Homework set 1 (David K. Cheng, Fundamentals of Engineering Electromagnetics)

P. 2-1 A rhombus is an equilateral parallelogram. Denote two neighboring sides of a rhombus by vectors A and B

a) Verify that the two diagonals are **A + B** and **A – B**.

b) Prove that the diagonals are perpendicular to each other.

P. 2-4 Let unit vectors $\mathbf{a}_A$ and $\mathbf{a}_B$ denote the directions of vectors A and B in the xy-plane that make angles α and β, respectively, with the x-axis.

a) Obtain a formula for the expansion of the cosine of the difference of two angles, $\cos(\alpha - \beta)$. By taking the scalar product $\mathbf{a}_A \cdot \mathbf{a}_B$.

b) Obtain a formula for $\sin(\alpha - \beta)$by taking the vector product $\mathbf{a}_B \times \mathbf{a}_A$.

P. 2-7 Given vector $\mathbf{A} = \mathbf{a}_x 5 - \mathbf{a}_y 2 + \mathbf{a}_z$, find the expression of

a) a unit vector $\mathbf{a}_B$ such that $\mathbf{a}_B \parallel \mathbf{A}$, and

b) a unit vector $\mathbf{a}_C$ in the xy-plane such that $\mathbf{a}_C \perp \mathbf{A}$.

P. 2-8 Decompose vector $\mathbf{A} = \mathbf{a}_x 2 - \mathbf{a}_y 5 + \mathbf{a}_z 3$ into two components, $\mathbf{A}_1$ and $\mathbf{A}_2$ that are, respectively, perpendicular and parallel to another vector $\mathbf{B} = -\mathbf{a}_x + \mathbf{a}_y 4$.

P. 2-13 Express the r-component, A_r, of a vector $\mathbf{A}$ at (r_1, ϕ_1, z_1)

a) in terms of A_x and A_y in Cartesian coordinates, and

b) in terms of A_R and A_θ in spherical coordinates.

6) By using the representations of summation convention and Levi-Civita symbol, prove that the scalar triple product $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ can be calculated by the following determinant in Cartesian coordinate system.

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \begin{vmatrix} \mathbf{A}_x & \mathbf{A}_y & \mathbf{A}_z \\ \mathbf{B}_x & \mathbf{B}_y & \mathbf{B}_z \\ \mathbf{C}_x & \mathbf{C}_y & \mathbf{C}_z \end{vmatrix}$$