 e^{-2t} - 8C_4 t e^{-2t}$

$- 4C_1 + 4C_2 \sin t - 4C_3 \cos t - 4C_4 e^{-2t} + 8C_4 t e^{-2t}$

$= t + \cos t + 2e^{-2t}$

$C_2 + 4C_2 = 0 \qquad \therefore C_2 = 0$

$-C_3 - 4C_3 = 1 \qquad \therefore C_3 = -\frac{1}{5}$

$8C_4 = 2 \qquad \therefore C_4 = \frac{1}{4}$

$-4C_1 = 1 \qquad \therefore C_1 = -\frac{1}{4}$

$\therefore y_p = -\frac{1}{4}t - \frac{1}{5}\sin t + \frac{1}{4} t e^{-2t}$ //

4. $y''' + y'' + y' + y = t + e^{-t}$

e^{-t}는 homogeneous equation의 solution

Let's $y_p = c_1 t + c_2 + c_3 t e^{-t}$

$$y_p' = c_1 + c_3 e^{-t} - c_3 t e^{-t}$$

$$y_p'' = -c_3 e^{-t} - c_3 e^{-t} + c_3 t e^{-t}$$

$$y_p''' = c_3 e^{-t} + c_3 e^{-t} + c_3 e^{-t} - c_3 t e^{-t}$$

주식에 대입

$$3c_3 e^{-t} - c_3 t e^{-t} - 2c_3 e^{-t} + c_3 t e^{-t} + c_1 + c_3 e^{-t} - c_3 t e^{-t}$$
$$+ c_1 t + c_2 + c_3 t e^{-t} = t + e^{-t}$$

$$\left.\begin{aligned} c_1 + c_2 &= 0 \\ c_1 &= 1 \\ 2c_3 &= 1 \end{aligned}\right\} \Rightarrow \begin{aligned} c_1 &= 1 \\ c_2 &= -1 \\ c_3 &= \tfrac{1}{2} \end{aligned}$$

$$\therefore y_p = t - 1 + \frac{1}{2} t e^{-t}$$ //