2016-2 항공기구조역학 HW6

기한 : 2016/12/16 18:00 장소 : 301동 1357호 제출함

Problem 9.14. Deflection of a simple square truss

Consider the square planar truss shown in fig. 9.44 and assume that all bars are of cross-sectional area, A, and modulus, E. Joints **D**, **E**, and **F** are pinned to the ground. This problem presents symmetries that may be helpful in simplifying the force calculations. *(1)* Find the vertical deflection at joint **A**. *(2)* Find the increase in horizontal distance between joints **B** and **C**.

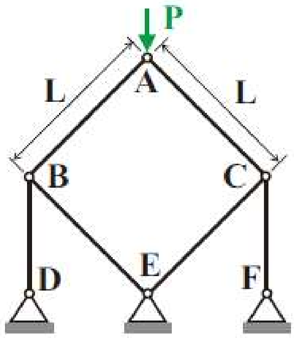


Fig. 9.44. Simple square planar truss with vertical load at joint **A**.

Problem 9.28. Redundant planar frame with tip load

Consider the cantilevered beam consisting of two segments of length L connected at a 90 degree angle, as shown in fig. 9.76. A simple support is located at point **B**, and a horizontal load, P, is applied at point **A**. *(1)* Find the magnitude and location of the maximum bending moment in the bent beam. *(2)* Find the horizontal deflection at point **A**.

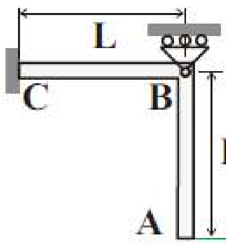


Fig. 9.76. Planar right angle frame with tip load.

Problem 10.2. Lever with sliding pivots

Bar **ABC** is of length $b + a$ and is constrained to move vertically at point **A** and horizontally at **B**, while a horizontal force, P, is applied at point **C**, as depicted in fig. 10.14. Point **A** is restrained by a vertical spring of stiffness constant k, which is relaxed when angle $\theta = 0$. Use the principle of minimum total potential energy to determine the equilibrium configurations of the system.

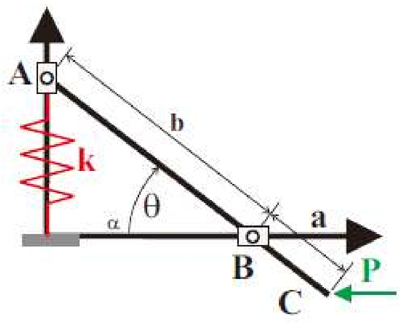


Fig. 10.14. Lever with spring-restrained sliding pivots.

Problem 10.6. Planar 3-bar truss

The hyperstatic, three bar truss depicted in fig. 10.18 is subjected to a load, P, applied at joint **A**, with a line of action at an angle $\theta = 45$ degrees with respect to the horizontal. All bars have the same Young's modulus, E, and cross-sectional area, $\mathcal{A}$. *(1)* Determine the displacement components, u_1 and u_2, of joint **A**. *(2)* Find the elongations in each bar. *(3)* Evaluate the forces in each bar.

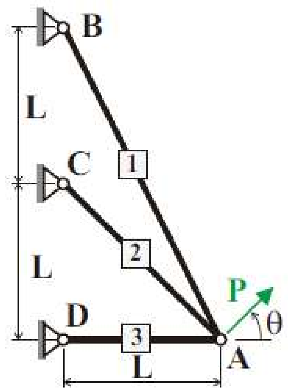
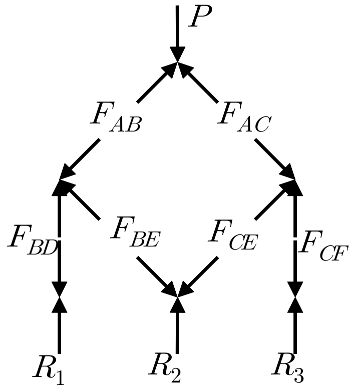


Fig. 10.18. Planar 3-bar truss with load applied at joint A.

