2013-2, 해양환경정보시스템

Homework 1 Solution

1.

(1) (10%)

$$f = 0.3Hz \Rightarrow \omega = 2\pi f = 0.6\pi \ rad/\sec$$

$$\omega^2 = gk \tanh kh \Rightarrow k = 0.407$$

$$\lambda = \frac{2\pi}{k} = \boxed{15.44m}$$

(2) (10%)

$$A = \frac{H}{2} = 0.15m$$

$$|u|_{\max} = |u|_{z=0} = \frac{gAk}{\omega}\frac{\cosh k(z+h)}{\cosh kh} = \frac{9.81\times0.15\times0.407}{0.6\pi}\frac{\cosh\left[0.407\times(0+3.5)\right]}{\cosh(0.407\times3.5)} = \boxed{0.318m/s}$$

$$|w|_{\max} = |w|_{z=0} = \frac{gAk}{\omega}\frac{\sinh k(z+h)}{\cosh kh} = \frac{9.81\times0.15\times0.407}{0.6\pi}\frac{\sinh\left[0.407\times(0+3.5)\right]}{\cosh(0.407\times3.5)} = \boxed{0.283m/s}$$

(3) (10%)

$$|p_d|_{z=-0.25} = \rho g A\frac{\cosh k(z+h)}{\cosh kh} = 1000\times9.81\times0.15\times\frac{\cosh\left[0.407\times(-0.25+3.5)\right]}{\cosh(0.407\times3.5)} = \boxed{1369.6N/m^2}$$

2.

(1) (10%)

$$k_{deep} = \frac{\omega^2}{g} = \boxed{0.082}$$

$$\lambda_{deep} = \frac{2\pi}{k_{deep}} = \boxed{76.504m}$$

(2) (10%)

$$\omega^2 = gk \tanh kh \Rightarrow k = 0.1376$$

$$\lambda = \frac{2\pi}{k} = \boxed{45.66m}$$

3. (20%)

$$\phi_1 = \frac{gA}{\omega}\frac{\cosh k(z+h)}{\cosh kh}\sin(kx - \omega t)$$

$$\phi_2 = \frac{gA}{\omega}\frac{\cosh k(z+h)}{\cosh kh}\sin(kx + \omega t)$$

$$\begin{aligned}\phi &= \phi_1 + \phi_2 \\ &= \frac{gA}{\omega}\frac{\cosh k(z+h)}{\cosh kh}\{\sin(kx-\omega t) + \sin(kx+\omega t)\} \\ &= \boxed{\frac{g(2A)}{\omega}\frac{\cosh k(z+h)}{\cosh kh}\sin kx \cos \omega t}\end{aligned}$$

4. (30%)

Kinematic free surface boundary condition (as shown in class)

$$\frac{dF(\mathbf{x},t)}{dt}=0 \Leftrightarrow -\frac{\partial \eta}{\partial t}-\frac{\partial \phi}{\partial x}\frac{\partial \eta}{\partial x}+\frac{\partial \phi}{\partial z}=0 \quad \text{on} \quad z=\eta(x,t)$$

Using the Taylor series expansion and retaining only the linear terms, $O(\varepsilon)$, we have

$$\left(-\frac{\partial \eta}{\partial t}+\frac{\partial \phi}{\partial z}\right)_{z=0}=0 \Rightarrow \boxed{\frac{\partial \eta}{\partial t}=\frac{\partial \phi}{\partial z}} \, on \; z=0$$

Dynamic free surface boundary condition

From Bernoulli equation:

$$\frac{\partial \phi}{\partial t}+\frac{1}{2}|\nabla \phi|^2+\frac{p(\vec{x},t)}{\rho}+gz=C(t)$$

If this equation is applied to the far field where $\phi \to 0,\ \eta \to 0$ and $p(\vec{x},t)=p_{atm}$:

$$C(t)=\frac{p_{atm}}{\rho}$$

Using the definition of surface tension, pressure on the free surface is

$$p=p_{atm}+\Delta p=p_{atm}+\left(-T\frac{\partial^2 \eta}{\partial x^2}\right)$$

Thus, we have dynamic free surface boundary condition as follows:

$$\frac{\partial \phi}{\partial t}+\frac{1}{2}|\nabla \phi|^2+\frac{\left(-T\frac{\partial^2 \eta}{\partial x^2}\right)}{\rho}+gz=0 \quad \text{on} \quad z=\eta(x,y,t)$$

Using the Taylor series expansion and retaining only the linear terms, $O(\varepsilon)$, we have

$$\left(\frac{\partial \phi}{\partial t}+\frac{1}{2}|\nabla \phi|^2+\frac{\left(-T\frac{\partial^2 \eta}{\partial x^2}\right)}{\rho}+gz\right)_{z=\eta}=\left(\frac{\partial \phi}{\partial t}+\frac{1}{2}|\nabla \phi|^2+\frac{\left(-T\frac{\partial^2 \eta}{\partial x^2}\right)}{\rho}+gz\right)_{z=0}$$
$$+(\eta-0)\frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial t}+\frac{1}{2}|\nabla \phi|^2+\frac{\left(-T\frac{\partial^2 \eta}{\partial x^2}\right)}{\rho}+gz\right)_{z=0}+\cdots=0$$

$$\therefore \boxed{\frac{\partial \phi}{\partial t}+\frac{\left(-T\frac{\partial^2 \eta}{\partial x^2}\right)}{\rho}+g\eta=0} \qquad \text{on } z=0$$