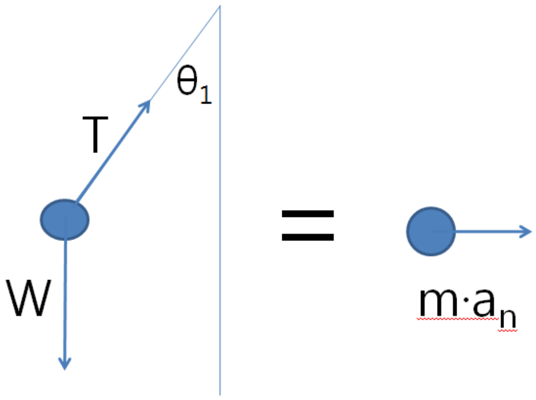
1. 길이 l_1의 줄이 회전 중 l_2의 길이만큼 줄어들 때, 그 각도를 구하는 문제

a) 관계식은? (**20 points**)

1) Draw Free Body Diagram (FBD)



2) Governing Equation

↑(+)$\sum F_y = 0$; $T\cdot\cos\theta_1 - W = 0$ $T = W/\cos\theta_1$

→(+)$\sum F_x = m\cdot a_n$; $T\cdot\sin\theta_1 = W\cdot\tan\theta_1 = mg\tan\theta_1 = mv_1^2/r$ Where $r = l\cdot\sin\theta$

3) 각속도 보존법칙 사용 or 계산

$\sum M_y = 0$; H_y = constant $\mathbf{r_1 m v_1 = r_2 m v_2}$ -------------(*)

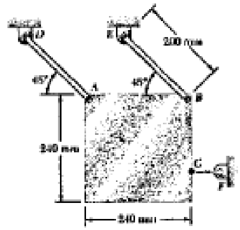
또, $v^2 = l\cdot g\cdot\sin\theta\cdot\tan\theta$ 이므로,

$v_1 = \sqrt{l_1 \cdot g \cdot \sin\theta_1 \cdot \tan\theta_1}$ 그리고, $v_2 = \sqrt{l_2 \cdot g \cdot \sin\theta_2 \cdot \tan\theta_2}$ -----(**)

4) (*)에 (**)을 넣어 정리하면,

$$\mathbf{l_1^3 \cdot \sin^3\theta_1 \cdot \tan\theta_1 = l_2^3 \cdot \sin^3\theta_2 \cdot \tan\theta_2}$$

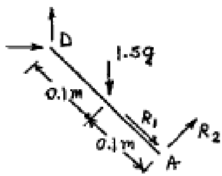
b) θ_2는? (**10 points**) $\mathbf{\theta_2 = 49.8°}$

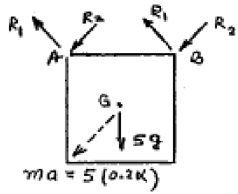


PROBLEM 16.103

A 5-kg uniform square plate is supported by two identical 1.5-kg uniform slender rods *AD* and *BE*. It is held in the position shown by rope *CF*. Determine, immediately after rope *CF* has been cut, (*a*) the acceleration of the plate, (*b*) the force exerted on the plate at point *B*.

SOLUTION





$$\Sigma M_G = 0: \quad R_2 = -R_3$$

$$\Sigma M_D = (1.5 \text{ kg})(g)(0.1 \text{ m})(0.707) - (0.2 \text{ m})R_2 = \frac{1}{3}(1.5 \text{ kg})(0.2 \text{ m})^2 \alpha$$

$$(0.15 \text{ kg}\cdot\text{m})(g)(0.707) - (0.2 \text{ m})R_2 = \left(0.02 \text{ kg}\cdot\text{m}^2\right)\alpha \tag{1}$$

$$+\!\!\!/ \ \Sigma F = 2R_2 + (5 \text{ kg})(g)(0.707) = (5 \text{ kg})(0.2 \text{ m})\alpha$$

$$2R_2 + (3.535 \text{ kg})g = (1 \text{ kg}\cdot\text{m})\alpha \tag{2}$$

Solve Eq. (1) and (2) for α and R_2

$$(0.2 \text{ m})R_2 - (0.02 \text{ kg}\cdot\text{m})\alpha = (0.10605 \text{ kg}\cdot\text{m})(g)$$

$$2R_2 - (1 \text{ kg}\cdot\text{m})\alpha = -(3.535 \text{ kg})(g)$$

$$\alpha = (3.8296/\text{m})\left(9.81 \text{ m/s}^2\right)$$

$$a = (0.2 \text{ m})\alpha = (0.2 \text{ m})(3.8296/\text{m})\left(9.81 \text{ m/s}^2\right)$$

$$= 7.5137 \text{ m/s}^2 \qquad \text{or} \quad a = 7.51 \text{ m/s}^2 \ \nearrow 45° \blacktriangleleft$$

