2016-2 항공기구조역학 HW5

기한 : 2016/12/2 18:00 장소 : 301동 1357호 제출함

Problem 8.49. Shearing and torsion of a high-lift device

The cross-section of a high lift device is shown in fig 8.59. The aerodynamic pressure acting on the lower panel of the device has a net resultant $V_3 = 100$ kN and its line of action is aligned with the left web, as indicated on the figure. Material properties are: $E = 73$ GPa, $G = 30$ GPa. *(1)* Find and sketch the shear stress distribution generated by the vertical shear force. *(2)* Find and sketch the shear stress distribution generated by the torque applied to the section. *(3)* Find and sketch the total shear stress distribution in the section. *(4)* Indicate the location and magnitude of the maximum shear stress in the section.

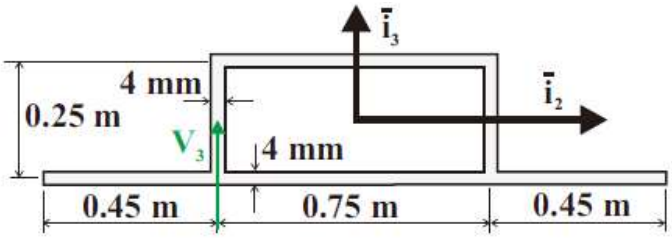


Fig. 8.59. High lift device subjected to a transverse shear force.

Problem 8.63. Warping function of an "I" beam

Consider the thin-walled, "I" shaped cross-section depicted in fig. 8.73. *(1)* Compute and plot the warping function $\Psi(s)$ for this section. *(2)* Determine the location of the twist center. *(3)* Compute the torsional stiffness of the section.

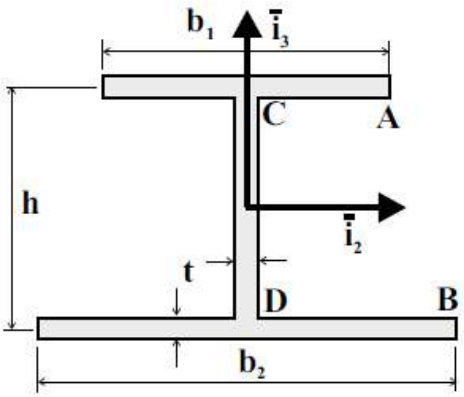


Fig. 8.73. Configuration of the I beam.

Problem 8.68. Shear flow in a multi-cellular sheet-stringer section

Consider the same thin-walled cross-section with two cells shown in fig. 8.45, but now construct an idealized sheet-stringer model as shown in fig. 8.86 where the stringer areas are computed under the assumption that only V_3 and M_2 are applied. Use this model to determine the shear flow in the sheets and to find the location of the shear center.

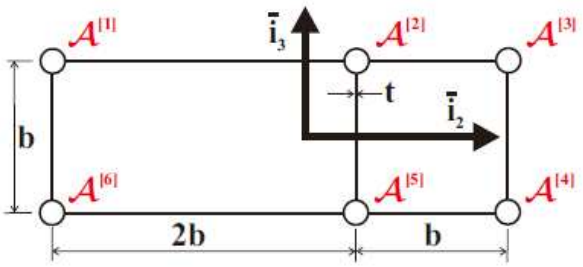


Fig. 8.86. Sheet-stringer beam with two cells.

Problem 9.5. Lever mechanism

A bar of length $3b$ is pinned at its lower end and supports a normal load, P, applied at its tip, as shown in fig. 9.21. A second bar, of length b, is pinned to the first bar as shown and to a slider that is constrained to move vertically on a frictionless rod. A weight, W, is supported at the slider. Use the principle of virtual work to determine the force, P, required to keep the weight, W, in equilibrium as a function of angle θ.

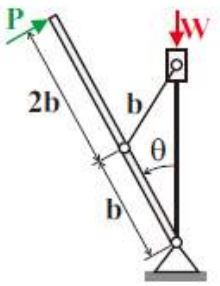


Fig. 9.21. Lever mechanism.

