Homework set 6 selected solution

P.5-6 A current I flows in the inner conductor of an infinitely long coaxial line and returns via the outer conductor. The radius of the inner conductor is a, and the inner and outer radii of the outer conductor are b and c, respectively. Find the magnetic flux density $\mathbf{B}$ for all regions and plot $|\mathbf{B}|$ versus r.

Solution)

Ampere's circuital law : $\oint_C \mathbf{B}\cdot d\mathbf{l} = \mu_0 I$

For $0 \le r \le a$

$$2\pi r B_\phi = \mu_0 \int_0^r \frac{I}{\pi a^2} 2\pi r dr = \frac{\mu_0 I r^2}{a^2}$$

$$\mathbf{B} = \hat{\phi} B_\phi = \hat{\phi}\frac{\mu_0 I r}{2\pi a^2}$$

For $a \le r \le b,$

$$2\pi r B_\phi = \mu_0 I$$

$$\mathbf{B} = \hat{\phi} B_\phi = \hat{\phi}\frac{\mu_0 I}{2\pi r}$$

For $b \le r \le c$

$$2\pi r B_\phi = \mu_0\left[I - \int_b^r \frac{I}{\pi\left(c^2-b^2\right)} 2\pi r dr\right] = \mu_0 I\left[1 - \frac{r^2-b^2}{c^2-b^2}\right] = \mu_0 I \frac{c^2-r^2}{c^2-b^2}$$

$$\mathbf{B} = \hat{\phi} B_\phi = \hat{\phi}\frac{\mu_0 I}{2\pi r}\left(\frac{c^2-r^2}{c^2-b^2}\right)$$

For $r \ge c$

$$2\pi r B_\phi = 0$$

$$\mathbf{B} = \hat{\phi} B_\phi = 0$$

P. 5-7 A thin conducting wire of length $3w$ forms a planar equilateral triangle. A direct current I flows in the wire. Find the magnetic flux density at the center of the triangle.

Solution)

Using the result of Example 5-3,

$$\mathbf{B}(0,0,0) = \hat{\phi}\frac{3\mu_0 I L}{2\pi r\sqrt{L^2+r^2}}$$

$\mathbf{B}$ at the center = $3\times\mathbf{B}$ produced by the current I flowing in each side wire of length w/2

$$\mathbf{B}(0,0,0) = \hat{z}\frac{3\mu_0 I(w/2)}{2\pi r\sqrt{(w/2)^2+r^2}} = z\frac{9\mu_0 I}{2\pi w} \quad \text{by} \quad r = w/2\sqrt{3}$$