2013-2, 해양환경정보시스템

Homework 2 Solution

1. (10%)

$$\omega^2 = gk\tanh kh$$

$$\Leftrightarrow 2\omega\frac{d\omega}{dk} = g\tanh kh + gkh\sec h^2 kh$$

$$\Leftrightarrow \frac{d\omega}{dk} = \frac{1}{2}\frac{\omega}{k}\frac{gk\tanh kh + gk^2 h\sec h^2 kh}{\omega^2} = \frac{1}{2}V_P\left(\frac{gk\tanh kh + gk^2 h\sec h^2 kh}{gk\tanh kh}\right)$$

$$= \frac{1}{2}V_P\left(1+\frac{kh}{\sinh kh\cosh kh}\right) = \frac{1}{2}V_P\left(1+\frac{2kh}{\sinh 2kh}\right) = V_g$$

2. (10%)

In shallow water,

$$kh \to 0$$

$$\omega^2 = gk^2 h$$

$$V_g = V_p = \sqrt{gh} = \sqrt{9.81\times 100} = 31.321\ (m/s)$$

$$t = \frac{180000\ \ (m)}{31.321\ \ (m/s)} = \boxed{5746.94\ (\text{sec}) = 1hr35\min}$$

3. (20%)

(1)

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$Section\ 1$: 0m~150m from wave maker ($depth\ h_1 = 15m$)

$Section\ 2$: 150m~300m from wave maker ($depth\ h_2 = 5m$)

$$\omega^2 = gk_1\tanh k_1 h_1 \to k_1 = 0.252\left(k_1 h_1 = 3.773 > \pi\ \ or\ \ 2h_1 > \lambda_1 \Rightarrow \text{deep water}\right)$$

$$\omega^2 = gk_2\tanh k_2 h_2 \to k_2 = 0.283\left(\frac{\pi}{10} < k_2 h_2 = 1.415 < \pi \Rightarrow \text{finite depth}\right)$$

$$V_{g,1} \simeq \frac{1}{2}V_p = \frac{1}{2}\frac{\omega}{k} = \frac{1}{2}\frac{\pi/2}{0.252} = 3.123\,m/\text{sec}$$

$$V_{g,2} = \left(\frac{1}{2}+\frac{kh}{\sinh 2kh}\right)V_p = \frac{\omega}{k} = 3.705\,m/\text{sec}$$

$$t_1 = \frac{L_1}{V_{g,1}} = \frac{150}{3.123} = \boxed{48.037\,\text{sec}}$$

$$t_{1+2} = \frac{L_1}{V_{g,1}} + \frac{L_2}{V_{g,2}} = \frac{150}{3.123} + \frac{150}{3.705} = \boxed{88.511\,\text{sec}}$$

(2)

$$0.3\times\frac{1}{2}\rho g A_I^2=\frac{1}{2}\rho g A_R^2$$

$$\Rightarrow A_R=\sqrt{0.3}A_I\simeq 0.548A_I$$

Wave elevation:

$$\eta_I=\boxed{A_I\cos(kx-\omega t)}$$

$$\eta_R=A_R\cos(kx+\omega t)=\boxed{\sqrt{0.3}A_I\cos(kx+\omega t)}$$

Max. & Min. amplitude

$$\eta=\eta_I+\eta_R=A_I\cos(kx-\omega t)+\sqrt{0.3}\cos(kx+\omega t)$$

$$=A_I(\cos kx\cos\omega t+\sin kx\sin\omega t)+\sqrt{0.3}A_I(\cos kx\cos\omega t-\sin kx\sin\omega t)$$

$$=\left(1+\sqrt{0.3}\right)A_I\cos kx\cos\omega t+\left(1-\sqrt{0.3}\right)A_I\sin kx\sin\omega t$$

$$\Rightarrow\boxed{Max=\left(1+\sqrt{0.3}\right)A_I=1.548A_I,\ \ Min=\left(1-\sqrt{0.3}\right)A_I=0.452A_I}$$

(3)

$$t_{trans}=\frac{L_2}{V_{g,2}}=\frac{150}{3.705}\simeq\boxed{40.481\text{sec}}$$

$$0.7\times\frac{1}{2}\rho g A_I^2=\frac{1}{2}\rho g A_T^2\Rightarrow\boxed{A_T=\sqrt{0.7}A_I\simeq 0.837A_I}$$

4. (30%)

(1)

Approach 1)

Phase velocity: $V_p = \frac{\omega}{k} = \sqrt{\frac{g}{k}}$ (deep water)

For steady ship wave (on ship fixed coordinate)

$$V_{ship} = V_p \quad \Rightarrow \quad U = \sqrt{\frac{g}{k}} \quad \Rightarrow \quad k = \frac{g}{U^2}$$

$$V_{ship} = V_p \quad \Rightarrow \quad U = \frac{\omega}{k} \quad \Rightarrow \quad \omega = kU = \frac{g}{U}$$

$$\therefore k = \frac{g}{U^2}, \quad \omega = \frac{g}{U}$$

Approach 2)

Linearized free surface boundary condition

$$\frac{\partial^2 \phi}{\partial t^2} - 2U \frac{\partial^2 \phi}{\partial x \partial t} + U^2 \frac{\partial^2 \phi}{\partial x^2} + g \frac{\partial \phi}{\partial z} = 0 \quad \text{on } z = 0$$

or $U^2 \frac{\partial^2 \phi}{\partial x^2} + g \frac{\partial \phi}{\partial z} = 0 \quad \text{on } z = 0 \quad \left(\text{Steady flow, } \frac{\partial}{\partial t} = 0 \right)$