Problem 9.13. Three springs in series

(1) Use the principle of complementary virtual work to determine the forces in each of the three springs connected in series depicted in fig. 9.39. *(2)* Find the solution of the same problem using the force method developed in section 4.3.3. *(3)* Compare the two solution approaches.

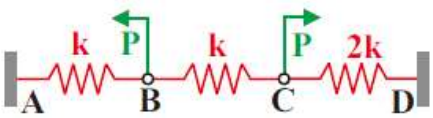
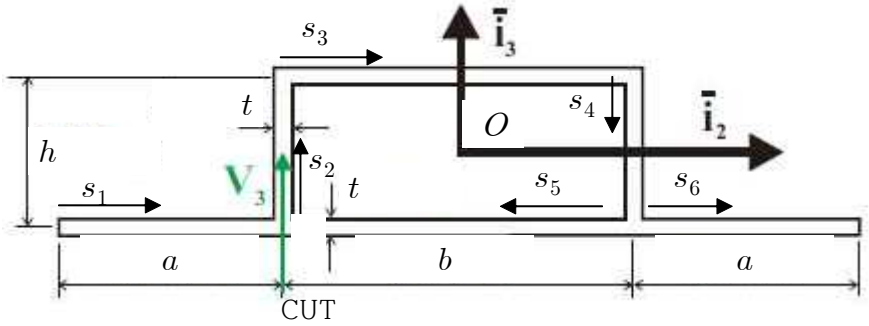


Fig. 9.39. Three-spring hyperstatic system.

문제	소문제	풀이
8.49	(1)	과정



Centroid

$$S=\int EdA$$
$$=2Et(a+b+h)$$
$$S_2=\int Ex_2dA$$
$$=0$$
$$S_3=\int Ex_3dA$$
$$=\frac{Eth(h-2a)}{2}$$

$$x_{2c}=\frac{S_2}{S}=0,\ x_{3c}=\frac{S_3}{S}=-\frac{ha}{2(a+b+h)}=-0.0388[m] \qquad (5.33)$$

Stiffness

$$H_{22}^c=\int E(x_3-x_{3c})^2dA$$
$$=\frac{Eth^2\{2(2h+3b)a+(h+b)(h+3b)\}}{6(a+b+h)}=1.0436\times10^7[Nm^2]$$
$$H_{33}^c=\int E(x_2-x_{2c})^2dA$$
$$=\frac{Et\{3b^2h+(a+b)(4a^2+2ab+b^2)\}}{6}=1.4011\times10^8[Nm^2]$$
$$H_{23}^c=0 \quad \text{(symmetry about } \vec{i}_3\text{)}$$

Shear flow

$$f(s)=c+\frac{Q_3(s)H_{23}^c-Q_2(s)H_{33}^c}{\Delta H}V_3-\frac{Q_3(s)H_{22}^c-Q_2(s)H_{23}^c}{\Delta H}V_2$$
$$=c-\frac{Q_2(s)}{H_{22}^c}V_3$$

(8.20)

$$Q_2(s) = \int_0^s Ex_3(s)t\,ds \qquad (8.21)$$

$$\begin{aligned}
Q_2(s_1) &= Et\int_0^{s_1}(-\frac{h}{2}-x_{3c})ds \\
&= Et\left(-\frac{h(b+h)}{2(a+b+h)}s_1\right) \\
Q_2(s_2) &= Et\int_0^{s_2}(-\frac{h}{2}-x_{3c}+s_2)ds \\
&= Et\left(\frac{s_2^2}{2}-\frac{h(b+h)}{2(a+b+h)}s_2\right) \\
Q_2(s_3) &= Et\int_0^{s_3}(\frac{h}{2}-x_{3c})ds \\
&= Et\frac{h(2a+b+h)}{2(a+b+h)}s_3 \\
Q_2(s_4) &= Et\int_0^{s_4}(\frac{h}{2}-x_{3c}-s_4)ds \\
&= Et\left(-\frac{s_4^2}{2}+\frac{h(2a+b+h)}{2(a+b+h)}s_4\right) \\
Q_2(s_5) &= Et\int_0^{s_5}(-\frac{h}{2}-x_{3c})ds \\
&= Et\left(-\frac{h(b+h)}{2(a+b+h)}s_5\right) \\
Q_2(s_6) &= Et\int_0^{s_6}(-\frac{h}{2}-x_{3c})ds \\
&= Et\left(-\frac{h(b+h)}{2(a+b+h)}s_6\right)
\end{aligned}$$

