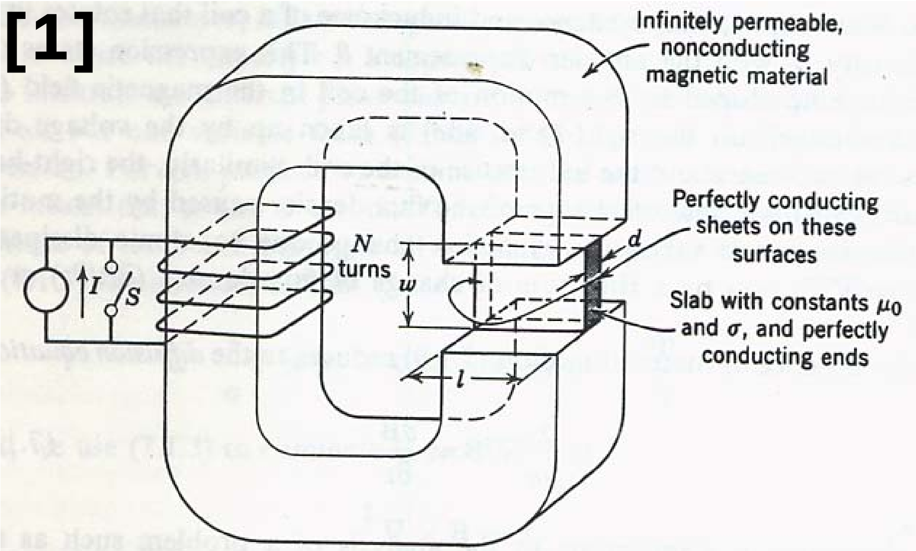
전자에너지변환론 기말고사 2007년 12월 17일 (월)

[1]



스위치가 열리는 순간 I가 가해지고 자장 B_0가 가해진다. 이 때 물건 (block) 안의 자계 분포를 구하라. 자계는 x 방향으로만 존재한다고 생각하고 z, t 만의 함수로 가정한다 (25 점).

(a) B_0를 구하라 (B_0는 공간의 자속밀도) (3 점).

(b) $B_x(z, t)$ at $0 < z < d$ (10 점)

(c) 기본 시정수 τ를 구하라 (2 점).

(d) 그래프를 그려라 (2 점).

(e) Eddy current를 구하고 그래프를 그려라 (4 점).

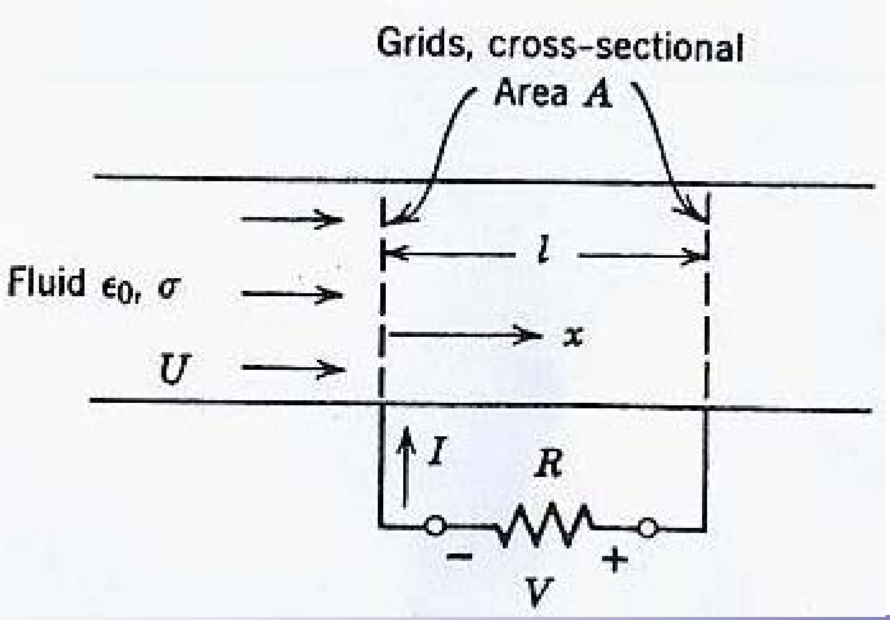
(f) Block을 one turn coil로 생각하고 집중정수상수 G, L을 구하라. 이 때 시정수는 (c)에서 구한 τ를 사용하라 (4 점).

