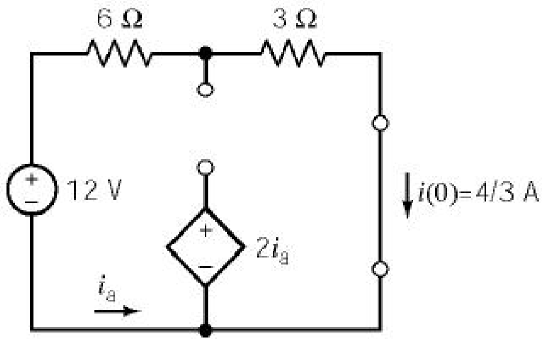
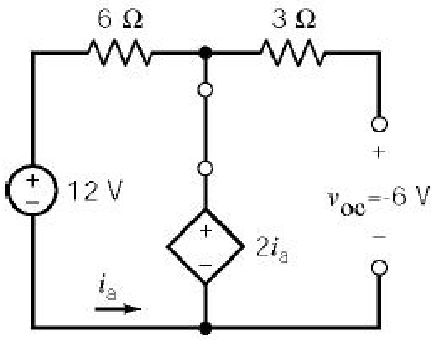
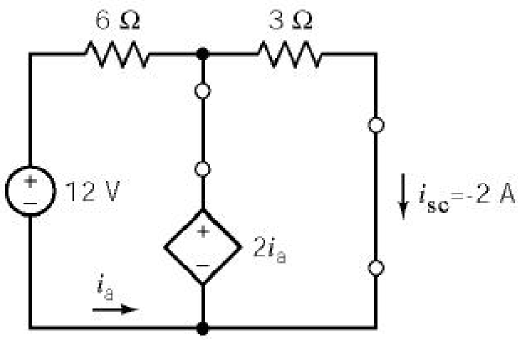
P8.3-4

Before the switch closes:



After the switch closes:





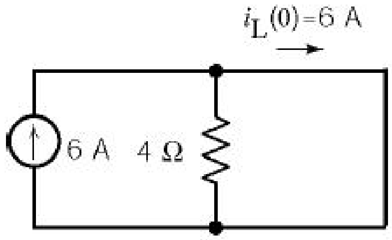
Therefore $R_t = \frac{-6}{-2} = 3\ \Omega$ so $\tau = \frac{6}{3} = 2$ s .

Finally, $i(t) = i_{sc} + (i(0) - i_{sc})\, e^{-\frac{t}{\tau}} = -2 + \frac{10}{3} e^{-0.5t}$ A for $t > 0$

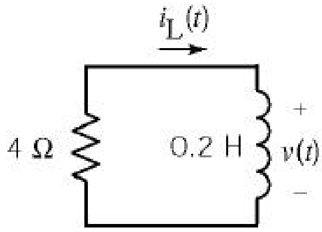
P8.3-7

At $t = 0^-$ (steady-state)

Since the input to this circuit is constant, the inductor will act like a short circuit when the circuit is at steady-state:



for t > 0



$$\underline{i_L(t) = i_L(0)\, e^{-(R/L)\,t} = 6\, e^{-20t} \text{ A}}$$

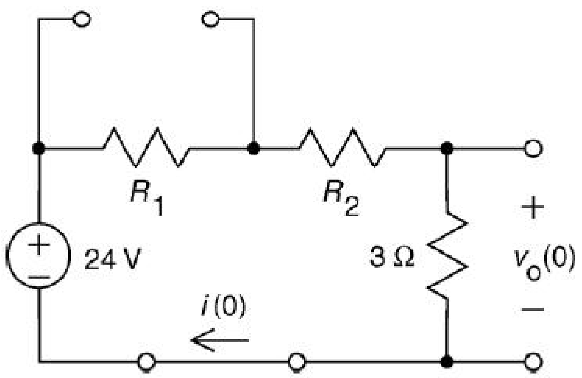
P8.3-9:

Before the switch closes, the circuit will be at steady state. Because the only input to this circuit is the constant voltage of the voltage source, all of the element currents and voltages, including the inductor current, will have constant values. Closing the switch disturbs the circuit by shorting out the resistor R_1. Eventually the disturbance dies out and the circuit is again at steady state. All the element currents and voltages will again have constant values, but probably different constant values than they had before the switch closed.

