4장

1. a) $y' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} y$, $y(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$ 의 eigenvalue 는 3 , -1

eigenvector $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$$\therefore y(t) = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-t}$$

$y(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ 대입

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} c_1 + c_2 \\ 2c_1 - 2c_2 \end{pmatrix} \Rightarrow \begin{matrix} c_1 = \frac{7}{4} \\ c_2 = \frac{1}{4} \end{matrix}$$

$$\therefore y(t) = \frac{7}{4} \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t} + \frac{1}{4} \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-t}$$

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b) $y' = \begin{pmatrix} 1 & -3 & 2 \\ 0 & -1 & 0 \\ 0 & -1 & -2 \end{pmatrix} y$, $y(0) = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$

eigenvalue 1 , -2 , -1

eigenvector $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$ $\begin{pmatrix} 5 \\ 2 \\ -2 \end{pmatrix}$

$$\therefore y(t) = c_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} e^{-2t} + c_3 \begin{pmatrix} 5 \\ 2 \\ -2 \end{pmatrix} e^{-t}$$

$y(0) = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$ 대입

$$\begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} c_1 - 2c_2 + 5c_3 \\ 2c_3 \\ 3c_2 - 2c_3 \end{pmatrix} \Rightarrow \begin{matrix} c_3 = 0 \\ c_2 = 1 \\ c_1 = 0 \end{matrix}$$

$$\therefore y(t) = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} e^{-2t}$$

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2. (a) $y' = \begin{pmatrix} -2 & -1 \\ 4 & -7 \end{pmatrix} y$

$y_1' = -2y_1 - y_2$

$y_2' = 4y_1 - 7y_2$

$\frac{dy_2}{dy_1} = \frac{y_2'}{y_1'} = \frac{4y_1 - 7y_2}{-2y_1 - y_2}$

임계점 : $(0,0)$

$p = (-2) + (-7) = -9$

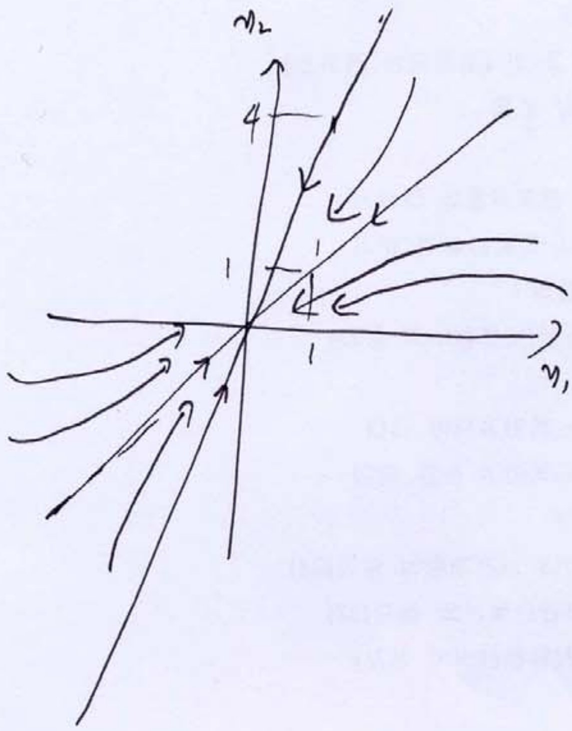
$q = 14 + 4 = 18$

$\Delta = 81 - 72 = 9$

$\therefore$ 마디점, 안정

eigenvalue $-3, \ -6$

eigenvector $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \ \begin{pmatrix} 1 \\ 4 \end{pmatrix}$



b) $y' = \begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix} y$

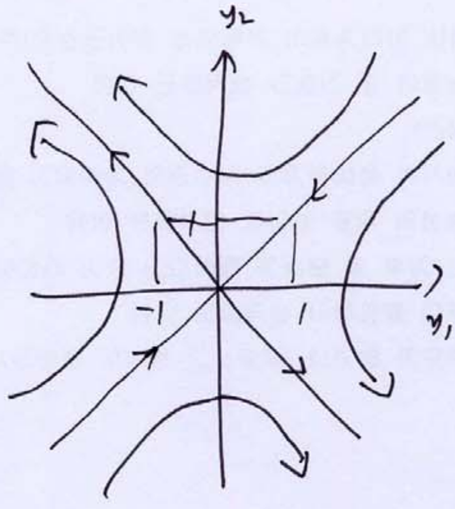
임계점 : $(0,0)$

$p = 2$, $q = -8$, $\Delta = 4 - 64 = -60$

$\therefore$ 안장점, 불안정

eigenvalue $-2, \ 4$

eigenvector $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \ \begin{pmatrix} -1 \\ 1 \end{pmatrix}$