$$\begin{aligned}
f_o(s_1) &= \frac{Et\,V_3}{H_{22}^c}\frac{h(b+h)}{2(a+b+h)}s_1 \\
f_o(s_2) &= \frac{Et\,V_3}{H_{22}^c}\left(-\frac{s_2^2}{2}+\frac{h(b+h)}{2(a+b+h)}s_2+\frac{ah(b+h)}{2(a+b+h)}\right) \\
f_o(s_3) &= \frac{Et\,V_3}{H_{22}^c}\left(-\frac{h(2a+b+h)}{2(a+b+h)}s_3+\frac{abh}{2(a+b+h)}\right) \\
f_o(s_4) &= \frac{Et\,V_3}{H_{22}^c}\left(\frac{s_4^2}{2}-\frac{h(2a+b+h)}{2(a+b+h)}s_4-\frac{bh}{2}\right) \\
f_o(s_5) &= \frac{Et\,V_3}{H_{22}^c}\frac{h(b+h)}{2(a+b+h)}(s_5-b) \\
f_o(s_6) &= \frac{Et\,V_3}{H_{22}^c}\frac{h(b+h)}{2(a+b+h)}(s_6-a)
\end{aligned}$$

$f_o(s_4=h)=f_o(s_5=0)+f_o(s_6=0)$도 성립함을 알 수 있다. (8.29)

$$f_c=-\frac{\int_C \frac{f_0(s)}{Gt}ds}{\int_C \frac{1}{Gt}ds} \qquad (8.45)$$

$$\int_C \frac{f_o(s)}{Gt}ds=\frac{1}{Gt}\left\{\int_0^h f(s_2)ds_2+\int_0^b f(s_3)ds_3+\int_0^h f(s_4)ds_4+\int_0^b f(s_5)ds_5\right\}$$
$$=-\frac{EV_3}{GH_{22}^c}\frac{bh(b+h)^2}{2(a+b+h)}$$

$$\int_C \frac{1}{Gt}ds=\frac{1}{Gt}\left\{\int_0^h ds_2+\int_0^b ds_3+\int_0^h ds_4+\int_0^b ds_5\right\}$$
$$=\frac{2(b+h)}{Gt}$$

$$f_c=\frac{Et\,V_3}{H_{22}^c}\frac{bh(b+h)}{4(a+b+h)}=9.0452\times10^4[N/m]$$

$$f=f_o+f_c$$
$$\tau=f/t$$

답

$$\tau(s_1)=\frac{EV_3}{H_{22}^c}\frac{h(b+h)}{2(a+b+h)}s_1$$

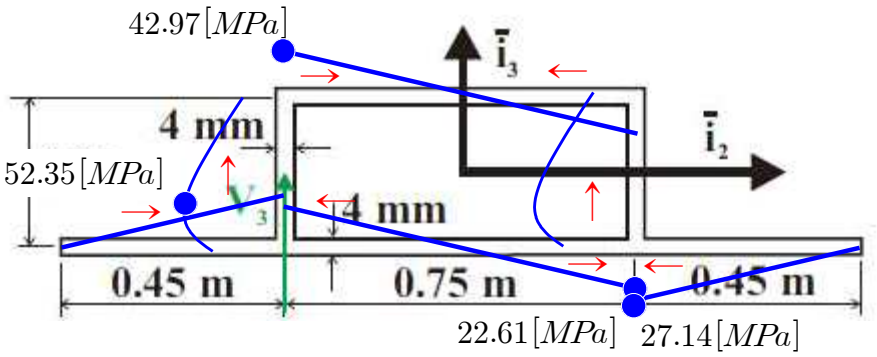
$$\tau(s_2)=\frac{EV_3}{H_{22}^c}\left\{-\frac{s_2^2}{2}+\frac{h(b+h)}{2(a+b+h)}s_2+\frac{h(b+h)(2a+b)}{4(a+b+h)}\right\}$$

