2016-2 항공기구조역학 HW2

기한 : 2016/10/7 18:00　　　　　　장소 : 301동 1357호 제출함

Problem 2.8. Yield criterion for a pressure vessel

A cylindrical pressure vessel of radius R and thickness t is subjected to an internal pressure p_i, as shown in fig. 1.20. At any point in the cylindrical portion of vessel wall, two stress components are acting: the hoop stress, $\sigma_h = Rp_i/t$ and the axial stress, $\sigma_a = Rp_i/(2t)$. The radial stress, acting in the direction perpendicular to the wall, is very small, $\sigma_r \approx 0$. The yield stress for the material is σ_y. *(1)* If the material is assumed to follow von Mises' criterion, find the maximum internal pressure the vessel can carry. *(2)* If the material is assumed to follow Tresca's criterion, find the maximum internal pressure the vessel can carry.

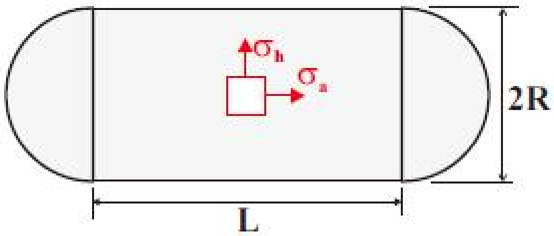


Fig. 1.20. Stresses acting in a pressure vessel.

Problem 4.1. Simple hyperstatic bars - displacement method solution

Three axially loaded bars, each of length L and all constructed from a material of elasticity modulus E, are arranged as shown in fig. 4.9. Two bars are connected in parallel and one of these has a cross-sectional area that is twice that of the other. A third bar is connected in series at the common point. An axial load, P, is applied at the junction of the three bars. Using the displacement method, determine *(1)* the displacement, d, of the connecting point between the three bars and *(2)* the forces in each of the three bars.

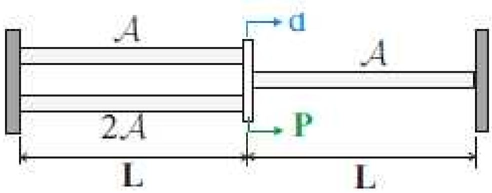


Fig. 4.9. Three bars in a parallel-series configuration.

Problem 4.2. Simple hyperstatic bars - force method solution

Solve problem 4.1 using using the force method.

Problem 4.3. Prestressed steel bar in an aluminum tube

A steel bar of cross-sectional area $\mathcal{A}_s = 800$ mm^2 fits inside an aluminum tube of cross-sectional area $\mathcal{A}_a = 1,500$ mm^2. The assembly is constructed in such a way that initially, the steel bar is prestressed with a compressive force, $-P$, while the aluminum tube is prestressed with a tensile load of equal magnitude, P. Next, the prestressed assembly is subjected to a tensile load F. *(1)* If no prestress is applied, *i.e.*, if $P = 0$, find the maximum external load, F, that can be applied to the assembly without exceeding allowable stress levels in either material. *(2)* Find the optimum prestress level to be applied. This optimum prestress is defined as that for which the allowable stress is reached simultaneously in both steel bar and aluminum tubes when subjected to the externally applied force, F. In other words, when optimally prestressed, both materials are used to their full capacity. *(3)* What improvement, in percent, is achieved by using the optimum prestress level as compared to not prestressing the assembly. Use the following data: $E_s = 210$ and $E_a = 73$ GPa; the yield stresses for steel and aluminum are $\sigma_y^s = 600$ and $\sigma_y^a = 400$ MPa, respectively.

Problem 4.6. Rotor blade hub connection

Figure 4.11 shows a potential design for the attachment of a rotor blade to the rotorcraft hub. The yoke consists of two separate pieces each of which connects the rotor blade to the hub, and the spindle also connects the rotor blade to the hub through an elastomeric bearing. As the rotor blade spins, a large centrifugal force F is applied to the assembly, which can be idealized as three parallel bars of length L, which connect the blade to the hub. The two bars modeling the yoke each have an axial stiffness $(E\mathcal{A})_y$, while the spindle has an axial stiffness $(E\mathcal{A})_s$. The elastomeric bearing is idealized as a very short spring of stiffness k_b in series with the spindle. *(1)* Calculate and plot the non-dimensional forces in the yoke, F_y/F, and in the spindle, F_s/F, as a function of the non-dimensional bearing stiffness, $0 \leq Lk_b/(E\mathcal{A})_s \leq 25$. *(2)* For what value of the stiffness constant k_b is all the centrifugal load carried by the yoke? *(3)* Find the maximum load that can be carried by the spindle. What is the corresponding value of k_b? *(4)* For what value on $Lk_b/(E\mathcal{A})_s$ do the yoke and spindle carry equal loads? Use the following data: $(E\mathcal{A})_y/(E\mathcal{A})_s = 0.8$