[2] 도전성 유체(s)가 단면적 ***A*** 인 절연파이프 안을 ***x*** 방향으로 흐르고 있다. 유체는 일정한 속도 ***U*** 로 흐르고 있다. 이온은 ***x* = 0** 에 있는 전원으로부터 전하밀도 ρ_o 로 주입되고 있다 **(25**점**).**

(a) 두 전극 사이에서의 전하 밀도 분포를 구하라 **(10**점**).**

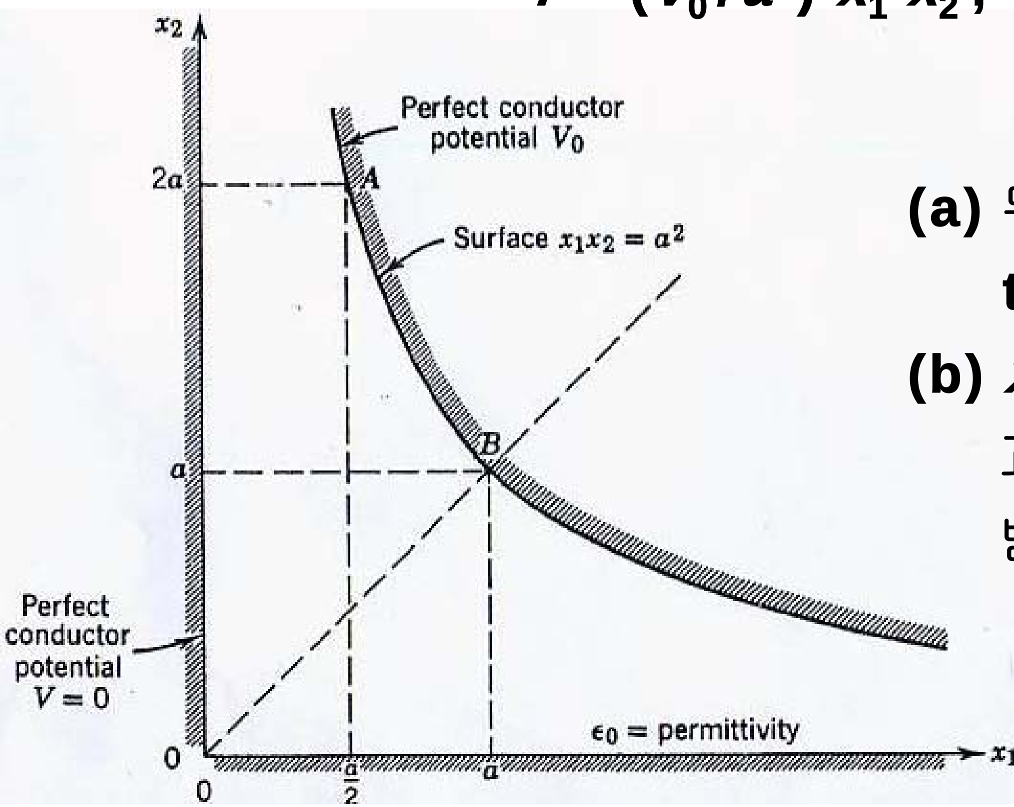
(b) 전극 사이의 전계의 세기를 구하라 **(10**점**).**

(c) 저항 ***R*** 사이에 인가되는 전압을 구하라 **(5**점**).**



[3] 그림과 같이 두 개의 등전위 전극이 있다. x_3 방향으로는 무한히 길다. 두 전극 사이의 포텐샬은 다음 식과 같고 전계의 세기는 아래의 식으로 규정한다 **(25 점).**