The inductor current is equal to the current in the 3 Ω resistor. Consequently,

$$i(t) = \frac{v_o(t)}{3} = \frac{6 - 3\,e^{-0.35\,t}}{3} = 2 - e^{-0.35\,t} \text{ A when } t > 0$$

In the absence of unbounded voltages, the current in any inductor is continuous. Consequently, the value of the inductor current immediately before $t = 0$ is equal to the value immediately after $t = 0$.



Here is the circuit before $t = 0$, when the switch is open and the circuit is at steady state. The open switch is modeled as an open circuit. An inductor in a steady-state dc circuit acts like a short circuit, so a short circuit replaces the inductor. The current in that short circuit is the steady state inductor current, $i(0)$. Apply KVL to the loop to get

$$R_1\, i(0) + R_2\, i(0) + 3\, i(0) - 24 = 0$$

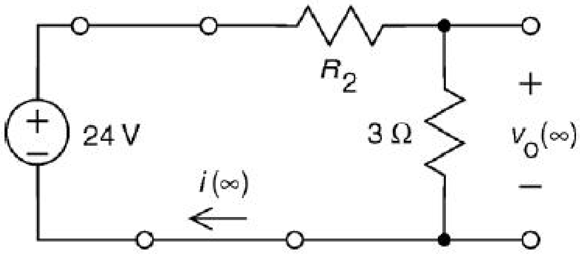
$$\Rightarrow \quad i(0) = \frac{24}{R_1 + R_2 + 3}$$

The value of $i(0)$ can also be obtained by setting $t = 0$ in the equation for $i(t)$. Do so gives

$$i(0) = 2 - e^0 = 1 \text{ A}$$

Consequently,

$$1 = \frac{24}{R_1 + R_2 + 3} \quad \Rightarrow \quad R_1 + R_2 = 21$$



Next, consider the circuit after the switch closes. Here is the circuit at $t = \infty$, when the switch is closed and the circuit is at steady state. The closed switch is modeled as a short circuit. The combination of resistor and a short circuit connected is equivalent to a short circuit. Consequently, a short circuit replaces the switch and the resistor R_1.

An inductor in a steady-state dc circuit acts like a short circuit, so a short circuit replaces the inductor. The current in that short circuit is the steady state inductor current, $i(\infty)$. Apply KVL to the loop to get

$$R_2\, i(\infty) + 3\, i(\infty) - 24 = 0 \quad \Rightarrow \quad i(\infty) = \frac{24}{R_2 + 3}$$

The value of $i(\infty)$ can also be obtained by setting $t = \infty$ in the equation for $i(t)$. Doing so gives

$$i(\infty) = 2 - e^{-\infty} = 2 \text{ A}$$

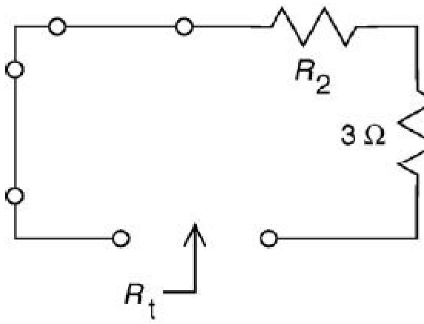
Consequently

$$2 = \frac{24}{R_2 + 3} \quad \Rightarrow \quad R_2 = 9 \;\; \Omega$$

Then

$$R_1 = 12 \;\; \Omega$$

Finally, the exponential part of $i(t)$ is known to be of the form $e^{-t/\tau}$ where $\tau = \dfrac{L}{R_t}$ and R_t is the Thevenin resistance of the part of the circuit that is connected to the inductor.



