1.

Given: $a = A - 6t^2$; at $t = 0$ s, $x = 8$ m, $v = 0$
at $t = 1$ s, $v = 30$ m/s

Find: (a) **t** when $v = 0$
(b) Total distance traveled when $t = 5$ s

We have: $a = A - 6t^2$; where A = constant

$$\frac{dv}{dt} = a = A - 6t^2$$

At $t = 0$, $v = 0$; $\int_0^v dv = \int_0^t (A - 6t^2)dt$

Or $v = At - 2t^2$ (m/s)

At $t = 1$, $v = 30$ m/s ; $30 = A(1) - 2(1)^3$

So, $A = 32\ m/s^2$

Therefore, $\mathbf{v = 32t - 2t^3}$ (m/s)

Also, $\frac{dx}{dt} = v = 32t - 2t^3$

At $t = 0$, $x = 8$ m; $\int_0^x dx = \int_0^t (32t - 2t^3)dt$

Therefore, $\mathbf{x = 8 + 16t^2 - \frac{1}{2}t^4}$ **(m)**

(a) When $v = 0$; $32t - 2t^3 = 2t(16 - t^2) = 0$

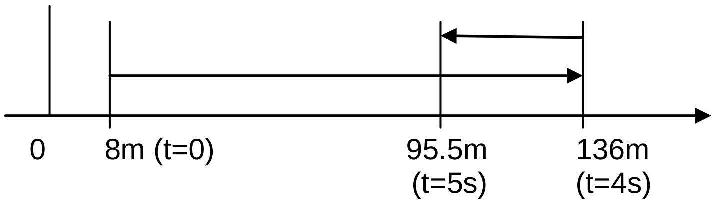
t = 0 and t = 4 s

(**b**) At $t = 4$ s ; $x_4 = 8 + 16(4)^2 - (1/2)(4)^4 = 136$ m

$t = 5$ s ; $x_5 = 8 + 16(5)^2 - (1/2)(5)^4 = 95.5$ m

Now we observe that $0 < t < 4s$; $v > 0$

$4 < t < 5s$; $v < 0$



Then, $x4 - x0 = 136 - 8 = 128$ m

$|x5 - x4| = |95.5 - 136| = 40.5$m

Therefore, total distance traveled = (128 + 40.5) = **168.5 m**

2. Given : $a = -\frac{5}{2v_0 - v} = \frac{dv}{dt}$ or $dt = -\frac{1}{5}(2v_0 - v)\,dv$

Integrating, using appropriate limits, $\int_0^t dt = -\int_{v_0}^{v} \frac{1}{5}(2v_0 - v)\,dv$

$$t = -\frac{2}{5}v_0 v + \frac{1}{10}v^2 \Big|_{v_0}^{v} = \frac{1}{10}v^2 - \frac{2}{5}v_0 v + \frac{3}{10}v_0^2$$

(a) At $t = 2s$, $v = \frac{1}{2}v_0$

(0.4점)

Then, $2 = \left(\frac{1}{40} - \frac{1}{5} + \frac{3}{10}\right)v_0^2 = \frac{1}{8}v_0^2$ or $v_0^2 = 16\ m^2/s^2$, $v_0 = \pm 4\ m/s$

(b) Time to come to rest. $v = 0$

(0.2점)

$t = 0 - 0 + \frac{3}{10}v_0^2 = \frac{3}{10}(4)^2$ $t = 4.8\ s$

(c) Position where velocity is $1\ m/s$.