Velocity potential: $\phi \propto \mathrm{Re}\{e^{kz - i(kx - \omega t)}\}$

Substitute velocity potential at linearized boundary condition

$$-U^2 k^2 + gk = 0 \quad \text{on } z = 0$$

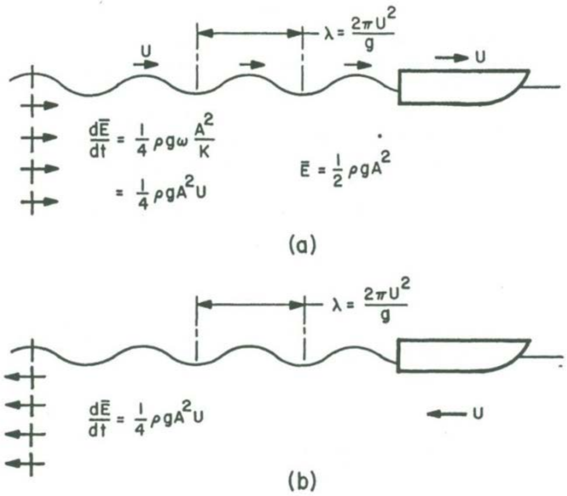
$$\therefore k = \frac{g}{U^2}, \ \omega = \sqrt{gk} = \frac{g}{U}$$

(2)

Group velocity: $V_g = \frac{1}{2} V_P = \frac{1}{2} U$

Energy flux: $\frac{d\overline{E}}{dt} = \overline{E} \cdot V_g = \frac{1}{2} \rho g A^2 \cdot \frac{1}{2} V_P = \frac{1}{4} \rho g A^2 U$

(3)



6.12
Two-dimensional ship waves as viewed by a fixed observer (a), and in a reference frame moving with the ship (b).

Approach 1) Earth fixed frame (그림 (a)의 경우)

Energy flux on section A: $\frac{d\bar{E}}{dt} = \bar{E} \cdot U = \frac{1}{2}\rho g A^2 U$

Energy flux on section B: $\frac{d\bar{E}}{dt} = \bar{E} \cdot V_g = \frac{1}{4}\rho g A^2 U$

$$\left.\frac{d\bar{E}}{dt}\right|_A = \left.\frac{d\bar{E}}{dt}\right|_B + \left.\frac{d\bar{E}}{dt}\right|_{loss} \quad => \quad \left.\frac{d\bar{E}}{dt}\right|_{loss} = \bar{E}U - \bar{E}V_g = \frac{1}{4}\rho g A^2 U = DU$$

$$\therefore D = \frac{1}{4}\rho g A^2$$

Approach 2) Body fixed frame (그림 (b)의 경우)

Energy flux on section A: $\frac{d\bar{E}}{dt} = DU$

Energy flux on section B: $\frac{d\bar{E}}{dt} = \bar{E} \cdot V_g = \frac{1}{4}\rho g A^2 U$

$$\left.\frac{d\bar{E}}{dt}\right|_A = \left.\frac{d\bar{E}}{dt}\right|_B \quad => \quad DU = \frac{1}{4}\rho g A^2 U$$

$$\therefore D = \frac{1}{4}\rho g A^2$$

(4)

Approach 1)

Let the point source and sink be located at x=L/2 and x=-L/2 respectively (L: ship length). Then, the generated wave can be expressed as

$$\eta_{source} = a\cos(kx_{source}) = a\cos\left\{k\left(x+\frac{L}{2}\right)\right\}$$

$$\eta_{sink} = -a\cos(kx_{sink}) = -a\cos\left\{k\left(x-\frac{L}{2}\right)\right\}$$

Total wave elevation is

$$\begin{aligned}\eta &= \eta_{source} + \eta_{sink}\\ &= a\cos\left\{k\left(x+\frac{L}{2}\right)\right\} - a\cos\left\{k\left(x-\frac{L}{2}\right)\right\}\\ &= a\left\{\cos(kx)\cos\left(\frac{kL}{2}\right) - \sin(kx)\sin\left(\frac{kL}{2}\right)\right\} - a\left\{\cos(kx)\cos\left(\frac{kL}{2}\right) - \sin(kx)\sin\left(-\frac{kL}{2}\right)\right\}\\ &= -2a\sin(kx)\sin\left(\frac{kL}{2}\right)\end{aligned}$$

The minimum wave elevation occurs at

$$\sin\left(\frac{kL}{2}\right) = 0 \Rightarrow \frac{kL}{2} = n\pi \Rightarrow L_{\min} = \frac{2n\pi}{k} = \frac{2n\pi U^2}{g} = n\lambda$$

The distance between the two singularities to minimize the wave elevation is $L_{\min} = n\lambda$

Approach 2)

5. (30%)

(1)

$$\Delta\omega_i = \Delta\omega = \frac{2.0-0.2}{10} = 0.18,\ \ \omega_i = 0.2 + \left(i - \frac{1}{2}\right)\Delta\omega,\ \ A_i = \sqrt{2S_\eta(\omega_i)\Delta\omega_i}\ \ (i = 1, 2, \cdots),$$

	w	S(w)	A
1	0.290	0.00012	0.00662
2	0.470	3.34467	1.09731
3	0.650	3.13490	1.06234
4	0.830	1.33059	0.69211
5	1.010	0.56215	0.44986
6	1.190	0.25984	0.30585
7	1.370	0.13140	0.21749
8	1.550	0.07171	0.16067
9	1.730	0.04167	0.12247
10	1.910	0.02550	0.09581

(2)

$$\eta(t) = \sum_{i=1}^{n} \eta_i(t) = \sum_{i=1}^{n} A_i \cos(\omega_i t + \theta_i)$$

Example plot