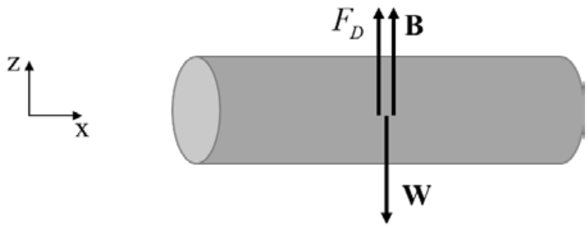
2013-2, 해양환경정보시스템

Homework 4 Solution

1.

(1)



v : velocity of the falling body

Morrison force: $F_D = -\rho C_M \forall \frac{dv}{dt} - \frac{1}{2}\rho C_D A_{projected} v|v|$

Buoyancy force: $B = \rho g \forall$

Equation of motion: $m\frac{dv}{dt} = -\rho C_M \forall \frac{dv}{dt} - \frac{1}{2}\rho C_D A_{projected} v|v| + \rho g \forall - W$

(2)

In equilibrium condition, $dv/dt = 0$ (no speed change).

$$m\cancel{\frac{dv}{dt}} = -\rho C_M \forall \cancel{\frac{dv}{dt}} - \frac{1}{2}\rho C_D A_{projected} v|v| + \rho g \forall - W$$

$$0 = -\frac{1}{2}\rho C_D A_{projected} v|v| + \rho g \forall - W$$

$$v|v| = \frac{\rho g \forall - W}{\frac{1}{2}\rho C_D A_{projected}} = \frac{2(\rho g \forall - W)}{\rho C_D A_{projected}} < 0 (\because \rho g \forall - W < 0)$$

$$\therefore v = -\sqrt{\frac{2(W - \rho g \forall)}{\rho C_D A_{projected}}} = -\sqrt{\frac{2\left(W - \rho g \frac{\pi d^2}{4} L\right)}{\rho C_D L d}}$$

2.

In deep water,

$$\phi = \frac{gA}{\omega} e^{kz} \sin(kx - \omega t)$$
$$u = A\omega e^{kz} \cos(kx - \omega t)$$
$$\frac{du}{dt} = A\omega^2 e^{kz} \sin(kx - \omega t)$$

Using u and $\frac{du}{dt}$ at x=0:

$$F_I = -\int_{-h}^{0} \rho C_M \frac{\pi d^2}{4} A\omega^2 e^{kz} \sin \omega t dz$$
$$= -\rho C_M \frac{\pi d^2}{4} A\omega^2 \sin \omega t \times \left[\frac{e^{kz}}{k} \right]_{-h}^{0}$$
$$= -\rho C_M \frac{\pi d^2}{4k} A\omega^2 \left[1 - e^{-kh} \right] \sin \omega t$$

$$F_D = \int_{-h}^{0} \frac{1}{2} \rho C_D d A^2 \omega^2 e^{2kz} \cos \omega t \left| \cos \omega t \right| dz$$
$$= \frac{1}{2} \rho C_D d A^2 \omega^2 \cos \omega t \left| \cos \omega t \right| \times \left[\frac{e^{2kz}}{2k} \right]_{-h}^{0}$$
$$= \frac{1}{4k} \rho C_D d A^2 \omega^2 \left[1 - e^{-2kh} \right] \cos \omega t \left| \cos \omega t \right|$$

Force at x = 0;

$$\therefore F = F_I + F_D = -\rho C_M \frac{\pi d^2}{4k} A\omega^2 \left[1 - e^{-kh} \right] \sin \omega t + \frac{1}{4k} \rho C_D d A^2 \omega^2 \left[1 - e^{-2kh} \right] \cos \omega t \left| \cos \omega t \right|$$