High Performance Concrete Engineering 2017-23536 박기슬 1

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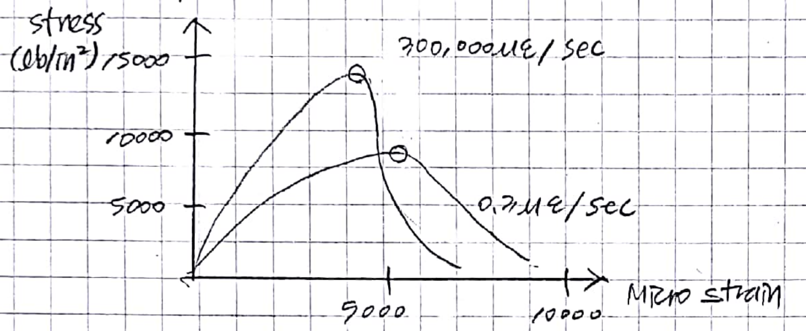
HW#5 (Deadline Nov 13th)

(a) What effect does increasing the rate of loading have on (i) concrete strength (ii) the strain corresponding to peak stress? [10 marks]

(i) It is agreed that the higher the rate of loading, the higher the observed strength and the elastic modulus
(ii) As the loading rate increases, the strain corresponding to peak stress decreases.



(Harsh. et al, 1989)

(b) Why does concrete response as described in answer to Prob. (a)? [10 marks]

There are two causes for strain rate sensitivity of concrete mechanical properties :

① The behavior of cement paste as viscoelastic material controlled by the viscous time-dependent movement of free water through voids and pores.
② The time-dependent nature of crack growth relative to the loading rate

→ It should be noted that the strain rate sensitivity of concrete is greatest when the materials are fully saturated and least when the materials are dry

→ Crack growth plays a role in strain rate response because cracks requires a finite time to propagate.
At high rates of loading, when cracks propagate much more slowly than the applied stress, the crack path is altered and crack length is shortened, which translates into improved compressive, tensile, and fracture properties at increasing load rates.

(c) Explain how ITZ of lightweight concrete can be denser than that of normal weight concrete. [10 marks]

The microstructure of the interfacial transition zone (ITZ) between cement paste and aggregates depends on the nature of aggregate, specifically its porousity and water absorption.
The substitution of lightweight aggregate (LWA) provides internal curing, thereby a nearly continuous microstructure is observed near LWA particle and partially penetrates into the LWA surface pores.

⊗ For normal weight aggregates, due to the size difference between cement & aggregates, a "wall effect" exists, so that there is a deficiency of cement particle near the aggregate surface relative to their concentration in non ITZ cement paste.
Due to wall effect that causes inefficient packing of cement particles near the aggregates, the ITZ regions will initially have a higher w/c ratio and a larger interparticle spacing than bulk cement paste. If sufficient water is not available at early ages, the concrete undergo self-dessication, with bulk cement paste regions imbibing water from the largest pores with in the ITZ, resulting in less hydration, greater porosity and larger empty pores being present in the ITZ

To sum up, adding pre-wetted lightweight aggregates that readily release water as needed for hydration, or to replace moisture lost through evaporation or self-dessication, enables internal curing and leads to denser microstructure of ITZ.

(d) Among below three cases, which pore system will have the weakest durability performance? Black area is open pore. [10 marks]

- porosity = (i) < (ii) < (iii)
- permeability = (i) < (iii) < (ii)
- tortuosity = (ii) < (iii)

Although the porosity of (iii) is greater than (ii), penetration of ion, etc. will be more difficult than that of B because of complexity of the connection between pores.
Therefore (ii) is the least durable, and (i), which is not connected between pores, is the most durable.