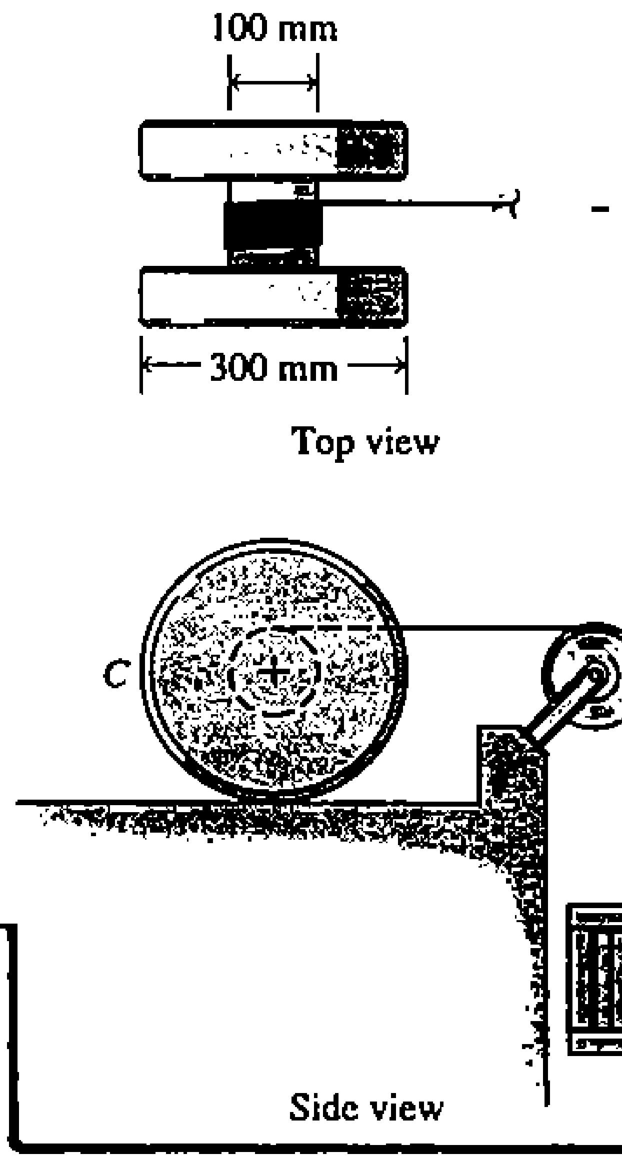
$$R_2 = \frac{1}{2}(-3.535 \text{ kg})\left(9.81 \text{ m/s}^2\right) + (3.8296 \text{ kg})\left(9.81 \text{ m/s}^2\right) = 1.4450 \text{ N}$$

Now

$$F_B = R_2 \cos 45 + (-R_3)\cos 45 = 2(1.4450)(0.707)$$

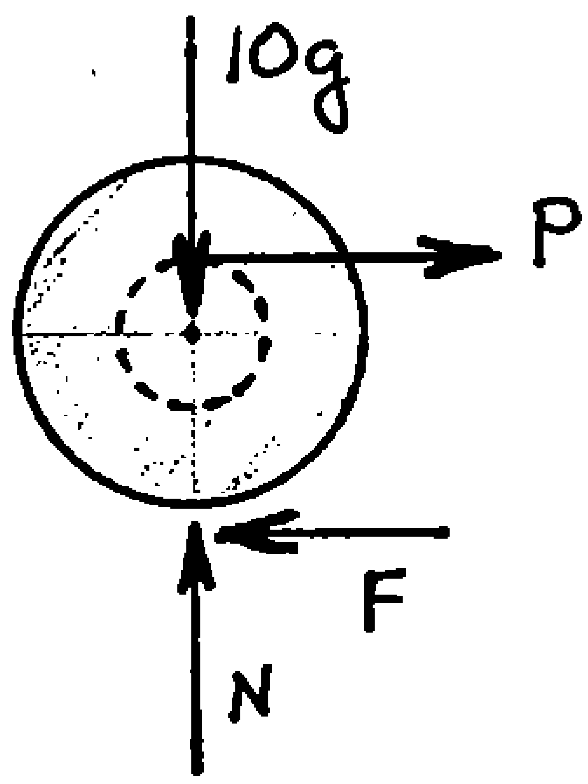
$$= 2.04323 \text{ N} \qquad F_B = 2.04 \text{ N} \downarrow \blacktriangleleft$$

18-26* The 10-kg spool C has a centroidal radius of gyration of 75 mm. A cord is attached to the center of the spool, passes over a small frictionless pulley, and is attached to a 25-kg crate A. If the system is released from rest and the spool rolls without slipping, determine the speed v_C and angular velocity ω_C of the spool and the speed v_A of the crate after the crate has dropped 2 m.



