7.5

Maximum stresses due to internal pressure

$$\sigma_h = \frac{p_i R}{t}$$
]
$$\sigma_a = \frac{p_i R}{2t}$$

Maximum shear due to internal pressure

$$H_{11} = 2\pi G R^3 t$$

$$\tau = G\frac{QR}{H_{11}} = \frac{Q}{2\pi R^2 t}$$

Von Mises criterion

$$\left(\frac{\sigma_h}{\sigma_{allow}}\right)^2 + \left(\frac{\sigma_a}{\sigma_{allow}}\right)^2 - \frac{\sigma_h \sigma_a}{\sigma_{allow}^2} + 3\left(\frac{\tau}{\sigma_{allow}}\right)^2 \le 1$$

$$\left(\frac{\frac{Q}{R^2 t \sigma_{allow}}}{\frac{2\pi}{\sqrt{3}}}\right)^2 + \left(\frac{\frac{Rp_i}{t\sigma_{allow}}}{\frac{2}{\sqrt{3}}}\right)^2 \le 1$$

7.6

(1) 1.81641 x 10^8

(2) 1.98066 x 10^8

(3) 0.000039328 $Q^2$ + 1.58623 × $10^{-9}$ $N^2$ + 6.87635 × $10^{-16} p^2$ <= 1

7.7

55724.8

7.10

(1)

$$H_{11} = \frac{1}{3} G(16b + h) t_0^3$$

(2)

$$\tau_{\max} = \frac{6Q}{(16b+h)t_0^2}$$

(3)

$$\tau_s = \frac{2GtM_1}{H_{11}}\frac{x_n}{t}$$

7.11

(1) 82

(2) 2.19512 x 10^8

(3) This occurs at the wall edges in the part of section with largest wall thickness which is in the flanges($t_f$)

(4) $\tau_s = 2Gt\dfrac{Q}{H_{11}}$

7.12

(1)

$$H_{11} = \frac{1}{3}G\pi Rt^3$$

(2)

$$\tau_s = 2G\frac{Q}{H_{11}}t_r = \frac{6Qt_r}{\pi Rt^3} \quad (-t/2 \le t_r \le t/2)$$

(3)

$$t_r = \pm\frac{t}{2}$$

7.13

(1)

$$H_{11} = \frac{5}{6}bGt^3$$

(2)

$$\tau_{\max} = \frac{6Q}{5bt^2}$$

This occurs at all points on the outer edges of the thin-walled section because t=constant.

7.14

(1)

$$H_{11} = \frac{13}{3} aGt^3$$

(2)

$$H_{11} = \frac{34}{3} aGt^3$$

(3)

$$\tau_{\max} = \frac{3Q}{17at^2}$$

Max stress occurs at outer edges of short horizontal leg (max thickness).

8.8

(1) From symmetry arguments, centroid is on axis i2 which is a symmetry axis. It is at the midpoint of the web because the centroids of the flanges and web all lie on this same vertical line.

(2)

$$H_{22} = E\left(\frac{1}{12}h^3 t + 2\left(\frac{1}{12}b^3 t \sin^2\alpha + bt2\left(\frac{h}{2}\right)^2\right)\right) = \frac{1}{12}tE(h^2(12b+h) + 2b^3\sin^2\alpha)$$

$$H_{33} = E\left(2\left(\frac{1}{12}b^3 t\cos^2\alpha\right) + 0\right) = \frac{1}{6}b^3 tE\cos^2\alpha$$

$$H_{23} = E\left(\frac{1}{12}b^3 t\cos(-\alpha)\sin(-\alpha) + \frac{1}{12}b^3 t\cos\alpha\sin\alpha\right) = 0$$

(3)

$$M_2 = PL$$

$$M_3 = 0$$

$$\Delta H = H_{22}H_{33} - H_{23}^2$$

$$\sigma(x_2, x_3) = E\left( -\frac{x_2 H_{23} - x_3 H_{33}}{\Delta H} M_2 - \frac{x_2 H_{22} - x_3 H_{23}}{\Delta H} M_3 \right)$$

$$\sigma_A = \sigma(-\frac{b}{2}\cos\alpha, \frac{h}{2} + \frac{b}{2}\sin\alpha) = \frac{6PL(h + b\sin\alpha)}{t(h^2(12b+h) + 2b^3\sin^2\alpha)}$$

$$\sigma_B = \sigma(0, \frac{h}{2}) = \frac{6PLh}{t(h^2(12b+h) + 2b^3\sin^2\alpha)}$$

$$\sigma_A = \sigma(\frac{b}{2}\cos\alpha, \frac{h}{2} - \frac{b}{2}\sin\alpha) = \frac{6PL(h - b\sin\alpha)}{t(h^2(12b+h) + 2b^3\sin^2\alpha)}$$