(e) Derive below equation for the critical tensile stress of fiber-reinforced cement composite. Notation is given in lecture slide. [20 marks]

$$\sigma_{cc} = \sigma_{mu}(1-V_f) + \alpha\tau V_f \frac{L}{d}$$

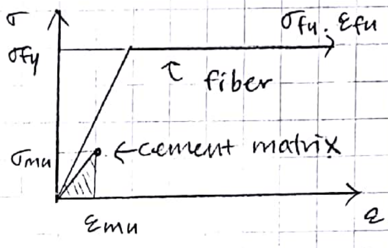
Under the loading, the total load sustained by the composite F_c,

$$F_c = F_m + F_f \quad - (1)$$

F_m : carried by matrix phase
F_f : carried by fiber phase

At the first crack, the stress of matrix phase reaches to the ultimate strength σ_{mu} while the fiber has not yet been fully utilized.

From the defintion of stress, $F=\sigma A$
equation (1) yields



$$\sigma_{cc}A_c = \sigma_{mu} A_m + \sigma_f A_f \quad -(2)$$

$$\sigma_{cc} = \sigma_{mu} \frac{A_m}{A_c} + \sigma_f \frac{A_f}{A_c}$$

$$= \sigma_{mu}(1-V_f) + \sigma_f V_f \quad (3)$$

if composite, matrix, fiber phase lengths are all equal the area fraction is equivalent to volume fraction.

Consider stress transfer in fibers to calculate σ_f

Force transferred through fiber by bond stress scheme is as shown. Since the bond-slip relation shows "elastic-perfectly plastic" behavior. Thus it can be assumed that when the minimum bond stress reaches τ, then crack initiates, thus

$$\sigma_f = \frac{\pi d}{\frac{\pi d^2}{4}} \times \int_0^{\frac{L}{2}} \tau \cdot \frac{2}{L} x\,dx = \frac{4}{d} \cdot \frac{2\tau}{L} \cdot \frac{1}{2} \cdot x^2 \Big|_{x=0}^{\frac{L}{2}} = \frac{L}{d}\tau$$

$$\therefore \sigma_{cc} = \sigma_{mu}(1-V_f) + \tau V_f \frac{L}{d} \quad (4)$$

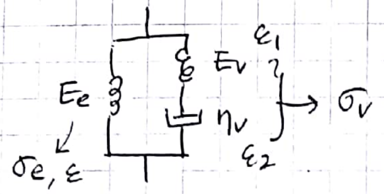
Since Eq (4) is derived based on the assumption that all fibers are oriented in the direction of the axial load
we should correct the σ_f part using α, then,

$$\sigma_{cc} = \sigma_{mu}(1-V_f) + \alpha\tau V_f \frac{L}{d}$$

$\alpha = \alpha_1\alpha_2\alpha_3$

α_1 : coefficient describing the average contribution of bond at onset of cracking
α_2 : efficiency factor of fiber orientation
α_3 : coefficient describing the reduction of bond strength due to an applied external stress radial to the surface

(f) Derive solution of below standard solid model for creep and relaxation [10 marks]



$$\sigma = \sigma_e + \sigma_v \quad : \text{equilibrium eq.} \qquad (1)$$

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \quad : \text{compatibility eq.} \qquad (2)$$

$$\left.\begin{aligned} \sigma_e &= E_e \varepsilon \\ \sigma_v &= E_v \varepsilon_1 = \eta \dot{\varepsilon}_2 \end{aligned}\right] \quad : \text{constitutive rel.} \qquad (3), (4)$$

From (2), (4)
$$\dot{\varepsilon}(t) = \dot{\varepsilon}_1(t) + \dot{\varepsilon}_2(t)$$
$$= \frac{\dot{\sigma}_v(t)}{E_v} + \frac{\sigma_v(t)}{\eta} \qquad (5)$$