Here is shows the circuit that is used to determine R_t. A short circuit has replaced combination of resistor R_1 and the closed switch. Independent sources are set to zero when calculating R_t, so the voltage source has been replaced by an short circuit.

$$R_t = R_2 + 3 = 9 + 3 = 12 \;\; \Omega$$

so

$$\tau = \frac{L}{R_t} = \frac{L}{12}$$

From the equation for $i(t)$

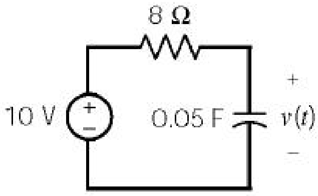
$$-0.35\, t = -\frac{t}{\tau} \quad \Rightarrow \quad \tau = 2.857 \text{ s}$$

Consequently,

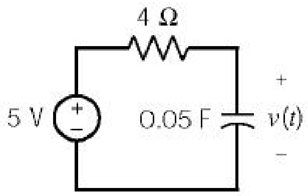
$$2.857 = \frac{L}{12} \quad \Rightarrow \quad L = 34.28 \;\; \text{H}$$

P8.4-1

Replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit to get:



$t < 0$ and $t > 1.5$ s



$0 < t < 1.5$ s

Before the switch closes at t = 0 the circuit is at steady state so $v(0) = 10$ V. For $0 < t < 1.5$s, $v_{oc} =$ 5 V and $R_t = 4\ \Omega$ so $\tau = 4 \times 0.05 = 0.2$ s . Therefore

$$v(t) = v_{oc} + (v(0) - v_{oc})\, e^{-t/\tau} = 5 + 5e^{-5t}\ \text{V} \quad \text{for}\ \ 0 < t < 1.5\ \text{s}$$

At t =1.5 s, $v(1.5) = 5 + 5e^{-0.05(1.5)} = 5$ V . For 1.5 s < t, $v_{oc} = 10$ V and $R_t = 8\ \Omega$ so $\tau = 8 \times 0.05 = 0.4$ s . Therefore

$$v(t) = v_{oc} + (v(1.5) - v_{oc})\, e^{-(t-1.5)/\tau} = 10 - 5\, e^{-2.5(t-1.5)}\ \text{V} \quad \text{for}\ \ 1.5\ \text{s} < t$$

Finally

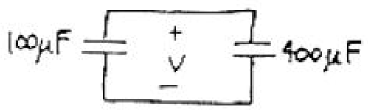
$$v(t) = \begin{cases} 5 + 5\, e^{-5t}\ \text{V} & \text{for}\ \ 0 < t < 1.5\ \text{s} \\ 10 - 5\, e^{-2.5(t-1.5)}\ \text{V} & \text{for}\ \ 1.5\ \text{s} < t \end{cases}$$

P8.4-4

At $t = 0^-$: Assume that the circuit has reached steady state so that the voltage across the 100 μF capacitor is 3 V. The charge stored by the capacitor is

$$q\left(0^-\right) = \left(100 \times 10^{-6}\right)(3) = 300 \times 10^{-6}\ \ \text{C}$$

0 < t < 10ms: With R negligibly small, the circuit reaches steady state almost immediately (i.e. at $t = 0^+$). The voltage across the parallel capacitors is determined by considering charge conservation:

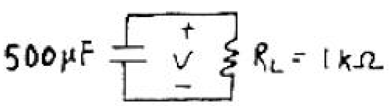


$$q\left(0^+\right) = (100\ \mu\text{F})\ v\left(0^+\right) + (400\ \mu\text{F})\ v\left(0^+\right)$$

$$v\left(0^+\right) = \frac{q\left(0^+\right)}{100 \times 10^{-6} + 400 \times 10^{-6}} = \frac{q\left(0^-\right)}{500 \times 10^{-6}} = \frac{300 \times 10^{-6}}{500 \times 10^{-6}}$$

$$\underline{v\left(0^+\right) = 0.6\ \text{V}}$$

10 ms < t < 1 s: Combine 100 μF & 400 μF in parallel to obtain



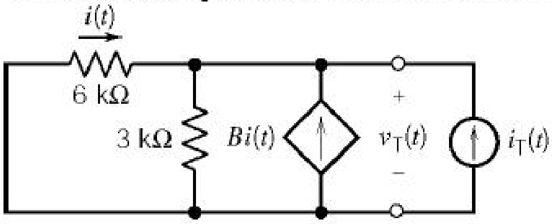
$$v(t) = v\left(0^+\right)\ e^{-(t-.01)/RC}$$

$$= 0.6e^{-(t-.01)/(10^3)\,(5\text{x}10^{-4})}$$

$$\underline{v(t) = 0.6\ e^{-2(t-.01)}\ \text{V}}$$

P8.5-3

The Thèvenin equivalent resistance of the circuit connected to the inductor is calculated as