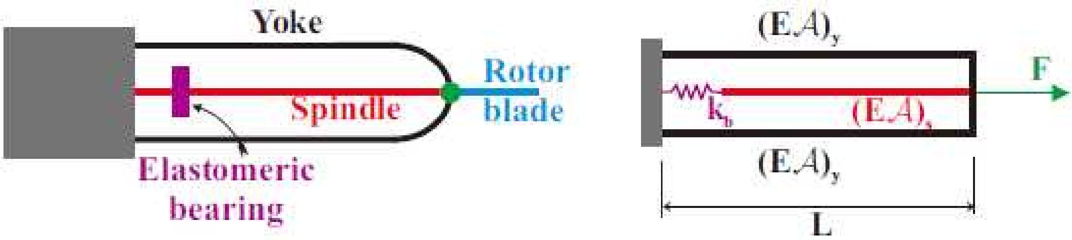
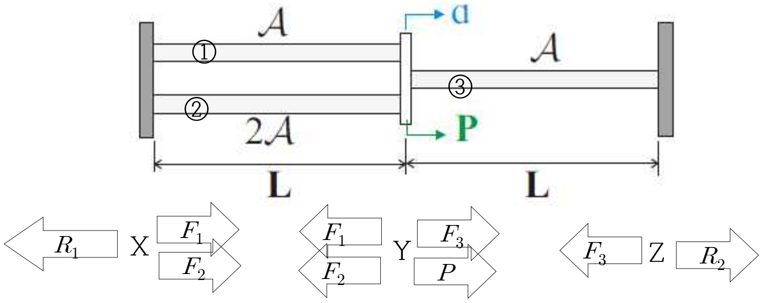


Fig. 4.11. Rotor blade connection to the hub by means of a yoke and spindle.

HW2_sol

<table>
<tr><th>문제</th><th>소문제</th><th>풀이</th></tr>
<tr><td rowspan="4">2.8</td><td rowspan="2">(1)</td><td>과정
$\tau_{ha}=0$이므로 σ_h와 σ_a는 principal stresses.
$\sigma_{von}=\sqrt{\sigma_h^2+\sigma_a^2-\sigma_h\sigma_a+3\tau_{ha}^2}=\dfrac{\sqrt{3}Rp_i}{2t}$ (2.36)
$\sigma_{von}=\sigma_Y$일 때, $p_{i,\max}=\dfrac{2t\sigma_Y}{\sqrt{3}R}$</td></tr>
<tr><td>답
$\dfrac{2t\sigma_Y}{\sqrt{3}R}$</td></tr>
<tr><td rowspan="2">(2)</td><td>과정
Tresca criterion에 따라 아래 식을 만족한다.
$\begin{cases}\sigma_h-\sigma_a\le\sigma_Y\\ \sigma_h\le\sigma_Y\\ \sigma_a\le\sigma_Y\end{cases}$ (2.29)
$\begin{cases}p_i\le\dfrac{2t\sigma_Y}{R}\\ p_i\le\dfrac{t\sigma_Y}{R}\\ p_i\le\dfrac{2t\sigma_Y}{R}\end{cases}$
따라서 $p_{i,\max}=\dfrac{t\sigma_Y}{R}$</td></tr>
<tr><td>답
$\dfrac{t\sigma_Y}{R}$</td></tr>
<tr><td>4.1</td><td></td><td>과정
1.


