Homework set 3 selected solution

P. 3-4 Three point chares $Q_1 = -9(\mu C), Q_2 = 4(\mu C),$ and $Q_3 = -36(\mu C)$ are arranged on a straight line,. The distance between Q_1 and Q_3 is 9 (cm). It is claimed that a location can be selected for Q_2 such that each charge will experience a zero force. Find this location.

For zero force on Q1, Q2, and Q3,

$$\frac{Q_1 Q_2}{4\pi\varepsilon_0 x^2} + \frac{Q_1 Q_3}{4\pi\varepsilon_0 9^2} = 0 \quad \cdots (1)$$

$$\frac{Q_1 Q_2}{4\pi\varepsilon_0 x^2} + \frac{Q_2 Q_3}{4\pi\varepsilon_0 (9-x)^2} = 0 \quad \cdots (2)$$

$$\frac{Q_1 Q_3}{4\pi\varepsilon_0 9^2} + \frac{Q_2 Q_3}{4\pi\varepsilon_0 (9-x)^2} = 0 \quad \cdots (3)$$

From (1), $x = 9\sqrt{\frac{Q_2}{-Q_3}} = 9\sqrt{\frac{4}{36}} = 3\ (cm)$

or from (2), $Q_3 x^2 + Q_1 (9-x)^2 = 0 \Rightarrow -36x^2 - 9(9-x)^2 = 0 \quad \Rightarrow x = 3$

or from (3), $Q_2 9^2 + Q_1 (9-x)^2 = 0 \Rightarrow 4 \cdot 9^2 - 9(9-x)^2 = 0 \quad \Rightarrow x = 3$

P. 3-7 A line charge of uniform density ρ_l forms a semicuircle of radius b in the upper half xy-plane. Determine the magnitude and direction of the electric field intensity at the center of the semicircle.

$$d\vec{E} = d\vec{E}_x + d\vec{E}_y$$

Where $d\vec{E}_x = -\hat{x}\frac{\rho_l (b d\phi)}{4\pi\varepsilon_0 b^2}\cos\phi$

$$d\vec{E}_y = -\hat{y}\frac{\rho_l (b d\phi)}{4\pi\varepsilon_0 b^2}\sin\phi$$

$$\vec{E} = \int d\vec{E} = \frac{\rho_l}{4\pi\varepsilon_0 b}\left[-\hat{x}\int_0^{\pi}\cos\phi d\phi - \hat{y}\int_0^{\pi}\sin\phi d\phi\right]$$

$$= -\hat{y}\frac{\rho_l}{2\pi\varepsilon_0 b}$$