Ohm's law: $i(t) = -\frac{v_T(t)}{6000}$

KCL: $i(t) + B\, i(t) + i_T(t) = \frac{v_T(t)}{3000}$

$$\therefore i_T(t) = -(B+1)\left(-\frac{v_T(t)}{6000}\right) + \frac{v_T(t)}{3000}$$

$$= \frac{(B+3)\, v_T(t)}{6000}$$

$$R_t = \frac{v_T(t)}{i_T(t)} = \frac{6000}{B+3}$$

<u>The circuit is stable when $B > -3$ A/A.</u>

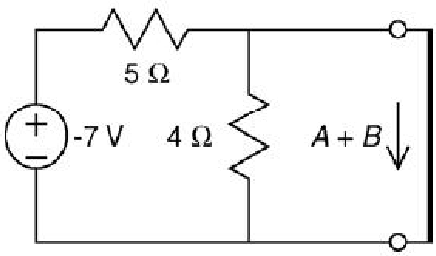
P8.6-3

The value of the input is one constant, -7 V, before time $t = 0$ and a different constant, 6 V, after time $t = 0$. The response of the first order circuit to the change in the value of the input will be

$$v_o(t) = A + B\,e^{-a t} \quad \text{for } t > 0$$

where the values of the three constants A, B and a are to be determined.

The values of A and B are determined from the steady state responses of this circuit before and after the input changes value.



The steady-state circuit for t < 0.

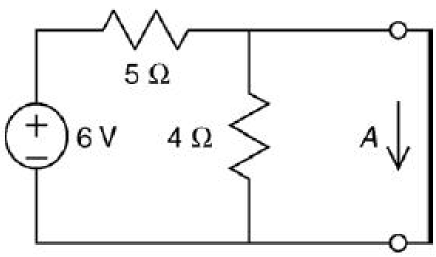
Inductors act like short circuits when the input is constant and the circuit is at steady state. Consequently, the inductor is replaced by a short circuit.

The value of the inductor current at time $t = 0$, will be equal to the steady state inductor current before the input changes. At time $t = 0$ the output current is

$$i_o(0) = A + B\,e^{-a(0)} = A + B$$

Consequently, the inductor current is labeled as $A + B$. Analysis of the circuit gives

$$A + B = \frac{-7}{5} = -1.4 \text{ A}$$



The steady-state circuit for t > 0.

Inductors act like short circuits when the input is constant and the circuit is at steady state. Consequently, the inductor is replaced by a short circuit.

The value of the inductor current at time $t = \infty$, will be equal to the steady state inductor current after the input changes. At time $t = \infty$ the output current is

$$i_o(\infty) = A + B\,e^{-a(\infty)} = A$$

Consequently, the inductor current is labeled as A. Analysis of the circuit gives

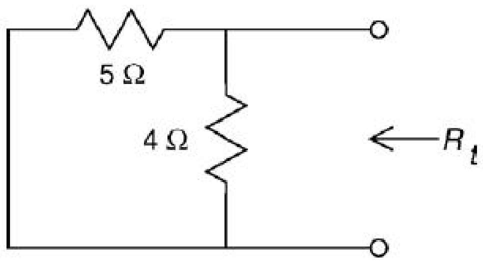
$$A = \frac{6}{5} = 1.2 \text{ A}$$

Therefore

$$B = -2.6 \text{ V}$$

The value of the constant a is determined from the time constant, τ, which is in turn calculated from the values of the inductance L and of the Thevenin resistance, R_t, of the circuit connected to the inductor.

