< 전자장2 Midterm-exam#1 Solution >

1. *The time harmonic wave equation for scalar potential*

$$\nabla V + k^2 V = -\frac{\rho}{\varepsilon}$$

The retarded scalar potential solution

$$V = \frac{1}{4\pi\varepsilon}\int \frac{\rho(t - R/c)}{R} dV^{'}$$

The phasor solution of the wave equation

$$V = \frac{1}{4\pi\varepsilon}\int \frac{\rho e^{-jkR}}{R} dV^{'} \quad where,\ k = w/c$$

$*$ *Quasi - static condiation*

$$if \quad kR \ll 1\ (\lambda \gg R\ or\ f \ll 1)\ ,\ then \quad e^{-jkR} \cong 1$$

$$\therefore V = \frac{1}{4\pi\varepsilon}\int \frac{\rho}{R} dV^{'}$$

2.

$$\nabla \times \overline{E} = -j\omega\mu\overline{H} \qquad \nabla \times \overline{H} = (\sigma + j\omega\varepsilon)\overline{E}$$

$$\nabla \times \nabla \times \overline{E} = \nabla(\nabla \quad \overline{E} \ \ - \nabla^2 \overline{E} = -j\omega\mu \ \sigma + j\omega\varepsilon \ \overline{E}$$

$$\nabla \bullet \overline{E} = 0$$

$$\therefore \ \nabla^2 \overline{E} + (\omega^2 \mu\varepsilon - j\omega\mu\sigma)\overline{E} = 0$$

3.

$$\overline{E} = E_0 e^{-j\beta z} a_x + E_0 e^{-j\beta z} a_y$$

$$= (ae^{-j\beta z} a_x + jae^{-j\beta z} a_y) + (be^{-j\beta z} a_x - jbe^{-j\beta z} a_y)$$

$$a + b = E_0, \quad a - b = -jE_0 \qquad \therefore a = \frac{E_0}{\sqrt{2}} e^{-j45^0},\ b = \frac{E_0}{\sqrt{2}} e^{j45^0}$$

$$\therefore \overline{E} = \frac{E_0}{\sqrt{2}}[(\cos(\omega t - \beta z - 45^0)a_x + \cos(\omega t - \beta z + 45^0)a_y)$$

$$\cos((\omega t - \beta z + 45^0)a_x + \cos(\omega t - \beta z - 45^0)a_y)]$$

$$\therefore \beta z = 45^0 \ plane$$

$$\overline{E} = \underbrace{\frac{E_0}{\sqrt{2}}[\sin \omega t a_x + \cos \omega t a_y]}_{left-handed} + \underbrace{\frac{E_0}{\sqrt{2}}[\cos \omega t a_x + \sin \omega t a_y]}_{right-handed}$$

4. a)

$$\delta = \frac{1}{\sqrt{\pi f \mu\sigma}}$$

$$\longrightarrow \sigma = \frac{1}{\pi f \mu\delta^2} = 9.89\times10^4 \quad (S/m)$$

b)

$$\alpha = \sqrt{\pi f \mu\sigma} = 1.976\times10^4 \ (Np/m) \quad for \ f = 1GHz$$

$$20\log e^{-\alpha z} = -30$$

$$\therefore z = -\frac{1}{\alpha}\ln 10^{-3/2} = 1.748\times10^{-4} m$$

5.

Reflected plain wave

$$E_r = -\hat{x}E_{10}e^{j\beta z} + \hat{y}jE_{10}e^{j\beta z}$$

$$H_r = -\hat{z}\frac{E_r}{\eta} = \frac{1}{\eta}(\hat{y}E_{10}e^{j\beta z} + \hat{x}jE_{10}e^{j\beta z})$$

Instantaneous expression

$$E_r(z;t) = \mathrm{Re}[E_r(z)e^{j\omega t}]$$

$$= -\hat{x}E_{10}\cos(\omega t + \beta z) - \hat{y}E_{10}\sin(\omega t + \beta z)$$

$\therefore$ *Left-handed circularly polarized plane wave*

$$H_r(z;t) = \mathrm{Re}[H_r(z)e^{j\omega t}]$$

$$= \frac{1}{\eta}(-\hat{x}E_{10}\sin(\omega t + \beta z) + \hat{y}E_{10}\cos(\omega t + \beta z))$$

$\therefore$ *Left-handed circularly polarized plane wave*