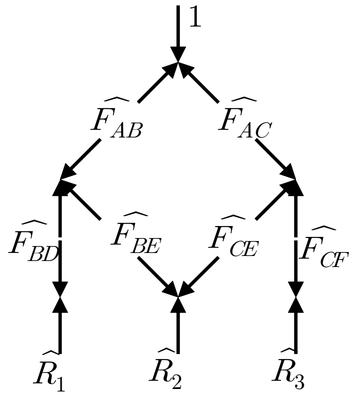
HW6_sol

문제	소문제	풀이
9.14	(1)	(see below)

과정



Free body diagram
- force equilibrium



Free body diagram
- unit load for A

수직 방향 force equilibrium

$$-P+\frac{\sqrt{2}}{2}F_{AB}+\frac{\sqrt{2}}{2}F_{AC}=0 \qquad -1+\frac{\sqrt{2}}{2}\widehat{F_{AB}}+\frac{\sqrt{2}}{2}\widehat{F_{AC}}=0$$

$$-\frac{\sqrt{2}}{2}F_{AB}+F_{BD}+\frac{\sqrt{2}}{2}F_{BE}=0 \qquad -\frac{\sqrt{2}}{2}\widehat{F_{AB}}+\widehat{F_{BD}}+\frac{\sqrt{2}}{2}\widehat{F_{BE}}=0$$

$$-\frac{\sqrt{2}}{2}F_{AC}+\frac{\sqrt{2}}{2}F_{CE}+F_{CF}=0 \qquad -\frac{\sqrt{2}}{2}\widehat{F_{AC}}+\frac{\sqrt{2}}{2}\widehat{F_{CE}}+\widehat{F_{CF}}=0$$

수평 방향 force equilibrium

$$\frac{\sqrt{2}}{2}F_{AB}-\frac{\sqrt{2}}{2}F_{AC}=0 \qquad \frac{\sqrt{2}}{2}\widehat{F_{AB}}-\frac{\sqrt{2}}{2}\widehat{F_{AC}}=0$$

$$-\frac{\sqrt{2}}{2}F_{AB}-\frac{\sqrt{2}}{2}F_{BE}=0 \qquad -\frac{\sqrt{2}}{2}\widehat{F_{AB}}-\frac{\sqrt{2}}{2}\widehat{F_{BE}}=0$$

$$\frac{\sqrt{2}}{2}F_{AC}+\frac{\sqrt{2}}{2}F_{CE}=0 \qquad \frac{\sqrt{2}}{2}\widehat{F_{AC}}+\frac{\sqrt{2}}{2}\widehat{F_{CE}}=0$$

$\hat{F}$는 F/P로 구해도 됨

Bar	F	$\hat{F}$	$e=LF/EA$
AB	$\sqrt{2}/2P$	$\sqrt{2}/2$	$\sqrt{2}/2(LP/EA)$
AC	$\sqrt{2}/2P$	$\sqrt{2}/2$	$\sqrt{2}/2(LP/EA)$
BD	P	1	$\sqrt{2}/2(LP/EA)$
BE	$-\sqrt{2}/2P$	$-\sqrt{2}/2$	$-\sqrt{2}/2(LP/EA)$
CE	$-\sqrt{2}/2P$	$-\sqrt{2}/2$	$-\sqrt{2}/2(LP/EA)$
CF	P	1	$\sqrt{2}/2(LP/EA)$

$$\Delta A=\sum\frac{\hat{F}FL}{EA} \qquad (9.65)$$

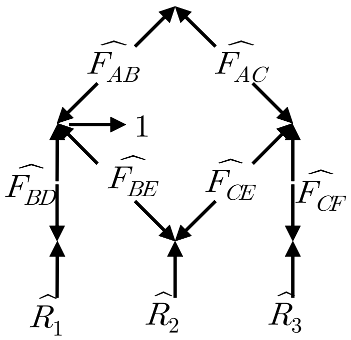
$$=(2+\sqrt{2})\frac{LP}{EA}$$

답

아래쪽으로 $(2+\sqrt{2})\dfrac{LP}{EA}$

(2)