$$\frac{1}{a} = \tau = \frac{L}{R_t}$$



Here is the circuit used to calculate R_t.

$$R_t = \frac{(5)(4)}{5+4} = 2.22\ \Omega$$

Therefore

$$a = \frac{2.22}{1.2} = 1.85\ \frac{1}{\text{s}}$$

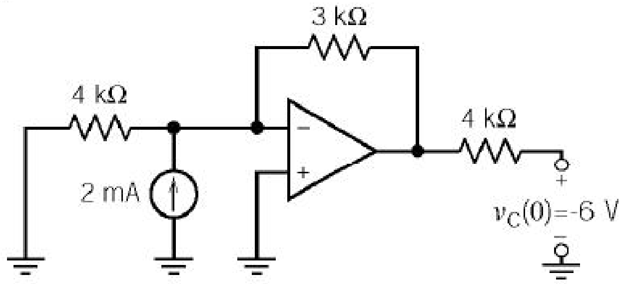
(The time constant is $\tau = \dfrac{1.2}{2.22} = 0.54$ s.)

Putting it all together:

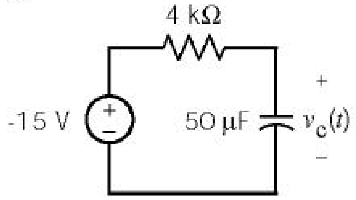
$$i_o(t) = \begin{cases} -1.4\ \text{A} & \text{for } t \le 0 \\ 1.2 - 2.6\,e^{-1.85t}\ \text{A} & \text{for } t \ge 0 \end{cases}$$

P8.6-8

For t < 0, the circuit is:



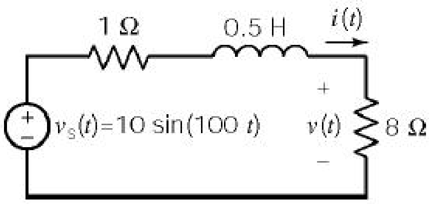
After t = 0, replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit to get:



$$\begin{aligned} v_c(t) &= -15 + (-6-(-15))\,e^{-t/(4000\times 0.00005)} \\ &= -15 + 9\,e^{-5t}\ \text{V} \end{aligned}$$

P8.7-5

Assume that the circuit is at steady state before $t = 0$. There are no sources in the circuit so $i(0) =$ 0 A. After $t = 0$, we have:



$$\text{KVL}: -10\sin 100t + i(t) + 5\frac{di(t)}{dt} + v(t) = 0$$

$$\text{Ohm's law}: i(t) = \frac{v(t)}{8}$$

$$\therefore \underline{\frac{dv(t)}{dt} + 18\,v(t) = 160\sin 100t}$$

$\therefore\ v_n(t) = Ae^{-18t}$, try $v_f(t) = B\cos 100t + C\sin 100t$, substitute into the differential equation and equate like terms $\Rightarrow B = -1.55\ \&\ C = 0.279 \Rightarrow v_f(t) = -1.55\cos 100t + 0.279\sin 100t$

$\therefore v(t) = v_n(t) + v_f(t) = Ae^{-18t} - 1.55\cos 100t + 0.279\sin 100t$

$v(0) = 8\,i(0) = 0 \quad \Rightarrow \quad v\,(0) = 0 = A - 1.55 \Rightarrow A = 1.55$

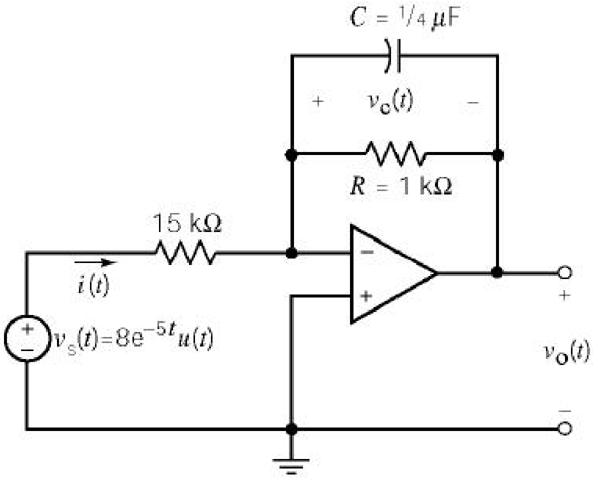
so $\underline{v(t) = 1.55e^{-18t} - 1.55\cos 100t + 0.279\ \sin 100t\ \text{ V}}$