$$\tau(s_3)=\frac{EV_3}{H_{22}^c}\left\{-\frac{h(2a+b+h)}{2(a+b+h)}s_3+\frac{bh(2a+b+h)}{4(a+b+h)}\right\}$$

$$\tau(s_4)=\frac{EV_3}{H_{22}^c}\left\{\frac{s_4^2}{2}-\frac{h(2a+b+h)}{2(a+b+h)}s_4-\frac{bh(2a+b+h)}{4(a+b+h)}\right\}$$

$$\tau(s_5)=\frac{EV_3}{H_{22}^c}\left\{\frac{h(b+h)}{2(a+b+h)}s_5-\frac{bh(b+h)}{4(a+b+h)}\right\}$$

$$\tau(s_6)=\frac{EV_3}{H_{22}^c}\left\{\frac{h(b+h)}{2(a+b+h)}s_6-\frac{ah(b+h)}{2(a+b+h)}\right\}$$



(2)

과정

$$H_{11}^{strip}=\frac{Glt^3}{3} \qquad (7.64)$$
$$=\frac{Gat^3}{3}=288[Nm^2/rad]$$

$$H_{11}^{box}=\frac{4GtA^2}{l} \qquad (8.68)$$
$$=\frac{2Gtb^2h^2}{b+h}=8.4375\times10^6[Nm^2/rad]$$

$$H_{11}=2H_{11}^{strip}+H_{11}^{box}$$
$$=2Gt\left(\frac{at^2}{3}+\frac{b^2h^2}{b+h}\right)=8.438\times10^6[Nm^2/rad]$$

Ch.8.8에서 Shear center와 twist center는 같고 $\vec{i_3}$축 위에 있다.

따라서 V_3에 의한 twist center에 작용하는 회전력의 moment arm은 $b/2$이다.

$$M_1=-\frac{bV_3}{2}=-37500[Nm]$$
$$\kappa_1=M_1/H_{11}=-0.0044[rad/m]$$

$$M_1^{strip}=H_{11}^{strip}\kappa_1=-1.2799[Nm]$$
$$M_1^{box}=H_{11}^{box}\kappa_1=-3.7497\times10^4[Nm]$$

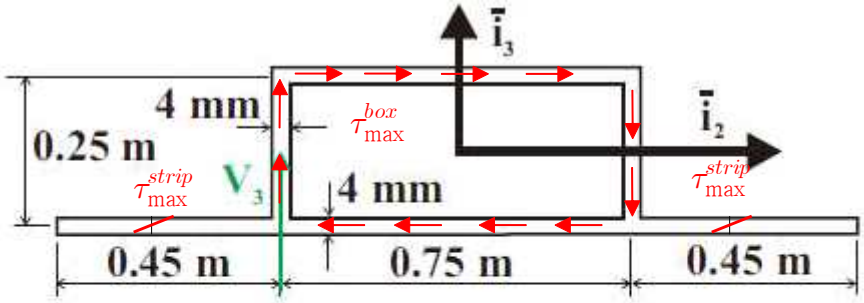
$$\tau_{\max}^{strip}=Gt\kappa_1 \qquad (7.62)$$
$$=-5.333\times10^5[Pa]$$

$$\tau_{\max}^{box}=\frac{M_1^{box}}{2At} \qquad (8.62)$$
$$=-2.4998\times10^7[Pa]$$

답

$\tau^{strip}(r) = -5.333 \times 10^5 \frac{r}{2t} [Pa]$

$\tau^{box} = -24.998 [MPa]$



과정

$\tau^{strip}_{\max}$는 다른 성분들의 1/100이므로 무시

(3)