과정



Free body diagram
- unit load for A

수직 방향 force equilibrium

$$\frac{\sqrt{2}}{2}\widehat{F_{AB}}+\frac{\sqrt{2}}{2}\widehat{F_{AC}}=0$$

$$-\frac{\sqrt{2}}{2}\widehat{F_{AB}}+\widehat{F_{BD}}+\frac{\sqrt{2}}{2}\widehat{F_{BE}}=0$$

$$-\frac{\sqrt{2}}{2}\widehat{F_{AC}}+\frac{\sqrt{2}}{2}\widehat{F_{CE}}+\widehat{F_{CF}}=0$$

수평 방향 force equilibrium

$$\frac{\sqrt{2}}{2}\widehat{F_{AB}}-\frac{\sqrt{2}}{2}\widehat{F_{AC}}=0$$

$$1-\frac{\sqrt{2}}{2}\widehat{F_{AB}}-\frac{\sqrt{2}}{2}\widehat{F_{BE}}=0$$

$$\frac{\sqrt{2}}{2}\widehat{F_{AC}}+\frac{\sqrt{2}}{2}\widehat{F_{CE}}=0$$

Bar	F	$\hat{F}$	$e=LF/EA$
AB	$\sqrt{2}/2P$	0	$\sqrt{2}/2(LP/EA)$
AC	$\sqrt{2}/2P$	0	$\sqrt{2}/2(LP/EA)$
BD	P	-1	$\sqrt{2}/2(LP/EA)$
BE	$-\sqrt{2}/2P$	$\sqrt{2}$	$-\sqrt{2}/2(LP/EA)$
CE	$-\sqrt{2}/2P$	0	$-\sqrt{2}/2(LP/EA)$
CF	P	0	$\sqrt{2}/2(LP/EA)$

$$\Delta B=\sum\frac{\hat{F}FL}{EA}$$
$$=-(1+\frac{\sqrt{2}}{2})\frac{LP}{EA}$$

답

왼쪽으로 $(2+\sqrt{2})\dfrac{LP}{EA}$

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<td>9.28</td>
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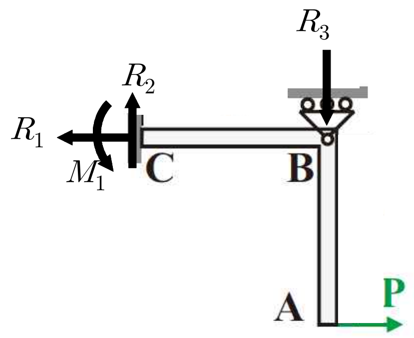
과정