$v\,dv = a\,dx$ or $dx = \frac{v\,dv}{a} = -\frac{1}{5}(2v_0 - v)v\,dv$

Integrating, using appropriate limits,

$$\int_0^x dx = -\frac{1}{5}\int_{v_0}^{v}(2v_0 v - v^2)\,dv = -\frac{1}{5}\left(v_0 v^2 - \frac{1}{3}v^3\right)\Big|_{v_0}^{v}$$

$$x = -\frac{1}{5}\left[v_0 v^2 - \frac{1}{3}v^3 - \frac{2}{3}v_0^3\right] = -\frac{1}{5}\left[\pm 4v^2 - \frac{1}{3}v^3 - \frac{2}{3}(\pm 64)\right]$$

With $v = 1\ m/s$,

(0.4점)

$x = -\frac{1}{5}\left[\pm 4 - \frac{1}{3} \mp \frac{128}{3}\right]$ $x = 7.80\ m$ and $-7.67\ m$

3.

constant acceleration. Choose $t=0$ at end of powered flight.

Then, $y_1 = 27.3\ m$ $\quad a = -g = -9.81\ m/s^2$

(a) When y reaches the ground, $y_f = 0$ and $t = 16s$.

$$y_f = y_1 + v_1 t + \frac{1}{2}at^2 = y_1 + v_1 t - \frac{1}{2}gt^2$$

$$v_1 = \frac{y_f - y_1 + \frac{1}{2}gt^2}{t} = \frac{0 - 27.3 + \frac{1}{2}(9.81)(16)^2}{16} = 76.8\ m/s \qquad (0.5점)$$

(b) When the rocket reaches its maximum altitude y_{max},

$$v = 0$$

$$v^2 = v_1^2 + 2a(y - y_1) = v_1^2 - 2g(y - y_1)$$

$$y = y_1 - \frac{v^2 - v_1^2}{2g}$$

$$y_{max} = 27.3 - \frac{0 - 76.8^2}{(2)(9.81)} \approx 328\ m \qquad (0.5점)$$

4. Given: $a_A = 7\,mm/s^2$ (constant, at $t=0$) – (upward)

a_B – constant, $v_{B_{t=0}} = 8\,mm/s$ – (downward)

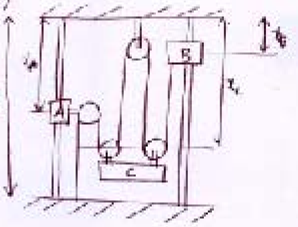
between $t=0$ and $t=2$, B moves through 20 mm

Find: a) a_B, a_C

b) the time at $v_C = 0$

c) the distance through which block C will have moved at that time.

sol)



d: value of x at lower support

total length of the cable = constant

$$(d - x_A) + (x_C - x_A) + 2x_C + (x_C - x_B) = \text{constant}$$

$$4v_C - v_B - 2v_A = 0 \quad \cdots (*)$$

$$4a_C - a_B - 2a_A = 0 \quad \cdots (**)$$

a) since a_B = constant

$$x_B - (x_B)_0 = (v_B)_0 t + \frac{1}{2} a_B t^2$$

$$a_B = \frac{2[x_B - (x_B)_0 - (v_B)_0 t]}{t^2} = \frac{2[(-20) - 0 - (-8)\cdot 2]}{2^2} = \frac{1}{2}(-20 + 16) = \underline{-2\,mm \text{ (upward)}}$$ 0.2

from (**)

$$a_C = \frac{1}{4}(a_B + 2a_A) = \frac{1}{4}(-2 + 2\cdot 7) = \underline{3\,mm/s \text{ (downward)}}$$ 0.2

b)

$$v_C = (v_C)_0 + a_C t, \quad t = \frac{v_C - (v_C)_0}{a_C}$$

from (*)

$$(v_C)_0 = \frac{1}{4}[(v_B)_0 + 2(v_A)_0] = \frac{1}{4}[(-8) + 2\cdot 0] = -2\,mm/sec \text{ (upward)}$$

$$\therefore t = \frac{0 - (-2)}{3} = \underline{\frac{2}{3}\text{ sec}}$$ 0.3

c)

$$x_C - (x_C)_0 = (v_C)_0 t + \frac{1}{2} a_C t^2 = (-2)\frac{2}{3} + \frac{1}{2}\cdot 3 \left(\frac{2}{3}\right)^2 = -\frac{4}{3} + \frac{2}{3} = \underline{-\frac{2}{3}\,mm \text{ (upward)}}$$