답

$$\tau(s_1) = \frac{EV_3}{H^c_{22}} \frac{h(b+h)}{2(a+b+h)} s_1$$

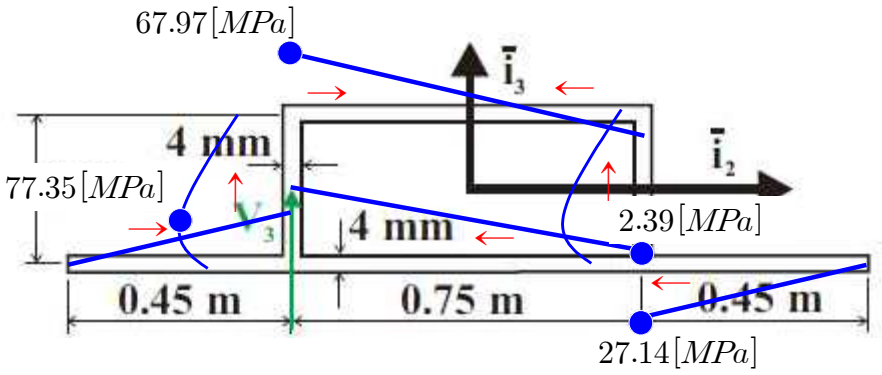
$$\tau(s_2) = \frac{EV_3}{H^c_{22}} \left\{ -\frac{s_2^2}{2} + \frac{h(b+h)}{2(a+b+h)} s_2 + \frac{h(b+h)(2a+b)}{4(a+b+h)} \right\} - \tau^{box}$$

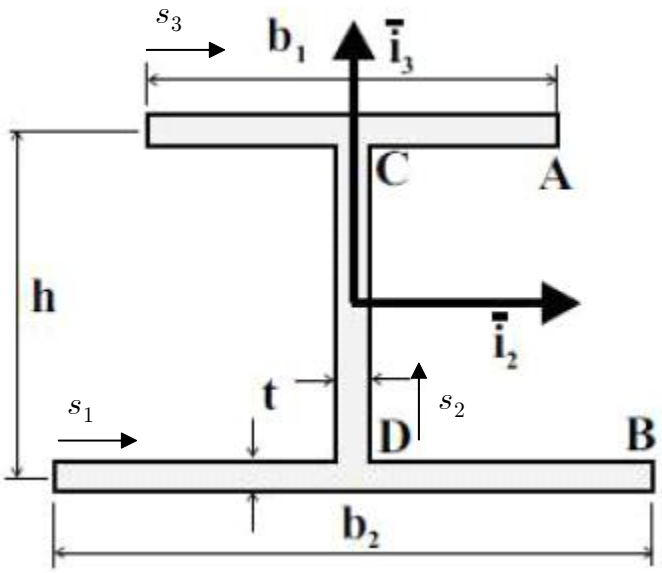
$$\tau(s_3) = \frac{EV_3}{H^c_{22}} \left\{ -\frac{h(2a+b+h)}{2(a+b+h)} s_3 + \frac{bh(2a+b+h)}{4(a+b+h)} \right\} - \tau^{box}$$

$$\tau(s_4) = \frac{EV_3}{H^c_{22}} \left\{ \frac{s_4^2}{2} - \frac{h(2a+b+h)}{2(a+b+h)} s_4 - \frac{bh(2a+b+h)}{4(a+b+h)} \right\} - \tau^{box}$$

$$\tau(s_5) = \frac{EV_3}{H^c_{22}} \left\{ \frac{h(b+h)}{2(a+b+h)} s_5 - \frac{bh(b+h)}{4(a+b+h)} \right\} - \tau^{box}$$

$$\tau(s_6) = \frac{EV_3}{H^c_{22}} \left\{ \frac{h(b+h)}{2(a+b+h)} s_6 - \frac{ah(b+h)}{2(a+b+h)} \right\}$$



	(4)	답 (3)의 그림 참고 $s_2 = \frac{h(b+h)}{2(a+b+h)} = 0.0862[m]$ $\tau_{\max} = 77.35[MPa]$
8.63	(1)	과정  Centroid $S = Et(b_1 + b_2 + h)$ $S_3 = \frac{Eth}{2}(b_1 - b_2)$ $x_{2c} = 0$(symmetry about $\vec{i_3}$), $x_{3c} = \frac{h(b_1 - b_2)}{2(b_1 + b_2 + h)}$ Centroid 위에 Principal coordinate 설정 ($\vec{i_{3p}} = \vec{i_3}$) Centroid로부터 각 segment까지의 거리 $r_o(s_1) = \frac{h}{2} + x_{3c}$ $r_o(s_2) = 0$ $r_o(s_3) = \frac{h}{2} - x_{3c}$