From (1), (3)
$$\sigma(t) = E_e \varepsilon(t) + \sigma_v(t) \qquad (6)$$

$$\Rightarrow \sigma_v(t) = \sigma(t) - E_e \varepsilon(t) \qquad (7)$$

Substitute (7) to (5), we obtain

$$\dot{\varepsilon}(t) = \frac{\dot{\sigma}(t) - E_e \dot{\varepsilon}(t)}{E_v} + \frac{\sigma(t) - E_e \varepsilon(t)}{\eta},$$

$$\left(1 + \frac{E_e}{E_v}\right) \dot{\varepsilon}(t) = \frac{\dot{\sigma}(t)}{E_v} + \frac{\sigma(t) - E_e \varepsilon(t)}{\eta},$$

$$(E_v + E_e)\, \dot{\varepsilon}(t) = \dot{\sigma}(t) + \frac{E_v}{\eta}\left(\sigma(t) - E_e \varepsilon(t)\right),$$

$$\dot{\varepsilon}(t) = \frac{1}{E_v + E_e} \times \frac{E_v}{\eta}\left(\frac{\eta}{E_v}\dot{\sigma}(t) + \sigma(t) - E_e \varepsilon(t)\right) \qquad (8)$$

Eq (8) can also be expressed as

$$\sigma(t) + \frac{\eta}{E_v}\dot{\sigma}(t) = E_e \varepsilon(t) + \frac{\eta(E_e + E_v)}{E_v}\dot{\varepsilon}(t) \qquad (9)$$

1) creep test : $\sigma(t) = \sigma_0$

$$\dot{\varepsilon}(t) = \frac{1}{E_v + E_e} \times \frac{E_v}{\eta} \left(\sigma_0 - E_e \varepsilon(t) \right)$$

$$\dot{\varepsilon}(t) + \frac{E_v E_e}{(E_v + E_e)\eta} \varepsilon(t) = \frac{E_v}{(E_v + E_e)\eta} \sigma_0$$

Assume $\varepsilon_h(t) = C e^{\lambda t}$

$$C\lambda e^{\lambda t} + \frac{E_v E_e}{(E_v + E_e)\eta} \cdot C e^{\lambda t} = C e^{\lambda t} \left(\lambda + \frac{E_v E_e}{(E_v + E_e)\eta} \right) = 0$$

$$\therefore \lambda = -\frac{E_v E_e}{(E_v + E_e)\eta}$$

Assume $\varepsilon_p(t) = C_1$

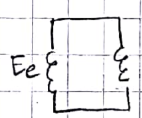
$$0 + \frac{E_v E_e}{(E_v + E_e)\eta} C_1 = \frac{E_v}{(E_v + E_e)\eta} \sigma_0$$

$$\therefore C_1 = \frac{\sigma_0}{E_e}$$

$$\Rightarrow \varepsilon(t) = \frac{\sigma_0}{E_e} + C \times \exp\left(-\frac{E_v E_e}{(E_v + E_e)\eta} t \right)$$

We know that immediately after applying the stress the strain will be entirely from spring ($\varepsilon_2 = 0$) and so,

$$\varepsilon(t=0) = \frac{\sigma_0}{E_e} + C = \frac{\sigma_0}{E_e + E_v}$$

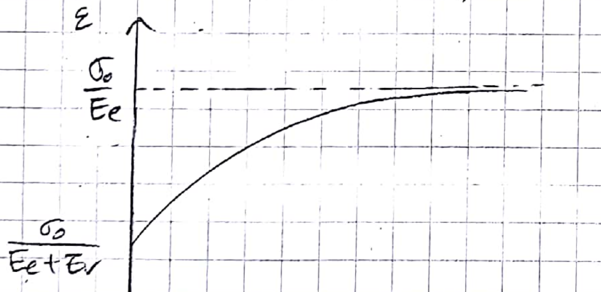


$E_e\varepsilon + E_v\varepsilon = \sigma_0$

$\Rightarrow \varepsilon = \frac{\sigma_0}{E_e + E_v}$

$$C = \frac{-E_v}{E_e(E_e + E_v)} \sigma_0$$

$$\therefore \varepsilon(t) = \frac{\sigma_0}{E_e} - \frac{E_v}{E_e(E_e + E_v)} \sigma_0 \times \exp\left(-\frac{E_e E_v}{(E_e + E_v)\eta} t \right)$$