$\begin{cases}F_1+F_2=R_1\\ F_3+P=F_1+F_2\\ R_2=F_3\end{cases}$

2.
$\begin{cases}E\epsilon_1=\dfrac{F_1}{A}\\ E\epsilon_2=\dfrac{F_2}{2A}\\ E\epsilon_3=\dfrac{F_3}{A}\end{cases}\rightarrow\begin{cases}F_1=AE\epsilon_1\\ F_2=2AE\epsilon_2\\ F_3=AE\epsilon_3\end{cases}$</td></tr>
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<tr><td></td><td></td><td>3.
$$\begin{cases} \epsilon_1 = \dfrac{d}{L} \\ \epsilon_2 = \dfrac{d}{L} \\ \epsilon_3 = \dfrac{-d}{L} \end{cases}$$
4. 3의 결과를 2에 대입
$$\begin{cases} F_1 = \dfrac{EA}{L}d \\ F_2 = \dfrac{2EA}{L}d \\ F_3 = -\dfrac{EA}{L}d \end{cases}$$
5. 4의 결과를 1에 대입
$$\begin{cases} \dfrac{3EA}{L}d = R_1 \\ \dfrac{4EA}{L}d = P \\ \dfrac{EA}{L}d = -R_2 \end{cases}$$
6.
$$d = \frac{LP}{4EA}$$
7. 6의 결과를 3에 대입
$$\begin{cases} \epsilon_1 = \dfrac{P}{4EA} \\ \epsilon_2 = \dfrac{P}{4EA} \\ \epsilon_3 = -\dfrac{P}{4EA} \end{cases}$$
8. 7의 결과를 4에 대입
$F_1 = \frac{P}{4}$, $F_2 = \frac{P}{2}$, $F_3 = -\frac{P}{4}$</td></tr>
<tr><td></td><td></td><td>답
(1) $\frac{LP}{4EA}$ (2) $F_1 = \frac{P}{4}$, $F_2 = \frac{P}{2}$, $F_3 = -\frac{P}{4}$</td></tr>
<tr><td>4.2</td><td></td><td>과정
1.
$$\begin{cases} F_1 + F_2 = R_1 \\ F_3 + P = F_1 + F_2 \\ R_2 = F_3 \end{cases}$$ 5 미지수, 3 식.
2. $N_R = 2$</td></tr>
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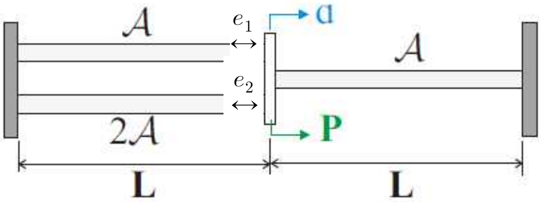
3.


4.
F_1, F_2를 redundant force로 결정.
$\begin{cases} R_1 = F_1 + F_2 \\ F_3 = F_1 + F_2 - P \\ R_2 = F_1 + F_2 - P \end{cases}$

5.
$\begin{cases} E\dfrac{d_1}{L} = \dfrac{F_1}{A} \\ E\dfrac{d_2}{L} = \dfrac{F_2}{2A} \\ E\dfrac{d_3}{L} = \dfrac{F_1 + F_2 - P}{A} \end{cases}$

6.
$\begin{cases} e_1 = d_1 + d_3 \\ e_2 = d_2 + d_3 \end{cases}$
$\begin{cases} e_1 = \dfrac{L}{EA}(2F_1 + F_2 - P) \\ e_2 = \dfrac{L}{EA}(F_1 + \dfrac{3}{2}F_2 - P) \end{cases}$

7.
$\begin{cases} 2F_1 + F_2 = P \\ 2F_1 + 3F_2 = 2P \end{cases}$
$F_1 = \dfrac{1}{4}P$, $F_2 = \dfrac{1}{2}P$