$$\frac{d\Gamma(s)}{ds} = -r_o \qquad (8.86)$$

$$\Gamma(s_1) = -\left(\frac{h}{2} + x_{3c}\right)s_1 + C_1$$

$$\Gamma(s_2) = C_2$$

$$\Gamma(s_3) = -\left(\frac{h}{2} - x_{3c}\right)s_3 + C_3$$

$C_2 = 0$으로 두면, $\Gamma(s_2) = 0$

$\Gamma(s_1 = \frac{b_2}{2}) = \Gamma(s_2 = 0)$, $\Gamma(s_3 = \frac{b_1}{2}) = \Gamma(s_2 = h)$이므로

$$\Gamma(s_1) = -\left(\frac{h}{2} + x_{3c}\right)(s_1 - \frac{b_2}{2}),\ \Gamma(s_2) = 0,\ \Gamma(s_3) = -\left(\frac{h}{2} - x_{3c}\right)(s_3 - \frac{b_1}{2})$$

$$c_1 = -\frac{1}{S}\int_C E\Gamma t ds \qquad (8.88)$$
$$= 0$$

$$x_{2r} = -\frac{1}{H_{22}^c}\int_C E\Gamma x_3 t ds \qquad (8.89)\ \text{(symmetry about } \vec{i_3}\text{)}$$
$$= 0$$

$$x_{3r} = -\frac{1}{H_{33}^c}\int_C E\Gamma x_2 t ds \qquad (8.90)$$
$$= \frac{Et}{12H_{33}^c}\left\{\left(\frac{h}{2} + x_{3c}\right)b_2^3 + \left(\frac{h}{2} - x_{3c}\right)b_1^3\right\}$$

$$\Psi(s) = \Gamma(s) + x_{2r}x_3 - x_{3r}x_2 + c_1 \qquad (8.87)$$

$$\Psi(s_1) = -\left(\frac{h}{2} + x_{3c}\right)(s_1 - \frac{b_2}{2}) - x_{3r}(s_1 - \frac{b_2}{2})$$

$$\Psi(s_2) = 0$$

$$\Psi(s_3) = -\left(\frac{h}{2} - x_{3c}\right)(s_3 - \frac{b_1}{2}) - x_{3r}(s_3 - \frac{b_1}{2})$$

답

$$\Psi(s_1) = -\left(\frac{h}{2} + x_{3c} + x_{3r}\right)(s_1 - \frac{b_2}{2})$$

$$\Psi(s_2) = 0$$

$$\Psi(s_3) = -\left(\frac{h}{2} - x_{3c} + x_{3r}\right)(s_3 - \frac{b_1}{2})$$

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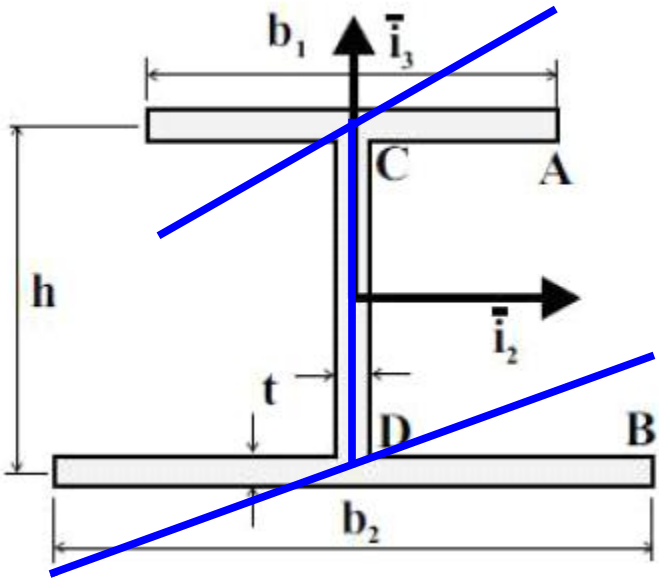
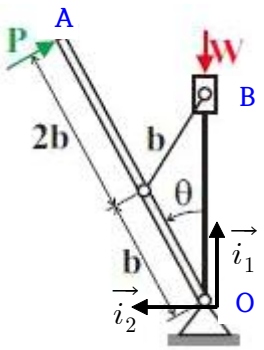
<tr><td></td><td></td><td> 
</td></tr>
<tr><td></td><td>(2)</td><td>답
$x_{2r}=0$
$x_{3r}=\dfrac{Et}{12H_{33}^{c}}\left\{\left(\dfrac{h}{2}+x_{3c}\right)b_2^3+\left(\dfrac{h}{2}-x_{3c}\right)b_1^3\right\}$
$=\dfrac{h\{(2b_1+h)b_2^3+(2b_2+h)b_1^3\}}{2(b_1^3+b_2^3)(b_1+b_2+h)}$</td></tr>
<tr><td></td><td>(3)</td><td>답
$H_{11}=\sum\dfrac{Glt^3}{3}=\dfrac{Gt^3(b_1+b_2+h)}{3}$</td></tr>
<tr><td>9.5</td><td></td><td>과정
 