8.9

(1)

$$\frac{2R}{\pi}$$

(2)

$$H_{22} = \frac{1}{2}\pi R^3 tE$$

$$H_{33} = \frac{(-8 + \pi^2)R^3 tE}{2\pi}$$

$$H_{23} = 0$$

(3)

$$(\sigma_A, \sigma_B, \sigma_C) = \left\{ -\frac{0.267436L^2 p_0}{R^2 t}, \frac{0.152652L^2 p_0}{R^2 t}, -\frac{0.267436L^2 p_0}{R^2 t} \right\}$$

(4)

$$\sigma_1 = \frac{L^2\pi(-\frac{2R}{\pi} + R\cos\theta)p_0}{4(-8+\pi^2)R^3 t}$$

8.10

(1)

$$\frac{2R^2}{2a+\pi R}$$

(2)

$$H_{22} = \frac{1}{6}(4a^3 + 12a^3 R + 12aR^2 + 3\pi R^3)tE$$

$$H_{33} = \frac{R^3(4a^2\pi + 4a(4+\pi^2)R + \pi(-8+\pi^2)R^2)tE}{2(2a+\pi R)^2}$$

$$H_{23} = 0$$

(3)

$$\left(\sigma_{1B}, \sigma_{1C}\right) = \left\{\frac{0.0603303L^2 p_0}{R^2 t}, -\frac{0.0563416L^2 p_0}{R^2 t}\right\}$$

8.11

(1)

$$\frac{h\alpha^2\beta}{1+2\alpha\beta}$$

(2)

$$H_{22} = \frac{1}{12}h^3 t(1+6\alpha\beta)E$$

$$H_{33} = \frac{h^3 t\alpha^3\beta(2+\alpha\beta)E}{3+6\alpha\beta}$$

$$H_{23} = 0$$

(3)

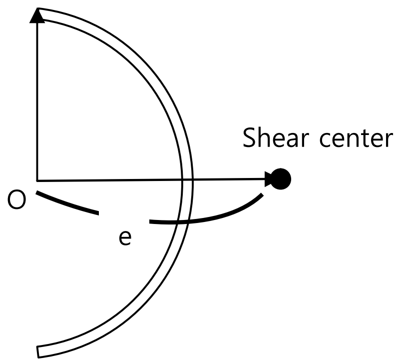
$$\left(\sigma_{1A}, \sigma_{1B}\right) = \left\{ \frac{3L^2(1+\alpha\beta)p_0}{8h^2 + t\alpha^2\beta(2+\alpha\beta)}, -\frac{3L^2 p_0}{16h^2 t\alpha + 8h^2 t\alpha^2\beta} \right\}$$

8.24

All shear flows flow forward to or from vertex B. Shear forces due to these shear flow can't make any moment about vertex B.

8.27

$$c = \frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} R\cos\theta R d\theta}{\pi R} = \frac{2R}{\pi}$$




$$H_{22} = E\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (R\sin\theta)^2 tRd\theta = \frac{1}{2}\pi R^3 tE$$

$$Q_{2\theta} = E\int_0^{\theta} R\sin\theta tRd\alpha = -R^2 tE(-1+\cos\theta)$$

$$f_\theta = -\frac{V_3 Q_{2\theta}}{H_{22}} + C_0$$

$$f_{\theta=\frac{\pi}{2}} = 0$$

$$C_0 = \frac{2V_3}{\pi R}$$

$$f_\theta = \frac{2V_3\cos\theta}{\pi R}$$

*Moment equipollence at semicircle center:*

$$M_\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f_\theta R^2 d\theta = \frac{4R}{\pi}$$

$$eV_3 = M_\theta$$

$$e = \frac{4R}{\pi}(outer\ of\ the\ semicircle)$$