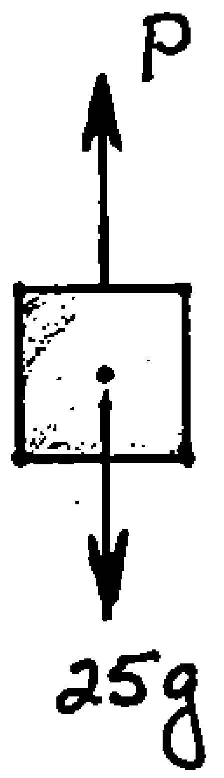
Solution

Neither N, F, nor W_C do work. The rope tension is an internal force; its work will cancel out when the work-energy equations for the crate A and spool C are added together. The weight W_A has a potential; the zero of gravitational potential energy is set at the initial position. If the spool rolls without slipping, then



$$v_C = 0.150\omega$$
$$v_A = 0.200\omega$$

and the kinetic energy of the system is



$$T = \frac{1}{2} m_C v_C^2 + \frac{1}{2} I_C \omega^2 + \frac{1}{2} m_A v_A^2$$
$$= \frac{1}{2}(10)(0.150\omega)^2 + \frac{1}{2}(10)(0.075)^2\omega^2$$
$$+ \frac{1}{2}(25)(0.200\omega)^2 = 0.64063\omega^2$$

Therefore, the work-energy equation $T_i + V_i + U = T_f + V_f$ gives

$$0 + 0 + 0 = 0.64063\omega^2 + 25(9.81)y$$

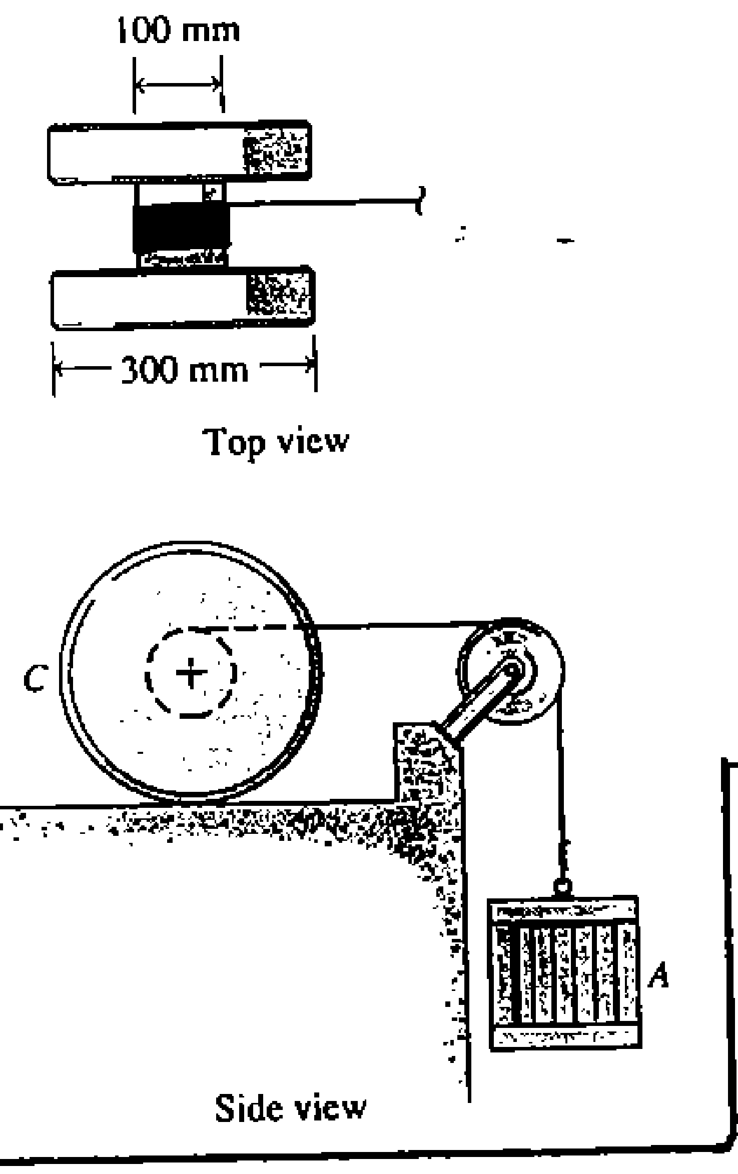
When the crate has dropped 2 m ($y = -2$ m)

$\omega_C = 27.7$ rad/s .... Ans.

$v_C = 4.15$ m/s .... Ans.