$\underline{u_A}=3b\cos\theta\vec{i_1}+3b\sin\theta\vec{i_2}$
$\underline{u_B}=2b\cos\theta\vec{i_1}$</td></tr>
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$$\underline{\delta u_A} = \underline{u_A}(\theta+\delta\theta) - \underline{u_A}(\theta)$$
$$= 3b\,(\cos(\theta+\delta\theta)-\cos\theta)\vec{i_1} + 3b\,(\sin(\theta+\delta\theta)-\sin\theta)\vec{i_2}$$
$$= 3b(\cos\theta\cos\delta\theta - \sin\theta\sin\delta\theta - \cos\theta)\vec{i_1}$$
$$+ 3b(\sin\theta\cos\delta\theta + \cos\theta\sin\delta\theta - \sin\theta)\vec{i_2}$$
$$= (-3b\sin\theta\vec{i_1} + 3b\cos\theta\vec{i_2})\delta\theta$$

($|\delta\theta| \ll 1$이므로 $\sin\delta\theta = 0,\ \cos\delta\theta = 1$)

$$\underline{\delta u_B} = 2b(\cos(\theta+\delta\theta)-\cos\theta)\vec{i_1}$$
$$= (-2b\sin\theta\vec{i_1})\delta\theta$$

$$\underline{P} = P\sin\theta\vec{i_1} - P\cos\theta\vec{i_2}$$
$$\underline{W} = -W\vec{i_1}$$

$$\delta W_E = \underline{P}\cdot\underline{\delta u_A} + \underline{W}\cdot\underline{\delta u_B} \qquad (9.17)$$
$$= (-3bP + 2bW\sin\theta)\delta\theta = 0$$

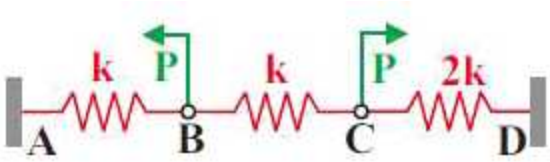
$$-3bP + 2bW\sin\theta = 0$$

답

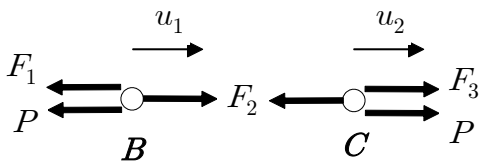
$$P = \frac{2\sin\theta}{3}W$$

9.13 (1)

과정



① FBD & force equilibrium



$F_1 + P = F_2$ ... (1)

$F_2 = F_3 + P$ ... (2)

② Strain(deformation)-displacement equation

e_1: spring AB의 인장된 길이, $e_1 = u_1$

e_2: spring BC의 인장된 길이, $e_2 = u_2 - u_1$

e_3: spring CD의 인장된 길이, $e_3 = -u_2$

$e_1 - u_1 = 0$ ... (3)

$e_2 + u_1 - u_2 = 0$ ... (4)

$e_3 + u_2 = 0$ ... (5)