0.3

5. Given : $a_A = 3t^2\ mm/s^2$ (upward)

a_B = constant, v_{B} (after moving 32 mm) = 8 mm/s (downward)

Find : a) a_C

b) the distance through which block C will have moved after 3s.

sol) a)

a_B = constant, then $vdv = adx \rightarrow \int vdv = a\int dx$

$$v_B^2 - (v_B)_0^2 = 2a_B[x_B - (x_B)_0]$$

$$a_B = \frac{v_B^2 - (v_B)_0^2}{2[x_B - (x_B)_0]} = \frac{8^2 - 0}{2(32 - 0)} = 1\ mm/s^2$$

from (x) - problem 4

$$a_C = \frac{1}{4}(a_B + 2a_A) = \frac{1}{4}[1 + 2(-3t^2)] = \underline{\frac{1}{4}(1 - 6t^2)}$$ 0.5

b) $v_C - \cancel{(v_C)_0} = \int_0^t a_C\,dt = \frac{1}{4}t - \frac{1}{2}t^3$

$$v_C = \frac{1}{4}t - \frac{1}{2}t^3 \rightarrow t(1 - 2t^2) = 0 \rightarrow t = \pm\frac{1}{\sqrt{2}},\ \underline{t = \frac{1}{\sqrt{2}} \rightarrow v_C = 0}$$

$$x_C = \int_0^3 \left(\frac{1}{4}t - \frac{1}{2}t^3\right)dt = \int_0^{\frac{1}{\sqrt{2}}} \left(\frac{1}{4}t - \frac{1}{2}t^3\right)dt + \int_{\frac{1}{\sqrt{2}}}^3 \left(\frac{1}{2}t^3 - \frac{1}{4}t\right)dt$$

$$= \left[\frac{1}{8}t^2 - \frac{1}{8}t^4\right]_0^{\frac{1}{\sqrt{2}}} + \left[\frac{1}{8}t^4 - \frac{1}{8}t^2\right]_{\frac{1}{\sqrt{2}}}^3$$

$$= 9 + \frac{1}{16} = \underline{9.0625\ mm}$$ 0.5

6. $\vec{r} = (Rt\cos\omega_n t)\hat{i} + ct\hat{j} + (Rt\sin\omega_n t)\hat{k}$

$$\vec{v} = \frac{d\vec{r}}{dt} = R(\cos\omega_n t - \omega_n t\sin\omega_n t)\hat{i} + c\hat{j} + R(\sin\omega_n t + \omega_n t\cos\omega_n t)\hat{k}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = R(-\omega_n\sin\omega_n t - \omega_n\sin\omega_n t - \omega_n^2 t\cos\omega_n t)\hat{i} + R(\omega_n\cos\omega_n t + \omega_n\cos\omega_n t - \omega_n^2 t\sin\omega_n t)\hat{k}$$

$$= R\left[(-2\omega_n\sin\omega_n t - \omega_n^2 t\cos\omega_n t)\hat{i} + (2\omega_n\cos\omega_n t - \omega_n^2 t\sin\omega_n t)\hat{k}\right]$$

$$|\vec{v}|^2 = [R(\cos\omega_n t - \omega_n t\sin\omega_n t)]^2 + (c)^2 + [R(\sin\omega_n t + \omega_n t\cos\omega_n t)]^2$$

$$= R^2[\cos^2\omega_n t - 2\omega_n t\sin\omega_n t\cos\omega_n t + \omega_n^2 t^2\sin^2\omega_n t] + c^2$$

$$+ R^2[\sin^2\omega_n t + 2\omega_n t\sin\omega_n t\cos\omega_n t + \omega_n^2 t^2\cos^2\omega_n t]$$

$$= R^2(1 + \omega_n^2 t^2) + c^2$$

$$\therefore |v| = \sqrt{R^2(1+\omega_n^2 t^2) + c^2}$$ 0.5

$$|\vec{a}|^2 = R^2\left[(-2\omega_n\sin\omega_n t - \omega_n^2 t\cos\omega_n t)^2 + (2\omega_n\cos\omega_n t - \omega_n^2 t\sin\omega_n t)^2\right]$$