P8.7-6

Assume that the circuit is at steady state before $t = 0$.

$$v_o(t) = -v_c(t)$$

$$v_C(0^+) = v_C(0^-) = -10\text{ V}$$



After $t = 0$, we have

$$i(t) = \frac{v_s(t)}{15000} = \frac{8\,e^{-5t}}{15000} = 0.533\,e^{-5t}\text{ mA}$$

The circuit is represented by the differential equation: $i(t) = C\dfrac{dv_C(t)}{dt} + \dfrac{v_C(t)}{R}$. Then

$$\left(0.533\times10^{-3}\right)e^{-5t} = \left(0.25\times10^{-6}\right)\frac{dv_c(t)}{dt} + \left(10^{-3}\right)v_c(t) \quad \Rightarrow \quad \frac{dv_c(t)}{dt} + 4000\,v_c(t) = 4000\,e^{-5t}$$

Then $v_n(t) = Ae^{-4000t}$. Try $v_f(t) = Be^{-5t}$. Substitute into the differential equation to get

$$\frac{d\left(B\,e^{-5t}\right)}{dt} + 4000\left(B\,e^{-5t}\right) = 4000\,e^{-5t} \quad \Rightarrow \quad B = \frac{4000}{-3995} = -1.00125 \cong -1$$

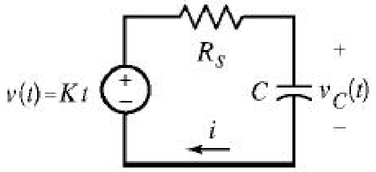
$$v_C(t) = v_f(t) + v_n(t) = e^{-5t} + Ae^{-4000t}$$

$$v_C(0) = -10 = 1 + A \Rightarrow A = -11 \Rightarrow \quad v_C(t) = 1\,e^{-2t} - 11\,e^{-4000t} \text{ V}$$

Finally

$$\underline{v_o(t) = -v_C(t) = 11e^{-4000t} - 1e^{-5t} \text{ V}, \text{ t} \ge 0}$$

P8.7-10



$$\text{KVL: } -kt + R_s\left[C\frac{dv_C(t)}{dt}\right] + v_C(t) = 0$$

$$\Rightarrow \frac{dv_C(t)}{dt} + \frac{1}{R_sC}v_C(t) = \frac{k}{R_sC}t$$

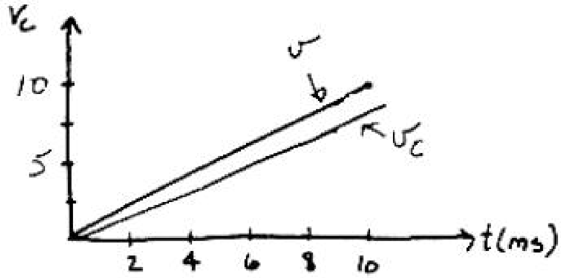
$v_c(t) = v_n(t) + v_f(t)$, where $v_c(t) = Ae^{-t/R_sC}$. Try $v_f(t) = B_0 + B_1 t$

& plug into D.E. $\Rightarrow \quad B_1 + \frac{1}{R_sC}\left[B_0 + B_1 t\right] = \frac{k}{R_sC}t$ thus $B_0 = -kR_sC$, $B_1 = k$.

Now we have $v_c(t) = Ae^{-t/R_sC} + k(t - R_sC)$. Use $v_c(0) = 0$ to get $0 = A - kR_sC \Rightarrow A = kR_sC$.