③ CVW

식(3)~(5)에 δF를 곱하여 complementary virtual work

$$\delta W' = -(e_1 - u_1)\delta F_1 - (e_2 + u_1 - u_2)\delta F_2 - (e_3 + u_2)\delta F_3 \quad \text{... (6)}$$
$$= -e_1\delta F_1 - e_2\delta F_2 - e_3\delta F_3 + (\delta F_1 - \delta F_2)u_1 + (\delta F_2 - \delta F_3)u_2 = 0$$

④ Compatibility equation

식(6)에서 statically admissible 가정

$\delta F_1 = \delta F_2$ ... (7)

$\delta F_2 = \delta F_3$ ... (8)

식(6)에 식(7)~(8)을 사용

$\delta W' = -e_1\delta F_1 - e_2\delta F_2 - e_3\delta F_3 = \delta W_I = 0$

$\delta W' = -(e_1 + e_2 + e_3)\delta F_2 = 0$ ... (9)

Compatibility equation $e_1 + e_2 + e_3 = 0$ ... (10)

⑤ Constitutive law

$F_1 = ke_1$ ... (11)

$F_2 = ke_2$ ... (12)

$F_3 = 2ke_3$ ... (13)

⑥ Solve for force

식(10)에 식(11)~(13) 대입

$$\frac{F_1}{k} + \frac{F_2}{k} + \frac{F_3}{2k} = 0,$$

$2F_1 + 2F_2 + F_3 = 0$ ... (14)

식(1)~(2)을 F_2에 대하여 푼 뒤, 식(14)에 대입

$F_1 = F_2 - P,\ F_3 = F_2 - P,$

$5F_2 - 3P = 0$

$$F_2 = \frac{3}{5}P$$

나머지 힘 $F_1 = -\frac{2}{5}P,\ F_3 = -\frac{2}{5}P$

답

$F_1 = -\frac{2}{5}P,\ F_2 = \frac{3}{5}P,\ F_3 = -\frac{2}{5}P$

(2)

과정

① FBD & force equilibrium

$F_1 + P = F_2$ ... (1)

$F_2 = F_3 + P$ ... (2)

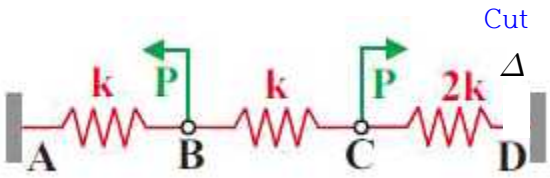
② Degrees of redundancy

미지수 $F_1,\ F_2,\ F_3$ 3개

식 (1), (2) 2개

$N_R = 1$

③ Cut



④ Redundant force

F_2를 redundant force로 선정

식(1)~(2)에서

$F_1 = F_2 - P$ ... (1')

$F_3 = F_2 - P$ ... (2')

⑤ Constitutive law

$F_1 = ke_1 \rightarrow e_1 = \frac{F_2 - P}{k}$ ... (3)

$F_2 = ke_2 \rightarrow e_2 = \frac{F_2}{k}$ ... (4)

$F_3 = 2ke_3 \rightarrow e_3 = \frac{F_2 - P}{2k}$ ... (5)

		⑥ Strain(deformation)-displacement equation $e_1 = u_1$ ... (6) $e_2 = u_2 - u_1$ ... (7) $e_3 = -u_2 - \Delta$ ... (8) 식(6)~(8)에서 $\Delta = -(e_1 + e_2 + e_3)$ ... (9) 식(3)~(5)를 식(9)에 대입 $\Delta = -(\frac{F_2 - P}{k} + \frac{F_2}{k} + \frac{F_2 - P}{2k})$ ... (10) $= \frac{-5F_2 + 3P}{2k}$ ⑦ Solve redundant force 식(10)에서 $\Delta = 0$ $F_2 = \frac{3}{5}P$ ⑧ Solve others $F_1 = -\frac{2}{5}P,\ F_3 = -\frac{2}{5}P$
		답 $F_1 = -\frac{2}{5}P,\ F_2 = \frac{3}{5}P,\ F_3 = -\frac{2}{5}P$
	(3)	답 redundancy를 줄이는 compatibility equation을 사용하는 공통점이 있다.