$$= R^2[4\omega_n^2\sin^2\omega_n t + 4\omega_n^3 t\sin\omega_n t\cos\omega_n t + \omega_n^4 t^2\cos^2\omega_n t + 4\omega_n^2\cos^2\omega_n t$$

$$- 4\omega_n^3 t\sin\omega_n t\cos\omega_n t + \omega_n^4 t^2\sin^2\omega_n t]$$

$$= R^2(4\omega_n^2 + \omega_n^4 t^2)$$

$$\therefore |a| = R\omega_n\sqrt{4 + \omega_n^2 t^2}$$ 0.5

7. Choose the origin at the center of the grinding wheel, so that the horizontal and vertical motions are:

$$x - x_0 = v_0 \cos\alpha t, \quad \text{or} \quad v_0 = \frac{x - x_0}{t\cos\alpha}$$

And

$$y - y_0 = v_0 \sin\alpha t - \frac{1}{2} g t^2 = (x - x_0)\tan\alpha - \frac{1}{2} g t^2$$

from which

$$t^2 = \frac{2[(y_0 - y) + (x - x_0)\tan\alpha]}{g}$$

Data: $\alpha = -6^\circ$, $x_0 = -20$ mm, $y_0 = 205$ mm, $r = \frac{1}{2} d = 175$ mm

$g = 9810$ mm/s^2

(a) Stream lands at B.

$$x = r\sin 10^\circ = 30.39 \text{ mm}$$

$$y = r\cos 10^\circ = 172.34 \text{ mm}$$

$$t^2 = \frac{2[(205 - 172.34) + (30.39 + 20)\tan(-6^\circ)]}{9810} = 0.005579 \; s^2$$

$$t = 0.07469 \text{ s}$$

$$v_0 = \frac{(30.39 + 20)}{(0.07469)\cos(-6^\circ)} = 678.37 \text{ mm/s}$$ (0.5점)

(b) Stream lands at C.

$$x = r\cos 30^\circ = 151.55 \text{ mm}$$

$$y = r\sin 30^\circ = 87.5 \text{ mm}$$

$$t^2 = \frac{2[(205 - 87.5) + (151.55 - (-20))\tan(-6^\circ)]}{9810} = 0.020279 \; s^2$$

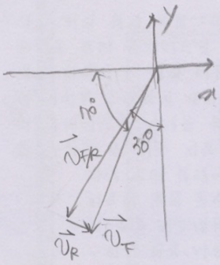
$$t = 0.14240 \text{ s}$$

$$v_0 = \frac{(151.55 + 20)}{(0.1424)\cos(-6^\circ)} = 1211.34 \text{ mm/s}$$ (0.5점)

8. $\vec{v}_F$: velocity of the ferry

$\vec{v}_{F/R}$: velocity of the ferry related to the river

$\vec{v}_r$: velocity of the river



$$\vec{v}_F = 9.8\,(\text{knots}) \measuredangle\, 70^\circ$$

$$= (-9.8\cos 70^\circ)\hat{i} + (-9.8\sin 70^\circ)\hat{j}$$

$$\vec{v}_{F/R} = 10\,(\text{knots}) \measuredangle\, 30^\circ$$

$$= (-10\sin 30^\circ)\hat{i} + (-10\cos 30^\circ)\hat{j}$$

$$\vec{v}_R = \vec{v}_F - \vec{v}_{FR}$$

$$= \{(-9.8\cos 70^\circ) - (-10\sin 30^\circ)\}\hat{i} + \{(-9.8\sin 70^\circ) - (-10\cos 30^\circ)\}\hat{j}$$

$$= \underline{1.648\,\hat{i} - 0.549\,\hat{j} \quad (\text{knots})}$$

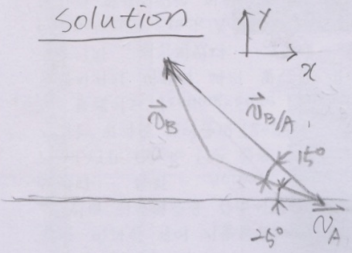
9.

