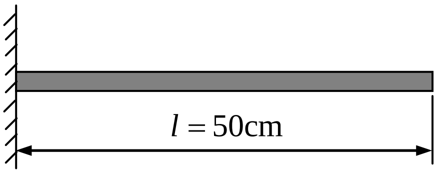
Aeroelasticity Homework No. 3

Due Date: November 11 (Fri)

1. Determine the first natural bending frequency of the cantilevered beam (Fig. 1) using both Galerkin and Rayleigh-Ritz methods. In both methods, use the following mode shape.

$$\gamma_1(x) = \frac{x^2}{l^2} + \frac{ax^3}{l^3} + \frac{bx^4}{l^4}, \left(a = -\frac{2}{3}, b = \frac{1}{6} \right)$$



$EI = 70GPa,$

$\rho = 2700 kg/m^3,$

$width = 5cm,$

$thickness = 1cm$

Figure 1. Cantilevered Beam

2. Using a finite element representation of the beam shown in Fig. 1, construct a structural dynamics model which accurately predicts (to within 1%) the first three natural frequencies of the beam. (Consider the deflection in weak bending direction of the given beam.) Also, determine the corresponding modes.

Note: Use a two-node beam element with two DOF's per node (translation and rotation). The stiffness and inertia matrix can be given by:

$$K = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}; \quad M = \frac{ml}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}; \quad q = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix}$$

where q_1, q_3 are translational D.O.F.'s and q_2, q_4 are rotational D.O.F.'s.

3. Assume that the lumped parameter model for a cantilevered beam as follows:

$$\begin{bmatrix} 0.25 & 0 & 0 \\ & 0.5 & \\ \text{Symm.} & & 0.25 \end{bmatrix}\begin{Bmatrix} \ddot{w}_1 \\ \ddot{w}_2 \\ \ddot{w}_3 \end{Bmatrix} + \begin{bmatrix} 2000 & -1000 & 0 \\ & 2000 & -1000 \\ \text{Symm.} & & 1000 \end{bmatrix}\begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$[M]\{\ddot{w}\} + [K]\{w\} = \{0\}$$

And, the eigenvectors of the system can be found as

$$\begin{bmatrix} \phi^{(1)}(x_1) & \phi^{(2)}(x_1) & \phi^{(3)}(x_1) \\ \phi^{(1)}(x_2) & \phi^{(2)}(x_2) & \phi^{(3)}(x_2) \\ \phi^{(1)}(x_3) & \phi^{(2)}(x_3) & \phi^{(3)}(x_3) \end{bmatrix} = \begin{bmatrix} -0.5637 & 1.0119 & -1.6304 \\ -1.0454 & 0.6041 & 0.7363 \\ -1.2233 & -1.4988 & -0.5072 \end{bmatrix}$$

Determine the first two natural frequencies.