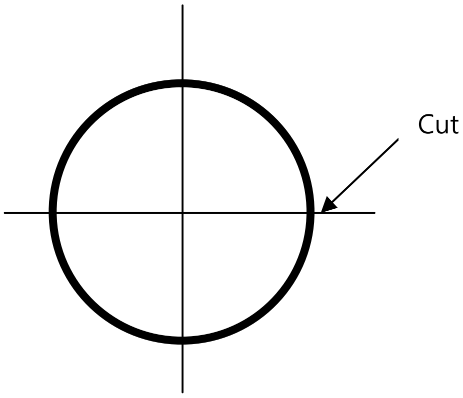
8.31 (page 332)



1) from symmetry

$H_{22} = H_{33}$ and $H_{23} = 0$

$$H_{22} = 2E\int_{-\pi/2}^{\pi/2} (R\sin\theta)^2 tRd\theta = \pi R^3 tE$$

2) Shear flow f0 in open(cut) system

$$f_0 = -\frac{V_3 E\int zdA}{H_{33}} = -\frac{V_3}{I_{yy}}\int_0^{\theta} (R\sin\theta)tRd\theta = -\frac{V_3 tR^2}{\pi R^3 t}(1-\cos\theta) = \frac{V_3}{\pi R}(\cos\theta - 1)$$

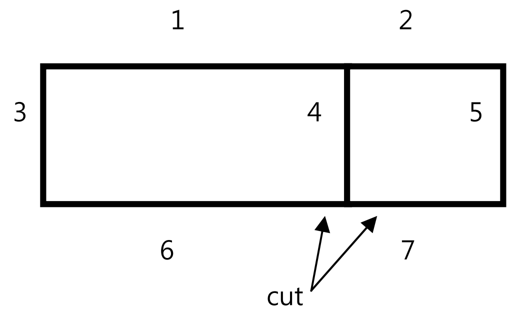
$$f_c = -\frac{\int_C \frac{f_0(s)}{Gt}ds}{\int_C \frac{1}{Gt}ds} = \frac{\int_0^{2\pi}\frac{V_3}{\pi R}(\cos\theta-1)Rd\theta}{2\pi R}$$

$$= \frac{\frac{V_3}{\pi}}{2\pi R}\int_0^{2\pi}(\cos\theta - 1)d\theta = \frac{\frac{V_3}{\pi}}{2\pi R}2\pi = \frac{V_3}{\pi R}$$

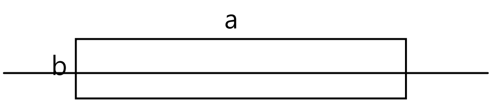
$$f(\theta) = f_0 + f_c = \frac{V_3\cos\theta}{\pi R}$$

3) $\theta = 0, \pi$ $\qquad f_{\max} = \frac{V_3}{\pi R}$

8.38 (page 333)



1) $H_{23}=0$



$$I_{yy} = \frac{ab^3}{12}$$

$$I_{yy,3} = I_{yy,4} = I_{yy,5} = \frac{ta^3}{12} \qquad I_{yy,1+2} = I_{yy,6+7} = \frac{3at^3}{12} + \left(\frac{a}{2}\right)^2 3at$$

$$H_{22} = E\left(I_{yy,3} + I_{yy,4} + I_{yy,5} + I_{yy,1+2} + I_{yy,6+7}\right) = E\left(\frac{1}{4}ta^3 + \frac{1}{2}at^3 + \frac{3}{2}ta^3\right)$$

$$\simeq \frac{7}{4}ta^3E$$

Sequences and path indices are followed as shown in fig. 8.40(page 331)

$$f_0(s_1) = \frac{V_3E\int_0^{s_1}\frac{a}{2}tds}{H_{22}} = \frac{2V_3s_1}{7a^2}$$

$$f_0(s_2) = f_{01}(2a) - \frac{V_3E\int_0^{s_2}(-\frac{a}{2}+s)tds}{H_{22}} = f_{01}(2a) - \frac{4V_3}{7a^3}\int_0^{s_2}\left(-\frac{a}{2}+s\right)ds$$