Free body diagram

Force-moment equilibrium

수평 $P - R_1 = 0$

수직 $R_2 - R_3 = 0$

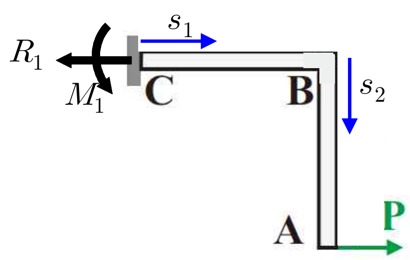
회전 $M_1 + PL - R_3 L = 0$

미지수 R_1, R_2, R_3, M_1 4개

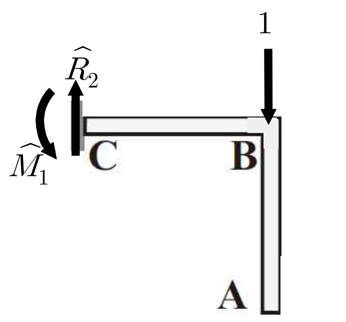
식 3개

$N_R = 1$

Isostatic system+Unit load force system



Isostatic system



Unit load force system

Isostatic system

$P - R_1 = 0$

$M_1 + PL = 0$

$\rightarrow R_1 = P,\ M_1 = -PL$

$M(s_1) = -M_1 = PL$

$M(s_2) = -M_1 - R_1 L = -Ps_1 + PL$

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Unit load force system

$$\widehat{R}_2 - 1 = 0$$

$$\widehat{M}_1 - 1L = 0$$

$$\rightarrow \ \widehat{R}_2 = 1, \ \widehat{M}_1 = L$$

$$\widehat{M}(s_1) = -\widehat{M}_1 + \widehat{R}_2 s_1 = s_1 - L$$

$$\widehat{M}(s_2) = 0$$

R_3 구하기

Isostatic system에서의 displacement $\Delta B_I = \int \frac{\widehat{M}M}{H} ds \qquad (9.91)$

$$= -\frac{PL^3}{2H}$$

Unit load force system에서의 displacement $\Delta B_U = \int \frac{(\widehat{M})^2}{H} ds$

$$= \frac{L^3}{3H}$$

$$R_3 = -\frac{\Delta B_I}{\Delta B_U} \qquad (9.94)$$

$$= \frac{3}{2}P$$

다시 force equilibrium

$$R_1 = P, \ R_2 = R_3 = \frac{3}{2}P, \ M_1 = \frac{PL}{2}$$

Moment distribution

$$M(s_1) = R_2 s_1 - M_1 = \frac{3}{2}Ps_1 - \frac{1}{2}PL$$

$$M(s_2) = -R_1 s_2 + R_3 L - M_1 = -Ps_2 + PL$$

답

점B에서 PL

(2)

과정

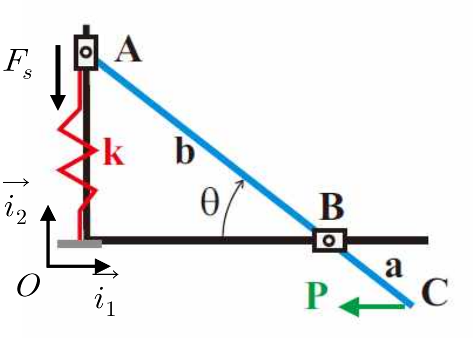
$P = 1$일 때,

$$\widehat{R_{1P}} = 1$$

$$\widehat{M_P}(s_1) = \frac{3}{2}s_1 - \frac{1}{2}L$$

$$\widehat{M_P}(s_2) = -s_2 + L$$

Bar BC의 인장 $\Delta A_1 = \frac{\widehat{R_{1P}} R_1 L}{EA} = \frac{PL}{EA}$

		점 C의 회전에 의한 변위 $M(s_1) = H\dfrac{d^2u}{ds_1^2}$, $\dfrac{du}{ds_1} = \dfrac{P(3s_1^2 - Ls_1)}{4H}$ $\left.\dfrac{du}{ds_1}\right\vert_{s_1 = L} = \dfrac{PL^2}{2H}$ $\Delta A_2 = \left.\dfrac{du}{ds_1}\right\vert_{s_1 = L} \times L = \dfrac{PL^3}{2H}$ Bar AB의 휨 $\Delta A_3 = \displaystyle\int_0^L \dfrac{\widehat{M}_P M}{H} ds_2 = \dfrac{PL^3}{3H}$ $\Delta A = \Delta A_1 + \Delta A_2 + \Delta A_3$ $\quad = \dfrac{PL}{EA} + \dfrac{5PL^3}{6H}$
		답 $\Delta A = \dfrac{PL}{EA} + \dfrac{5PL^3}{6H}$
10.2		과정  $\vec{u}_C = \overrightarrow{OC} = (a+b)\cos\theta \vec{i}_1 - a\sin\theta \vec{i}_2$ $\vec{u}_A = \overrightarrow{OA} = b\sin\theta \vec{i}_2$ $\vec{P} = -P\vec{i}_1$ $\vec{F}_s = -F_s\vec{i}_2 = -kb\sin\theta \vec{i}_2$ $\Pi = A + \Phi = \sum \dfrac{ke^2}{2} - \sum \vec{P} \cdot \vec{u}$ $\quad = \dfrac{k}{2}(b\sin\theta)^2 + P(a+b)\cos\theta$ $\dfrac{\partial \Pi}{\partial \theta} = kb^2 \sin\theta\cos\theta - P(a+b)\sin\theta = 0$