2) Relaxation test : $\varepsilon(t) = \varepsilon_0$, $\dot{\varepsilon}(t) = 0$

$$\sigma(t) + \frac{\eta}{E_v}\dot{\sigma}(t) = E_e \varepsilon_0$$

$$\sigma_h(t) = Ce^{\lambda t}, \quad Ce^{\lambda t} + \frac{\eta}{E_v}\cdot \lambda Ce^{\lambda t} = 0$$

$$Ce^{\lambda t}\left(1 + \frac{\eta}{E_v}\lambda\right) = 0$$

$$\therefore \lambda = -\frac{E_v}{\eta}$$

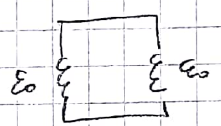
$$\sigma_p(t) = c_1, \quad c_1 = E_e \varepsilon_0$$

$$\Rightarrow \sigma(t) = E_e\varepsilon_0 + C\cdot \exp\left(-\frac{E_v}{\eta}t\right)$$

we know that immediately after applying the strain the stress will be resisted by spring and so,

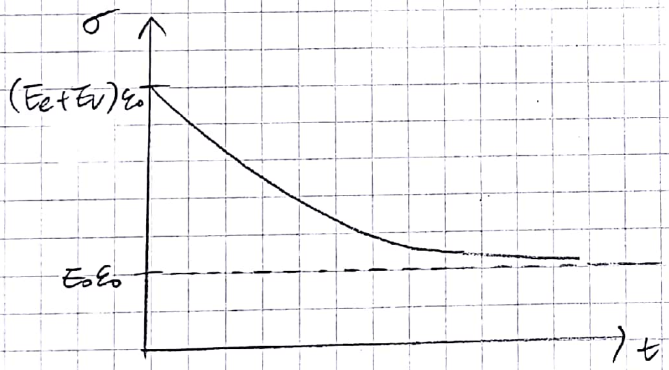
$$\sigma(0) = E_e\varepsilon_0 + C = E_e\varepsilon_0 + E_v\varepsilon_0$$

$$C = E_v\varepsilon_0$$



$$\therefore \sigma(t) = E_e\varepsilon_0 + E_v\varepsilon_0 \times \exp\left(-\frac{E_v}{\eta}t\right)$$

$$\sigma_0 = \varepsilon_0 E_e + \varepsilon_0 E_v = \varepsilon_0(E_e + E_v)$$



(g) If the tensile strength of concrete at early age is 1.5MPa, will this concrete structure be at risk of thermal cracking under the below condition? [10 marks]

Maximum tempatuture difference = 25°C
Degree of restraint = 1
Elastic modulus = 10GPa
Creep coefficient = 0.5
Coefficient of thermal expansion = 10×10^{-6} /°C

Tensile stress induced by shrinkage due to temp. change

$$\sigma_t = k_r \frac{E}{1+\varphi} \alpha \Delta T$$

$$= 1 \times \frac{10 \times 10^3 \text{ MPa}}{1+0.5} \times 10 \times 10^{-6} \times 25$$

$$= 1666.67 \times 10^{-3} \text{ MPa} = 1.67 \text{ MPa} > 1.5 \text{ MPa}$$

⇒ This structure is at risk of thermal cracking since tensile stress due to temparature change is larger than tensile strength of concrete.

(h) A concrete column with a 28-day cylinder strength of $\boxed{32\text{MPa}}$ is calculated to bear an applied compressive load equivalent to 0.3 of its strength. If the creep coefficient after one year is found to be 1.85, calculate the strain in the column. What is its specific creep? Solve this question based on ACI and FIB code, respectively. [20 marks]