$$= \frac{4V_3}{7a} + \frac{2V_3}{7a^3}\left(as_2 - s_2^2\right) = \frac{2V_3\left(2a^2 + as_2 - s_2^2\right)}{7a^3}$$

$$f_0(s_3) = \frac{2V_3(2a - s_3)}{7a^2}$$

$$f_0(s_4) = \frac{2V_3s_4}{7a^2}$$

$$f_0(s_5) = \frac{2V_3(a^2 + as_5 - s_5^2)}{7a^3}$$

$$f_0(s_6) = \frac{2V_3(a - s_6)}{7a^2}$$

$$f_0(s_7) = \frac{2V_3(as_7 - s_7^2)}{7a^3}$$

$$warp1 = \int_0^c \frac{f_{01} - f_{c1}}{Gt} ds_1 + \int_0^c \frac{f_{02} - f_{c1}}{Gt} ds_2 + \int_0^c \frac{f_{03} - f_{c1}}{Gt} ds_3 - \int_0^c \frac{f_{07} + f_{c1} - f_{c2}}{Gt} ds_7 = 0$$

$$warp2 = \int_0^c \frac{f_{04} + f_{c2}}{Gt} ds_4 + \int_0^c \frac{f_{05} + f_{c2}}{Gt} ds_5 + \int_0^c \frac{f_{06} + f_{c2}}{Gt} ds_6 - \int_0^c \frac{f_{07} + f_{c1} - f_{c2}}{Gt} ds_7 = 0$$

$$-42 f_{c1} + 7 f_{c2} + 12 \frac{V_3}{a} = 0$$

$$-7 f_{c1} + 28 f_{c2} + 4 \frac{V_3}{a} = 0$$

$$f_{c1} = \frac{44 V_3}{161 a}$$

$$f_{c2} = -\frac{12 V_3}{161 a}$$

$$f_1 = f_{01} - f_{c1} = \frac{2(-22a + 23 s_1) V_3}{161 a^2}$$

$$f_2 = f_{02} - f_{c1} = \frac{2(24 a^2 + 23 a s_2 - 23 s_s^s) V_3}{161 a^3}$$

$$f_3 = f_{03} - f_{c1} = \frac{2(24a - 23 s_3) V_3}{161 a^2}$$

$$f_4 = f_{04} + f_{c2} = \frac{2(-6a + 23 s_4) V_3}{161 a^2}$$

$$f_5 = f_{05} + f_{c2} = \frac{2(17 a^2 + 23 a s_5 - 23 s_5^2) V_3}{161 a^3}$$

$$f_6 = f_{06} + f_{c2} = \frac{2(17a - 23 s_6) V_3}{161 a^2}$$

$$f_7 = f_{07} + f_{c1} - f_{c2} = \frac{2(28 a^2 + 23 a s_7 - 23 s_7^2) V_3}{161 a^3}$$

2)

$$f_1 + f_4 + f_7 = 0 \qquad s_1 = s_4 = s_7 = 0$$

$$f_1 - f_2 = 0 \qquad s_1 = 2a, s_2 = 0$$

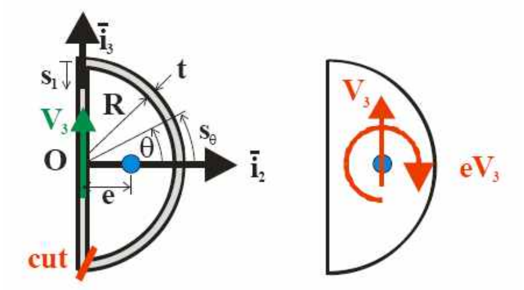
$$f_2 - f_3 = 0 \qquad s_2 = a, s_3 = 0$$

$$f_4 - f_5 = 0 \qquad s_4 = a, s_5 = 0$$

$$f_5 - f_6 = 0 \qquad s_5 = a, s_6 = 0$$

$$f_3 + f_6 + f_7 = 0 \qquad s_3 = 2a, s_6 = s_7 = a$$

8.54 (page 353)



1) $H_{22} = E\left(\int_{-\pi/2}^{\pi/2} \left(R\sin\theta\right)^2 tRd\theta + \frac{1}{12}(2R)^3 t\right) = \frac{1}{6}(4+3\pi)R^3 tE$