$\therefore v_c(t) = k[t - R_sC(1 - e^{-t/R_sC})]$. Plugging in k=1000 , R_s= 625 kΩ & C=2000 pF get

$\underline{v_c(t) = 1000[t - 1.25\times10^{-3}(1 - e^{-800\,t})]}$



$v(t)$ and $v_C(t)$ track well on a millisecond time scale.

DP 8-1

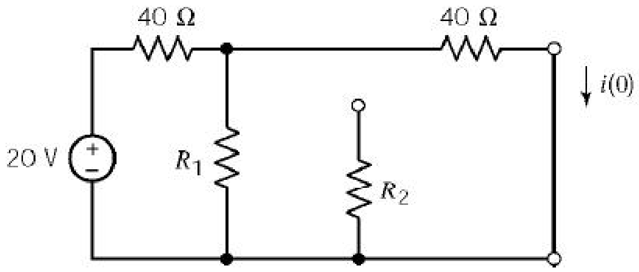
Steady-state response when the switch is open: $6 = \dfrac{R_3}{R_1 + R_2 + R_3} 12 \quad \Rightarrow \quad R_1 + R_2 = R_3$.

Steady-state response when the switch is open: $10 = \dfrac{R_3}{R_1 + R_3} 12 \quad \Rightarrow \quad R_1 = \dfrac{R_3}{5}$.

$10 \text{ ms} = 5\,\tau = \left(R_1 \,||\, R_3\right) C = \dfrac{R_3}{6} C$

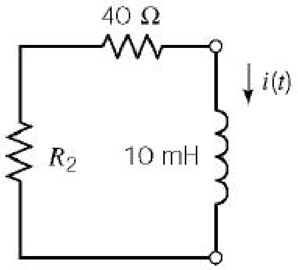
Let $C = 1\ \mu\text{F}$. Then $R_3 = 60\ \text{k}\Omega$, $R_1 = 30\ \text{k}\Omega$ and $R_2 = 30\ \text{k}\Omega$.

DP 8-5



$$i(0) = \frac{20}{40+\dfrac{40R_1}{40+R_1}} \times \frac{R_1}{R_1+40}$$

For $t > 0$:



$i(t) = i(0)\, e^{-t/\tau}$ where $\tau = \dfrac{L}{R_t} = \dfrac{10^{-2}}{40+R_2}$

At $t < 200\,\mu s$ we need $i(t) > 60$ mA and $i(t) < 180$ mA

First let's find a value of R_2 to cause $i(0) < 180$ mA.

Try $\underline{R_2 = 40\,\Omega}$. Then $i(0) = \dfrac{1}{6}$ A = 166.7 mA so $i(t) = 0.1667\ e^{-t/\tau}$.

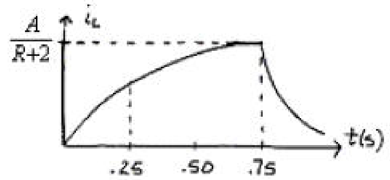
Next, we find a value of R_2 to cause $i(0.0002) > 60$ mA.

Try $\underline{R_2 = 10\Omega}$, then $\tau = \dfrac{10^{-2}}{50} = 0.2$ ms $= \dfrac{1}{5000}$ s.

$i(0.0002) = 166.7\times10^{-3} e^{-5000\times0.0002} = 166.7\times10^{-3} e^{-1} = 61.3$ mA

DP 8-6

The current waveform will look like this:



We only need to consider the rise time:

$$i_L(t) = \frac{V_s}{R+2}(1-e^{-t/\tau}) = \frac{A}{R+2}(1-e^{-t/\tau})$$

where

$$\tau = \frac{L}{R_t} = \frac{0.2}{3} = \frac{1}{15} \text{ s}$$

$$\therefore i_L(t) = \frac{A}{3}(1-e^{-15t})$$

Now find A so that $i_L^2 R_{fuse} \geq 10$ W during $0.25 \leq t \leq 0.75$ s

$\therefore$ we want $[i_L^2(0.25)]R_{fuse} = 10$ W $\Rightarrow$ $\frac{A^2}{9}(1-e^{-15(.25)})^2(1)=10 \Rightarrow \underline{A = 9.715 \text{ V}}$