3. $\dfrac{y_2'}{y_1'} = \dfrac{-y_1(1+y_1+y_2)}{y_2(1+y_1+y_2)}$

임계점 : $(0,0)$ and $1+y_1+y_2=0$ 인 모든점.
$(x, -1-x)$ ~~x~~ $x \in \mathbb{R}$

i) $(0,0)$

$y_1' = +y_2$
$y_2' = -y_1$

$\therefore \begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

$p = 0$, $q = 1$, $\Delta = -4$

$\therefore$ 중심

임계점 $(x, -1-x)$

$y_1 = x + \tilde{y}_1$, $y_2 = -1-x+\tilde{y}_2$

$\tilde{y}_1' = (-1-x+\tilde{y}_2)(1+x+\tilde{y}_1-1-x+\tilde{y}_2)$
$= (-1-x+\tilde{y}_2)(\tilde{y}_1+\tilde{y}_2) \approx (-1-x)\tilde{y}_1 + (-1-x)\tilde{y}_2$

$\tilde{y}_2' = -(x+\tilde{y}_1)(1+x+\tilde{y}_1-1-x+\tilde{y}_2)$
$= -(x+\tilde{y}_1)(\tilde{y}_1+\tilde{y}_2) \approx -x\tilde{y}_1 - x\tilde{y}_2$

$\therefore \begin{pmatrix} \tilde{y}_1' \\ \tilde{y}_2' \end{pmatrix} = \begin{pmatrix} -1-x & -1-x \\ -x & -x \end{pmatrix} \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}$

$p = -1-2x$, $q = 0$

$q=0$ 임으로 임계점의 유형 판단 불가능.

4. $$y' = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} y + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{2t}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix}$$

$\lambda_1 = 1$, eigenvector $\begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$

λ_2 and $\lambda_3 = 1 \pm j2$

i) $\lambda_1 = 1$

$$y_1 = e^{t} \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$$

ii) λ_2와 λ_3는 complex conjugate.

$\therefore$ λ_2만 고려

$\lambda_2 = 1 + j2$, eigenvector $\begin{pmatrix} 0 \\ 1 \\ -j \end{pmatrix}$

$$\begin{pmatrix} 0 \\ 1 \\ -j \end{pmatrix} e^{(1+j2)t} = e^{t}(\cos 2t + j \sin 2t) \left[\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - j \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right]$$

$$= e^{t} \left[\cos 2t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \sin 2t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right]$$

$$+ j e^{t} \left[\sin 2t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - \cos 2t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right]$$

$$\therefore y_2 = e^{t} \begin{pmatrix} 0 \\ \cos 2t \\ \sin 2t \end{pmatrix} , \quad y_3 = e^{t} \begin{pmatrix} 0 \\ \sin 2t \\ -\cos 2t \end{pmatrix}$$

$\therefore$ homogeneous equation의 solution은

$$y_h = e^{t} \left(c_1 \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ \cos 2t \\ \sin 2t \end{pmatrix} + c_3 \begin{pmatrix} 0 \\ -\sin 2t \\ \cos 2t \end{pmatrix} \right)$$

Let's $y_p = b e^{2t}$

y_p 를 준식에 대입

$$2 b e^{2t} = A b e^{2t} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{2t}$$

$$2b = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} b + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\begin{pmatrix} 2b_1 - 1 \\ 2b_2 \\ 2b_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ 2b_1 + b_2 - 2b_3 \\ 3b_1 + 2b_2 + b_3 \end{pmatrix}$$

$$\therefore b_1 = 1, \quad b_2 = -\frac{4}{5}, \quad b_3 = \frac{7}{5}$$

$$\therefore y_p = \begin{pmatrix} 1 \\ -\frac{4}{5} \\ \frac{7}{5} \end{pmatrix} e^{2t}$$

$$\therefore y(t) = y_h + y_p$$

$$= e^{t} \left(c_1 \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ \cos 2t \\ \sin 2t \end{pmatrix} + c_3 \begin{pmatrix} 0 \\ -\sin 2t \\ \cos 2t \end{pmatrix} \right)$$

$$+ \begin{pmatrix} 1 \\ -\frac{4}{5} \\ \frac{7}{5} \end{pmatrix} e^{2t}$$