A. ACI code

1) material constants

age of concrete

$$\underbrace{f_c'(t)}_{\text{predicted concrete strength}} = \frac{t}{a+bt} \underbrace{f_{c,28}'}_{\text{concrete mean strength}} = \frac{365}{4+0.85 \times 365} \times 32\text{MPa}$$

$$= 37.18 \text{ MPa}$$

(문제에서 주어진 28-day cylinder strength가 mean value인 것으로 가정)

$$E_c(t) = 0.043 \times 2300^{1.5} \times \sqrt{37.18} = 28921.1 \text{ MPa}$$

unit weight 가정

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2) elastic strain

$$\varepsilon_e = \frac{32MPa \times 0.3}{E_c} = \frac{32 \times 0.3}{28921.1} = 3.32 \times 10^{-4}$$

3) shrinkage

$$\varepsilon_{sh}(t, t_0) = \frac{t - t_0}{\alpha + (t - t_0)} \times \varepsilon_{sh,u}$$

$\alpha = 35$; assume moisture curing

$\varepsilon_{sh,u} = 780 \times 10^{-6} \gamma_{sh}$

$\gamma_{sh} = \gamma_{cp} \cdot \gamma_{\lambda} \cdot \gamma_h \cdot \gamma_s \cdot \gamma_\psi \cdot \gamma_c \cdot \gamma_\alpha = 1$

; no information for γ_{sh} in problem.

Since we assume $\gamma_{cp} = 1$, initial moisture curing period is 7 days.

$$\therefore \varepsilon_{sh}(t, t_0) = \frac{365 - 7}{35 + (365 - 7)} \times 780 \times 10^{-6} \times 1$$

$$= 710.5 \times 10^{-6}$$

4) Creep

$$J(t, t') = \frac{1}{E_c(t')} + \underbrace{\frac{\phi(t, t')}{E_c(t)}}_{\text{specific creep}}$$

$\phi(t, t')$ = creep coefficient = 1.85

t' = time of loading = 365 days. = target time t (assumption)

$E_c(t') = E_c(t) = 28921.1$

$$\text{specific creep} = \frac{\phi(t, t')}{E_c(t)} \times \sigma(t') = \frac{1.85}{28921.1} \times 32 \times 0.3 = 6.14 \times 10^{-4}$$

B. fib code

1) material constants.

$f_{cm} = f_{ck} + \Delta f = 32\,\text{MPa}$ ← 앞에서와같이 주어진강도 32MPa이 mean value인 것으로 가정

$f_{cm}(t) = \beta_{cc}(t) \cdot f_{cm}$

$\beta_{cc}(t) = \exp\left\{ s\left(1 - \sqrt{\frac{28}{t/t_1}}\right)\right\}$ $t = 365\text{d}$, $t_1 = 1\text{d}$, $s = 0.35$ (type 1, moist curing)

$= \exp\left\{ 0.35\left(1 - \sqrt{\frac{28}{365}}\right)\right\}$

$= 1.288$

$\therefore f_{cm}(t) = 1.288 \times 32 = 41.216\,\text{MPa}$

$E_{ci} = E_{c0}\left(\frac{f_{cm}}{f_{cm0}}\right)^{\frac{1}{3}}$

$= 2.15\times10^4\,\text{MPa} \times \left(\frac{32\,\text{MPa}}{10\,\text{MPa}}\right)^{\frac{1}{3}} = 3.17\times10^4\,\text{MPa}$

$E_{ci}(t) = \beta_E(t)\, E_{ci}$

$= \sqrt{\beta_{cc}(t)}\, E_{ci} = \sqrt{1.288} \times 3.17\times10^4$

$= 3.598\times10^4\,\text{MPa}$

⇒ fib code가 ACI code에 비해 콘크리트 강도. 강성을 크게 예측함

2) creep

$\varepsilon_{cc}(t,t') = \frac{\sigma_c(t')}{E_{ci}}\phi(t,t')$; specific creep

$\phi(t,t') = 1.85$ (from problem)

$\sigma_c(t') = 32\,\text{MPa} \times 0.3 = 9.6\,\text{MPa}$

$\varepsilon_{cc}(t,t') = \frac{9.6\,\text{MPa}}{3.17\times10^4\,\text{MPa}} \times 1.85 = \underline{5.6\times10^{-4}}$