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$$P=\frac{kb^2\cos\theta}{a+b}$$

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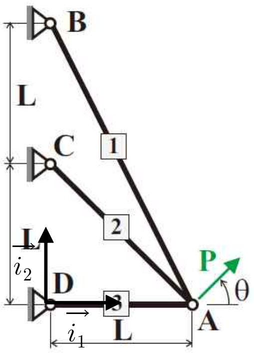
과정



힘 벡터 $\vec{P}=P\cos\theta\vec{i_1}+P\sin\theta\vec{i_2}$

점A의 변위 $\vec{u_A}=u_1\vec{i_1}+u_2\vec{i_2}$

Bar의 인장

$$e_1=\frac{\sqrt{5}}{5}u_1-\frac{2\sqrt{5}}{5}u_2$$

$$e_2=\frac{\sqrt{2}}{2}u_1-\frac{\sqrt{2}}{2}u_2$$

$$e_3=u_1$$

$$\begin{aligned}A&=\sum\frac{k}{2}e^2\\&=\frac{1}{2}\frac{EA}{\sqrt{5}L}\left(\frac{\sqrt{5}}{5}u_1-\frac{2\sqrt{5}}{5}u_2\right)^2+\frac{1}{2}\frac{EA}{\sqrt{2}L}\left(\frac{\sqrt{2}}{2}u_1-\frac{\sqrt{2}}{2}u_2\right)^2+\frac{1}{2}\frac{EA}{L}u_1^2\\&=\frac{EA}{2L}\left\{u_1^2+\frac{\sqrt{2}}{4}(u_1-u_2)^2+\frac{\sqrt{5}}{25}(u_1-2u_2)^2\right\}\end{aligned}$$

(10.21)

$$\begin{aligned}\Phi&=-\sum\vec{P}\bullet\vec{u}\\&=-Pu_1\cos\theta-Pu_2\sin\theta\end{aligned}$$

$$\Pi=A+\Phi$$

$$\frac{\partial\Pi}{\partial u_1}=\frac{EA}{2L}\left\{2u_1+\frac{\sqrt{2}}{2}(u_1-u_2)+\frac{2\sqrt{5}}{25}(u_1-2u_2)\right\}-P\cos\theta=0$$

$$\frac{\partial\Pi}{\partial u_1}=\frac{EA}{2L}\left\{-\frac{\sqrt{2}}{2}(u_1-u_2)-\frac{4\sqrt{5}}{25}(u_1-2u_2)\right\}-P\sin\theta=0$$

$$\begin{bmatrix}1+\frac{\sqrt{2}}{4}+\frac{\sqrt{5}}{25} & -\frac{\sqrt{2}}{4}-\frac{2\sqrt{5}}{25}\\ -\frac{\sqrt{2}}{4}-\frac{2\sqrt{5}}{25} & \frac{\sqrt{2}}{4}+\frac{4\sqrt{5}}{25}\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}=\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}\frac{PL}{EA}$$

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		답 $u_1 = 1.1838\frac{PL}{EA}$, $u_1 = 1.8801\frac{PL}{EA}$
	(2)	답 $e_1 =- 1.1523\frac{PL}{EA}$, $e_2 =- 0.4924\frac{PL}{EA}$, $e_3 = 1.1838\frac{PL}{EA}$
	(3)	답 $F_1 =- 0.5153P$, $F_2 =- 0.3482P$, $F_3 = 1.1838P$