HW#7 - Selected solution

9-20. For an air-filled rectangular copper cavity resonator,

a) Calculate its Q for the TE_{101} mode if its dimensions $a = d = 1.8b = 3.6\ (cm)$ and
b) Determine how much b should be increased in order to make Q 20% higher.

a) $$Q_{101} = \frac{\pi f_{101}\mu_0 abd\left(a^2+d^2\right)}{R_s\left(2b\left(a^3+d^3\right)+ad\left(a^2+d^2\right)\right)}$$

$$= \frac{\sqrt{\pi f_{101}\mu_0\sigma}\,abd\left(a^2+d^2\right)}{2b\left(a^3+d^3\right)+ad\left(a^2+d^2\right)} = \sqrt{\pi f_{101}\mu_0\sigma}\,\frac{ab}{a+2b} \qquad ①$$

$$= \sqrt{\pi\left(5.9\times10^9\right)\left(4\pi\times10^{-7}\right)\left(1.57\times10^7\right)}\,\frac{0.036\times0.02}{0.036+2\times0.02} \approx 5682$$

$$R_s = \sqrt{\frac{\pi f_{101}\mu_0}{\sigma}}\ ,\ a = d$$

$$f_{101} = \frac{c}{2}\sqrt{a^{-2}+d^{-2}} = 5.9\times10^9$$

b) From ①, $Q_{101} = A\frac{ab}{a+2b}$ *where* $A = 6\times10^5$

$$b = \frac{1}{\frac{A}{Q_{101}} - \frac{2}{a}}$$

For $Q_{101}^{'} = 1.2Q_{101}$,

$$b^{'} = \frac{1}{\frac{A}{1.2Q_{101}} - \frac{2}{a}} = \frac{1}{\frac{6\times10^5}{1.2\times5682} - \frac{2}{0.036}} \approx \frac{1}{32.44} \approx 3.1\ cm$$