8.
$d_1 = d_2 = -d_3 = \dfrac{LP}{4EA}$
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<td>
답
(1) $\dfrac{LP}{4EA}$ (2) $F_1 = \dfrac{P}{4}$, $F_2 = \dfrac{P}{2}$, $F_3 = -\dfrac{P}{4}$
</td>
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<tr>
<td>4.3</td>
<td>(1)</td>
<td>
과정
$\begin{cases} E_s\epsilon_s = \dfrac{F_s}{A_s} \\ E_a\epsilon_a = \dfrac{F_a}{A_a} \\ F_s + F_s = F \\ \epsilon_s = \epsilon_a \end{cases}$ 4 미지수(F_s, F_a, ϵ_s, ϵ_a), 4 식
$F_s = \dfrac{A_sE_sF}{A_sE_s + A_aE_a}$, $F_a = \dfrac{A_aE_aF}{A_sE_s + A_aE_a}$
</td>
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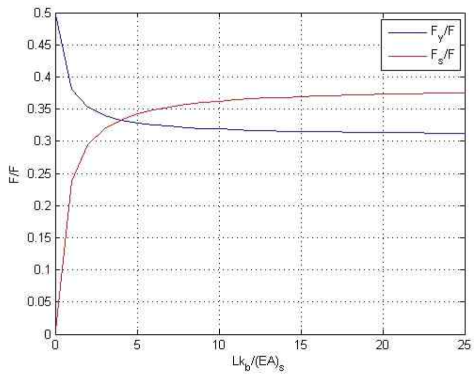
		$\sigma_s = \dfrac{E_s F}{A_s E_s + A_a E_a}$, $\sigma_a = \dfrac{E_a F}{A_s E_s + A_a E_a}$ $\sigma_s \le \sigma_{sY}$, $\sigma_a \le \sigma_{aY}$ $F_{s,\max} = \dfrac{(A_s E_s + A_a E_a)\sigma_{sY}}{E_s} = 792.8\,N$ $F_{a,\max} = \dfrac{(A_s E_s + A_a E_a)\sigma_{aY}}{E_a} = 1520.5\,N$
		답 $792.8\,N$
	(2)	과정 $\begin{cases} E_s \epsilon_{s0} = \dfrac{-P}{A_s} \\ E_a \epsilon_{a0} = \dfrac{P}{A_a} \\ E_s(\epsilon_s + \epsilon_{s0}) = \dfrac{F_s - P}{A_s} \\ E_a(\epsilon_a + \epsilon_{a0}) = \dfrac{F_a + P}{A_a} \\ F_s + F_s = F \\ \epsilon_s = \epsilon_a \end{cases}$ 6 미지수(F_s, F_a, ϵ_s, ϵ_a, ϵ_{s0}, ϵ_{a0}), 6 식 ϵ_{s0}, ϵ_{a0}를 소거하면 (1)과 같은 식이므로 $F_s = \dfrac{A_s E_s F}{A_s E_s + A_a E_a}$, $F_a = \dfrac{A_a E_a F}{A_s E_s + A_a E_a}$ $\begin{cases} \sigma_s = \dfrac{F_s - P}{A_s} = \dfrac{E_s F}{A_s E_s + A_a E_a} - \dfrac{P}{A_s} = \sigma_{sY} \\ \sigma_a = \dfrac{F_a + P}{A_a} = \dfrac{E_a F}{A_s E_s + A_a E_a} + \dfrac{P}{A_a} = \sigma_{aY} \end{cases}$ $P = \dfrac{A_s A_a (E_a \sigma_{sY} - E_s \sigma_{aY})}{A_s E_s - A_a E_a} = 173.8\,N$ $F_{\max} = A_s \sigma_{sY} + A_a \sigma_{aY} = 1080\,N$
		답 $173.8\,N$
	(3)	과정 $F_{\max 1} = 792.8\,N$ $F_{\max 2} = 1080\,N$ $\dfrac{F_{\max 2} - F_{\max 1}}{F_{\max 1}} \times 100(\%) = 36.2\%$
		답 36.2%
4.6	(1)	과정 용수철의 길이는 매우 작다고 하였으므로 용수철의 초기 길이는 0이다.

		$\begin{cases} d_y = \dfrac{LF_y}{(EA)_y} \\ d_s = \dfrac{LF_s}{(EA)_s} \\ d_b = \dfrac{F_s}{k_b} \\ 2F_y + F_s = F \\ d_s + d_b = d_y \end{cases}$ 5 미지수(d_y, d_s, d_b, F_y, F_s), 5식 변위들 소거 $\begin{cases} 2F_y + F_s = F \\ \dfrac{LF_s}{(EA)_s} + \dfrac{F_s}{k_b} = \dfrac{LF_y}{(EA)_y} \end{cases}$ $\begin{cases} 2\dfrac{F_y}{F} + \dfrac{F_s}{F} = 1 \\ \left(\dfrac{Lk_b}{(EA)_s} + 1\right)\dfrac{F_s}{F} = \dfrac{(EA)_s}{(EA)_y}\dfrac{Lk_b}{(EA)_s}\dfrac{F_y}{F} \end{cases}$
		답 $\begin{cases} \dfrac{F_y}{F} = \dfrac{\dfrac{Lk_b}{(EA)_s} + 1}{\left(2 + \dfrac{(EA)_s}{(EA)_y}\right)\dfrac{Lk_b}{(EA)_s} + 2} \\ \dfrac{F_s}{F} = \dfrac{\dfrac{(EA)_s}{(EA)_y}\dfrac{Lk_b}{(EA)_s}}{\left(2 + \dfrac{(EA)_s}{(EA)_y}\right)\dfrac{Lk_b}{(EA)_s} + 2} \end{cases}$ 
	(2)	답 $k_b = 0$
	(3)	과정 $\begin{cases} \overline{F_y} = \dfrac{\overline{k_b} + 1}{3.25\overline{k_b} + 2} \\ \overline{F_s} = \dfrac{1.25\overline{k_b}}{3.25\overline{k_b} + 2} \end{cases}$ 항상 $\dfrac{d\overline{F_y}}{d\overline{k_b}} > 0$ 이므로 $\overline{k_b} = 25$일 때, $\overline{F_s} = 0.3754$
		답 $k_b = \dfrac{25(EA)_s}{L}$, $F_s = 0.3754F$
	(4)	과정 $\overline{F_y} = \overline{F_s}$ $\overline{k_b} = 4$
		답 $\dfrac{4(EA)_s}{L}$