For open section

$$Q_{2\theta} = E\int_{-\pi/2}^{\theta} \left(R\sin\phi\right) tRd\phi$$

$$f_{0\theta} = -\frac{V_3 Q_{2\theta}}{H_{22}} = \frac{6V_3\cos\theta}{(4+3\pi)R}$$

$$Q_{21} = Es_1 t(R - \frac{s_1}{2})$$

$$f_{01} = -\frac{V_3 Q_{21}}{H_{22}} + f_{0\theta} = -\frac{3V_3(2R - s_1)s_1}{(4+3\pi)R^3}$$

Constant shear flow

$$warp = \int_{-\pi/2}^{\pi/2} \frac{f_{0\theta} + f_c}{Gt} R d\theta + \int_0^{2R} \frac{f_{01} + f_c}{Gt} R ds_1 = 0$$

$$f_c = -\frac{8V_3}{(2+\pi)(4+3\pi)R}$$

Moment about O

$$M_0 = \int_{-\pi/2}^{\pi/2} (f_{0\theta} + f_c) R^2 d\theta = \frac{4(6+\pi)RV_3}{8+10\pi+3\pi^2}$$

$$e = \frac{M_0}{V_3} = \frac{4(6+\pi)R}{8+10\pi+3\pi^2} = 0.529757R$$

2)

$$f_{s\theta} = f_{0\theta} + f_c = \frac{2V_3(-4+3(2+\pi)\cos\theta)}{(8+10\pi+3\pi^2)R}$$

$$f_{s1} = f_{01} + f_c = \frac{V_3(-8-6(2+\pi)Rs_1 + 3(2+\pi)s_1^2)}{(8+10\pi+3\pi^2)R^3}$$

3) moment due to V_3 at web

$$Q_1 = -eV_3$$

$$A = \frac{1}{2}\pi R^2$$

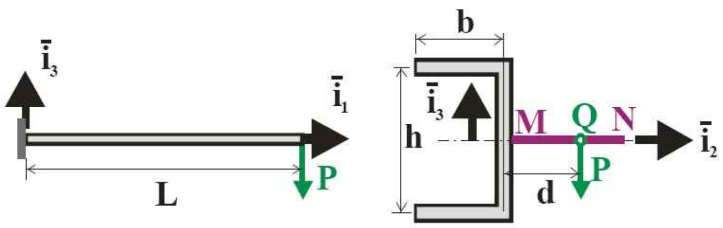
$$f_t = \frac{Q_1}{2A} = -\frac{4(6+\pi)V_3}{\pi(8+10\pi+3\pi^2)R}$$

4)

$$f_\theta = f_{s\theta} + f_t = \frac{6V_3(-2+\pi\cos\theta)}{\pi(4+3\pi)R}$$

$$f_1 = f_{s1} + f_t = \frac{3V_3(-4R^2 - 2\pi R s_1 + \pi s_1^2)}{\pi(4+3\pi)R^3}$$

8.60 (page 361)



1)

Cantilever beam

$$u_3 = -\frac{PL^3}{3H_{22}}$$

2)

$$H_{11} = \frac{1}{3}G(2b+h)t^3 \qquad \text{from 7.61}$$

$$e = \frac{3b}{6+h/b} \qquad \text{from 8.41}$$

$$Q_1 = -(d-e)P$$

$$\kappa_1 = \frac{Q_1}{H_{11}} = \frac{d\Phi}{dx}$$

$$\Phi_{1,tip} = \frac{3\left(-d+\dfrac{3b^2}{6b+h}\right)LP}{G(2b+h)t^3} = \frac{6\left(-d+\dfrac{3b^2}{6b+h}\right)LP(1+\nu)}{E(2b+h)t^3}$$

3)

$$u_{3,Q} = u_3 + (d-e)\sin(\Phi_{1,tip})$$

4)

$$u_{3,Q} = -0.00214041 + (-0.075 + 0.2\eta)\sin(20.869(0.075 - 0.2\eta))$$

$$\eta = \frac{d}{b}$$

5) d/b = 0.375