HW#6 - Selected solution

9-7. A standard air-filled S-band rectangular waveguide has dimensions $a = 7.21(cm)$and $b = 3.40(cm)$. What mode types can be used to transmit electromagnetic waves having the following wavelengths?

a) λ = 10 (cm) b) λ = 5 (cm)

Cutoff wavelength for TE_{mn} and TM_{mn} modes:

$$(\lambda_c)_{mn} = \frac{u}{(f_c)_{mn}} = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}} \quad (m)$$

$$TE_{10} : (\lambda_c)_{10} = \frac{2}{\sqrt{(1/0.0721)^2}} = 14.42\ cm$$

$$TE_{01} : (\lambda_c)_{01} = \frac{2}{\sqrt{(1/0.034)^2}} = 6.8\ cm$$

$$TE_{20} : (\lambda_c)_{20} = \frac{2}{\sqrt{(2/0.0721)^2}} = 7.21\ cm$$

$$TM_{11} : (\lambda_c)_{11} = \frac{2}{\sqrt{(1/0.0721)^2 + (1/0.034)^2}} \approx 6.15\ cm$$

a) For $\lambda = 10\ cm,\quad (\lambda_c)_{mn} > \lambda \longrightarrow TE_{10}$

b) For $\lambda = 5\ cm,\quad (\lambda_c)_{mn} > \lambda \longrightarrow TE_{10},\ TE_{01},\ TE_{20},\ TM_{11}$

9-15. An electromagnetic wave is to propagate along an air-filled $a \times b$ rectangular waveguide at the dominant mode. Assume $a = 2.50(cm)$ and the usable bandwidth to be between $1.15(f_c)_{10}$ and 15% below the cutoff frequency of the next higher mode.

a) Calculate and compare the permissible bandwidth for $b = 0.25a$, $b = 0.50a$, and $b = 0.75a$.
b) Calculate and compare the average powers transmitted along the three guides in part (a) at 7 (GHz) if the maximum electric intensity is 10 (kV/m). Neglect the losses.

$$(f_c)_{mn} = \frac{c}{2}\sqrt{(m/a)^2 + (n/b)^2}, \qquad (f_c)_{10} = \frac{c}{2a} = \frac{3\times10^8}{2\times0.025} = 6\ GHz \longrightarrow \text{Dominant mode: } TE_{10}$$

a) For $b = 0.25a$,

$$\begin{cases} (f_c)_{01} = \dfrac{c}{2b} = \dfrac{3\times10^8}{2\times0.25\times0.025} = 24\ \ GHz \\ (f_c)_{20} = \dfrac{c}{a} = 12\ GHz \\ (f_c)_{11} = \dfrac{c}{2a}\sqrt{1+\left(\dfrac{a}{b}\right)^2} = \dfrac{3\times10^8}{2\times0.025}\sqrt{1+\left(\dfrac{1}{0.25}\right)^2} = 24.7\ GHz \end{cases}$$

$\longrightarrow$ Next higher mode: TE_{20}

Usable band width: $1.15(f_c)_{10} < f < 0.85(f_c)_{20}$

$\longrightarrow 6.9 < f < 10.2\ GHz$

Possible band width $= 10.2 - 6.9 = 3.3\ \ GHz$

For $b = 0.5a$,

$$\begin{cases} (f_c)_{01} = \dfrac{c}{2b} = 12\ \ GHz \\ (f_c)_{20} = \dfrac{c}{a} = 12\ GHz \\ (f_c)_{11} = \dfrac{c}{2a}\sqrt{1+\left(\dfrac{a}{b}\right)^2} = 13.4\ GHz \end{cases}$$

$\longrightarrow$ Next higher mode: TE_{01}, TE_{20}

Usable band width: $6.9 < f < 10.2\ GHz$

Possible band width $= 10.2 - 6.9 = 3.3\ \ GHz$

For $b = 0.75a,$ $\begin{cases} (f_c)_{01} = \dfrac{c}{2b} = 8\ GHz \\ (f_c)_{20} = \dfrac{c}{a} = 12\ GHz \\ (f_c)_{11} = \dfrac{c}{2a}\sqrt{1+\left(\dfrac{a}{b}\right)^2} = 10\ GHz \end{cases}$

$\longrightarrow$ Next higher mode: TE_{01}

Usable band width: $6.9 < f < 6.8\ GHz$ $\longrightarrow$ No possible band

b) Using (9-101) for $f = 7\ GHz,\ E_0 = 10kV/m,\ (f_c)_{10} = 6.9\ GHz$

$$P_{av} = \frac{E_0^2 ab}{4\eta_0}\sqrt{1-\left(\frac{f_c}{f}\right)^2} \approx 85.3b$$

For $b = 0.25a = 0.25 \times 0.025,\qquad P_{av} \approx 5.33\ W$

$b = 0.5a = 0.5 \times 0.025,\qquad P_{av} \approx 10.7\ W$