$$f = (V_0 / a^2)\, x_1 x_2, \quad \mathbf{E} = -\,\mathrm{grad}\, f$$



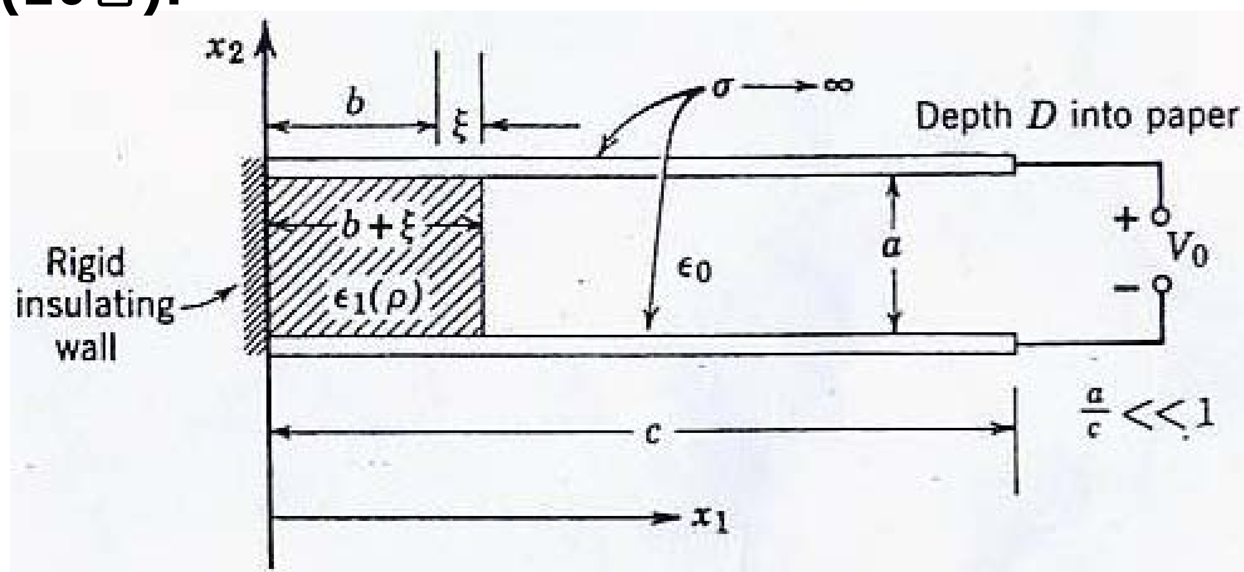
(a) 두 전극 사이의 모든 방향의 **stress tensor** 성분을 구하라 **(10점).**

(b) x_3 방향으로는 길이가 D 이고 **A** 점과 **B** 점 사이의 전극의 곡면 부분이 받는 전체 힘을 구하라 **(15 점).**

[4] 탄성 물질이 왼쪽이 고정되어 있고 두 전극 사이에 놓여 있다. 오른 쪽은 자유롭게 놓여 있고 유전율은 밀도의 함수 이다. $e_1 = e_1(\rho)$. 두 전극에는 일정 전압이 걸려있다 **(25점).**

(a)분극 물질에 적용하는 **Maxwell stress tensor**를 이용하여 두 전극 사이의 물질의 오른 쪽 면에 가해지는 전체 힘을 구하라 **(15점).**

(b) Energy method(가상 변위법**)**를 이용해서 **(a)** 의 결과를 확인하라 **(10점).**



[1]

(a) $\oint_c \vec{H} \cdot dl = NI$.

$$H_0 w = NI \rightarrow H_0 = \frac{NI}{w}$$

$$\therefore B_0 = \mu_0 H_0 = \frac{\mu_0 NI}{w}.$$

(b) $-\frac{1}{\mu\sigma}\vec{\nabla}^2\vec{B} + \frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{V} \times \vec{B})$ $\vec{V}=0$이므로

$\Rightarrow -\frac{1}{\mu\sigma}\vec{\nabla}^2\vec{B} + \frac{\partial \vec{B}}{\partial t} = 0$., x성분만 고려하면 $\frac{1}{\mu_0\sigma}\frac{\partial^2 \vec{B}_x}{\partial z^2} = \frac{\partial B_x}{\partial t}$

초기조건에서 $\left(\begin{array}{l} t = 0^+ \rightarrow B_x = 0 \\ t \rightarrow \infty \rightarrow B_x = B_0 \end{array} \right.$

일반해 $B_x = \vec{B}(z) \cdot e^{-\alpha t} + B_0$.

$$\Rightarrow \frac{1}{\mu_0\sigma}\frac{d^2\hat{B}}{dz^2} = -\alpha\hat{B}$$

$$\Rightarrow \hat{B} = C_1 \sin\sqrt{\mu_0\sigma\alpha}\, z + C_2 \cos\sqrt{\mu_0\sigma\alpha}\, z.$$

$$\therefore B_x = (C_1 \sin\sqrt{\mu_0\sigma\alpha}\, z + C_2 \cos\sqrt{\mu_0\sigma\alpha}\, z)e^{-\alpha t} + B_0.$$

$C_2 = 0$ 일때 $\rightarrow \sqrt{\mu_0\sigma\alpha_n}\, d = n\pi \Rightarrow \alpha_n = \frac{n^2\pi^2}{\mu_0\sigma d^2}$

* $t = 0^+$ 일때,

$$0 = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi z}{d} + B_0$$

$$\Rightarrow \int_0^d -B_0 \sin\frac{m\pi z}{d}\, dz = \int_0^d \sum_{n=1}^{\infty} a_n \sin\frac{n\pi z}{d}\cdot \sin\frac{m\pi z}{d}\cdot dz.$$

$m \neq n$ 이면 right term $=0$ 이므로.