constraints

$$2x_A + x_{B/A} = const$$
$$2v_A + v_{B/A} = 0$$
$$2a_A + a_{B/A} = 0$$

$$\Rightarrow \quad |\overrightarrow{v_{B/A}}| = 400\,mm/s$$
$$|\overrightarrow{a_{B/A}}| = 300\,mm/s^2$$

Solution



$$\vec{v}_A = 200\,mm/s \ \nwarrow 25°$$
$$= (200\cos 25°)\hat{i} + (-200\sin 25°)\hat{j}$$
$$\vec{v}_{B/A} = 400\,mm/s \ \nwarrow 40°$$
$$= (-400\cos 40°)\hat{i} + (400\sin 40°)\hat{j}$$

(a) $$\vec{v}_B = \vec{v}_A + \overrightarrow{v_{B/A}}$$
$$= (200\cos 25° - 400\cos 40°)\hat{i} + (-200\sin 25° + 400\sin 40°)\hat{j}$$
$$= -125.157\,\hat{i} + 172.591\,\hat{j} \ (mm/s)$$

(b) $$\vec{a} = 150\,mm/s^2 \ \searrow 25° = 150\cos 25°\,\hat{i} - 150\sin 25°\,\hat{j}$$
$$\vec{a}_{B/A} = 300\,mm/s^2 \ \nwarrow 40° = -300\cos 40°\,\hat{i} + 300\sin 40°\,\hat{j}$$
$$\vec{a}_B = \vec{a}_A + \overrightarrow{a_{B/A}}$$
$$= (150\cos 25° - 300\cos 40°)\hat{i} + (-150\sin 25° + 300\sin 40°)\hat{j}$$
$$= -93.867\,\hat{i} + 129.444\,\hat{j} \ (mm/s^2)$$

10.

constraints

i) $x_B + (x_B - x_A) + 2(d - x_A) = Const.$

, where d is distance of the A block from the left wall.

$$\left.\begin{aligned} 2v_B - 3v_A &= 0 \\ 2a_B - 3a_B &= 0 \end{aligned}\right\} \Rightarrow \quad \begin{aligned} v_A &= \frac{2}{3} v_B \\ a_A &= \frac{2}{3} a_B = \frac{2}{3} \times 300 = 200 \, (mm/s^2) \\ \vec{a}_A &= 200 \, \hat{i} \, (mm/s^2) \end{aligned}$$

Solution

a) $2(d - x_A) + y_{C/A} = Const.$

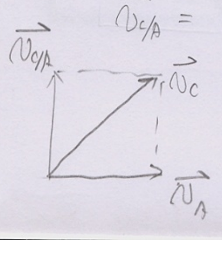
$$\left\{\begin{aligned} -2v_A + v_{C/A} &= 0 \\ -2a_A + a_{C/A} &= 0 \end{aligned}\right\} \Rightarrow \quad a_{C/A} = 400 \, (mm/s^2)$$

$$\therefore \vec{a}_{C/A} = 400 \, \hat{j} \, (mm/s^2)$$

b)

$$v_A = v_{0A} + a_A t$$
$$= 0 + 200 \times 2 = 400 \, (mm/s)$$
$$\vec{v}_A = 400 \, \hat{i} \, (mm/s)$$

$$v_{C/A} = v_{0C/A} + a_{C/A} t$$
$$= 0 + 400 \times 2 = 800 \, mm/s$$
$$\vec{v}_{C/A} = 800 \, \hat{j} \, mm/s$$



$$\vec{v}_C = \vec{v}_A + \vec{v}_{C/A}$$
$$= 400 \, \hat{i} + 800 \, \hat{j} \, (mm/s)$$