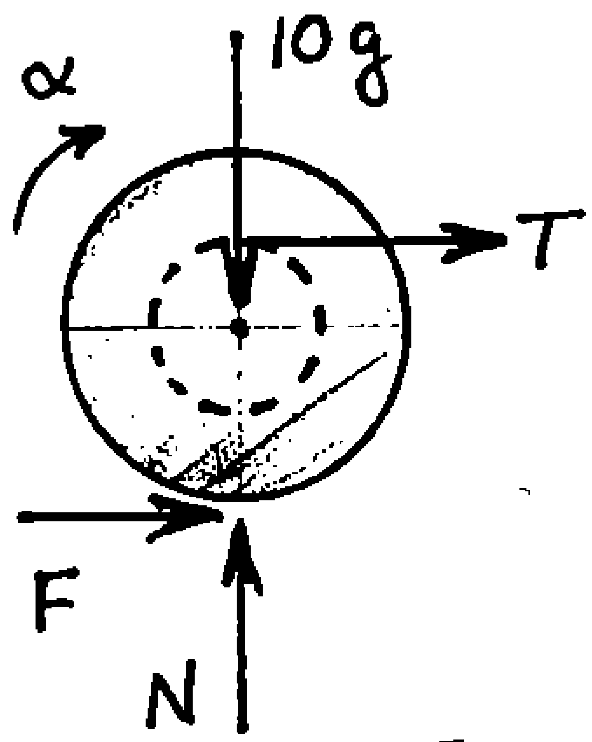
$v_A = 5.53$ m/s .... Ans.

16-76 The 10-kg spool C has a centroidal radius of gyration of 75 mm. A cord connects the spool to a 25-kg crate. If the system is released from rest and the spool rolls without slipping, determine the speed v_C and the angular velocity ω_C of the spool and the speed v_A of the crate after the crate has dropped 2 m.



Solution

Separate free-body diagrams must be drawn of the spool and the crate since their motions are different. The equations of motion are



$$+\rightarrow \Sigma F_x = \quad F + T = 10a_{Gx} \tag{a}$$

$$+\uparrow \Sigma F_y = \quad N - 10(9.81) = 0 \tag{b}$$

$$\circlearrowright + \Sigma M_G = 0.05T - 0.15F = I_G\alpha \tag{c}$$

$$+\downarrow \Sigma F_y = \quad 25(9.81) - T = 25a_A \tag{d}$$

where $a_{Gy} = 0$,

$$I_G = 10(0.075)^2 = 0.05625 \text{ kg}\cdot\text{m}^2$$

and the accelerations a_{Gx}, a_A, and α are related by

$$a_{Gx} = 0.15\alpha$$

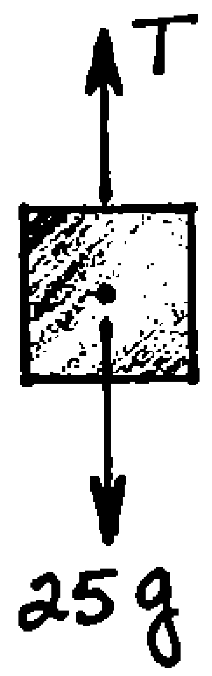
$$a_A = 0.20\alpha$$

Therefore

$$N = 98.1 \text{ N}$$

and adding Eqs. *a* and *d* gives

$$245.25 + F = 10(0.15\alpha) + 25(0.20\alpha)$$



$$F = 6.500\alpha - 245.25 \tag{e}$$

$$T = 245.25 - 5.00\alpha \tag{f}$$

(Problem 16-76 continues ...)

(Problem 16-76 - cont.)

Then, combining Eqs. *c*, *e*, and *f* gives

$$0.05(245.25 - 5\alpha) - 0.15(6.5\alpha - 245.25) = 0.05625\alpha$$

$$\alpha = 38.28293 \text{ rad/s}^2 \circlearrowright = \text{constant}$$

$$F = 3.58902 \text{ N} \longrightarrow$$

$$T = 53.83537 \text{ N}$$

$$a_{Gx} = 5.74244 \text{ m/s}^2 \longrightarrow = \text{constant}$$

$$a_A = 7.65659 \text{ m/s}^2 \downarrow = \text{constant}$$

Finally, integrating the (constant) acceleration of the crate with respect to time gives its velocity and position

$$v_A = 7.65659t \text{ m/s} \downarrow$$

$$y_A = 3.82829t^2$$

where y_A is positive downward (the same direction and a_A) and the constants of integration are both zero since the crate starts from rest when $y_A = 0$. Then the crate will have dropped 2 m when

$$y_A = 3.82829t^2 = 2 \text{ m}$$

$t = 0.723$ s $V_A = 5.53$ m/s .... Ans.

at which time

$v_{Gx} = 5.74244t = 4.15$ m/s $\longrightarrow$ .... Ans.

$\omega = 38.28293t = 27.7$ rad/s $\circlearrowright$ .... Ans.