$$\therefore \int_0^d -B_0 \sin\frac{n\pi z}{d}\cdot dz = \int_0^d a_n \sin^2\frac{n\pi z}{d}\, dz.$$

여기서
$$\begin{cases} a_n = -\dfrac{4}{n\pi}B_0 & n \text{ odd.} \\ a_n = 0 & n \text{ even} \end{cases}$$

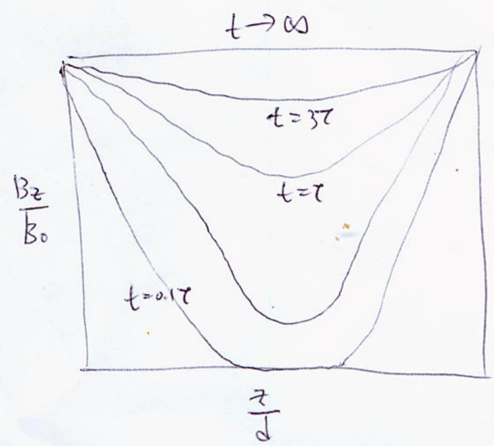
$$\therefore B_x = B_0\left(1 - \sum_{n\,odd} \frac{4}{n\pi}\sin\frac{n\pi z}{d}\, e^{-\lambda_n t}\right)$$

$$\tau = \frac{1}{\lambda_1} = \frac{\mu_0 \sigma d^2}{\pi^2}$$

$$\therefore B_x = B_0\left(1 - \sum_{n\,odd} \frac{4}{n\pi}\sin\frac{n\pi z}{d}\, e^{-n^2 t/\tau}\right)$$

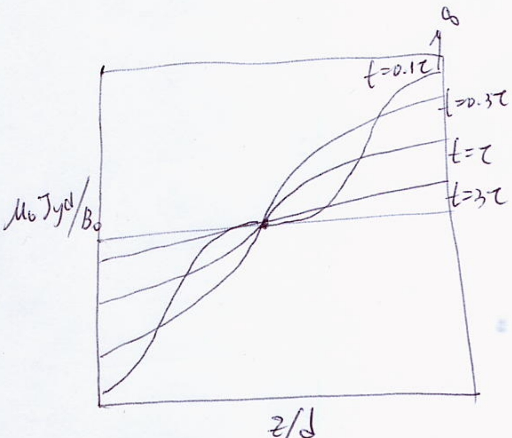
(c) $\tau = \dfrac{\mu_0 \sigma d^2}{\pi^2}$

(d)

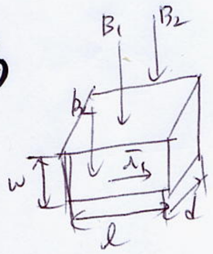


(e) $\mu_0 \vec{J} = \vec{\nabla} \times \vec{B}$

$$\Rightarrow J_y = \frac{1}{\mu_0}\frac{dB_x}{dz} = -\frac{B_0}{\mu_0 d}\sum_{n\,\mathrm{odd}} 4\cos\frac{n\pi z}{d}\, e^{-n^2 t/\tau}$$



(f)



$$B_2 = \frac{\mu_0 N i_2}{w}$$

$$B_1 = \frac{\mu_0 N i_2}{w} - \frac{\mu_0 i_1}{w}$$

$$\lambda_1 = -dl B_1 = L i_1 - M i_2$$

$$\therefore L = \frac{dl\mu_0}{w}, \quad M = \frac{dl\mu_0 N}{w}$$

$$GL\frac{di_1}{dt} + i_1 = GM\frac{di_2}{dt}$$

$i_1 = IN e^{-t/GL}$ (i_2 : magnitude I of step function)

$t>0$, outside the conductors : $B = B_2 = \frac{\mu_0 N I}{w}$

between the conductors : $B = B_1 = \frac{\mu_0 N I}{w}(1 - e^{-t/GL})$

$$\therefore G = \frac{\sigma(2d/\pi^2)w}{2l} \qquad GL = \frac{\mu_0\sigma d^2}{\pi^2}$$

[2]

(a) $x=0$ 에서 ρ_0가 주입 (ρ_0 = constant)

$$\frac{\sigma}{\varepsilon_0}\rho_f + U\frac{d\rho_f}{dt} = 0.$$

$$\Rightarrow \rho_f(x) = \rho_0 e^{-\frac{\sigma}{\varepsilon_0 U}x} = \rho e^{-\frac{1}{Re}\frac{x}{l}} \quad (0 \le x \le l)$$

(여기서 $Re = \frac{\varepsilon_0 U}{\sigma l}$

(b) $\vec{J}_f = \sigma\vec{E} + \rho_f\vec{U}$ 에서 x 방향만 고려.

$$J_x = \sigma E_x + \rho_f U, \quad J_x = \frac{I}{A}$$

$$E_x = \frac{1}{\sigma}(J_x - \rho_f U)$$

$$= \frac{I}{\sigma A} - \frac{\rho_0}{\sigma}Ue^{-\frac{1}{Re}\frac{x}{l}}$$

$$= \frac{I}{\sigma A} - \frac{\rho_0 U}{\sigma}e^{-\frac{\sigma}{\varepsilon_0 U}x}$$

$\Rightarrow$ open circuit 일때 Van de Graaff Generator와 동일

$$(I=0) \quad V_x = -\int_0^l E_x dx = \int_0^l \frac{\rho_0 U}{\sigma}e^{-\frac{1}{Re}\frac{x}{l}}dx$$

$$= \left[-\frac{\rho_0}{\sigma}Re\, l\, e^{-\frac{1}{Re}\frac{x}{l}}\right]_0^l = \frac{\rho_0}{\sigma}Re\, l(1-e^{-\frac{1}{Re}})$$

$$= \frac{\rho_0\varepsilon_0}{\sigma^2}U^2(1-e^{-\frac{1}{Re}})$$

$$= \underline{\rho_0 \cdot Re^2 \cdot \frac{l^2}{\varepsilon_0}(1-e^{-\frac{1}{Re}}) = V_{oc}}$$

(C) 저항이 삽입되면,

$$I = \frac{V}{R}$$

$$E_x = \frac{V}{\sigma A R} - \frac{\rho_0}{\sigma} u e^{-\frac{1}{R_e}\frac{x}{l}} \qquad \left(R' = \frac{1}{\sigma}\frac{l}{A} \right)$$

$$= \frac{R'}{R}\frac{V}{l} - \frac{\rho_0}{\sigma} u e^{-\frac{1}{R_e}\frac{x}{l}}$$

$$= \frac{R'}{R}\frac{V}{l} + E_{oc}$$

$$V = -\int_0^l \left(\frac{R'}{R}\frac{V}{l} \right) dx + \int_0^l \frac{\rho_0}{\sigma} e^{-\frac{1}{R_e}\frac{x}{l}}\, dx$$

$$= -\frac{R'}{R} \cdot V + V_{oc} \qquad \left(V_{oc} = \frac{\rho_0 \varepsilon_0}{\sigma^2} u^2 \left(1 - e^{-\frac{1}{R_e}} \right) \right)$$

$$V = \left(\frac{R}{R+R'} \right) V_{oc} = \frac{\frac{\rho_0\varepsilon_0}{\sigma^2} u^2 \left(1 - e^{-\frac{1}{R_e}} \right)}{\left(1 + \frac{l}{\sigma A R} \right)} = \frac{\frac{\rho_0\varepsilon_0}{\sigma^2} u^2 \left(1 - e^{-\frac{\sigma l}{\varepsilon_0 u}} \right)}{1 + \frac{l}{\sigma A R}}$$

또, $E_x = \frac{R'}{R} \cdot \frac{1}{l} \cdot \frac{R}{R+R'} V_{oc} + E_{oc} = \frac{R'}{R+R'} \frac{V_{oc}}{l} + E_{oc}$ //

open circuit이면 $R \to \infty$ $V = V_{oc}$, $E_x = E_{oc}$.

short circuit이면 $R \to 0$. $V = 0$, $E_x = \frac{V_{oc}}{l} + E_{oc} = 0$.

따라서 $J_x = \rho_f u$가 되어 모든 전류는 외부로 나가므로 유기전압 $V = 0$ 이 된다.

만약 $R_e \to \infty$ 이면,

$$E_{oc} = \lim_{R_e \to \infty} -\frac{R_e l}{\varepsilon_0} \rho_0 e^{-\frac{1}{R_e}\frac{x}{l}} = \infty \qquad \frac{1}{R_e} = x$$

$$V_{oc} = \lim_{R_e \to \infty} R_e^2 \rho_0 \frac{l^2}{\varepsilon_0} \left(1 - e^{-\frac{1}{R_e}} \right) = \lim_{x \to 0} \frac{\rho_0 l^2 (1 - e^{-x})}{\varepsilon_0 x^2} = \infty$$

$R_e \to 0$ 이면

$$E_{oc} = \lim_{R_e \to 0} -\frac{l \rho_0}{\varepsilon_0} R_e e^{-\frac{1}{R_e}\frac{x}{l}} = 0$$

$$V_{oc} = \lim_{R_e \to 0} R_e^2 \rho_0 \frac{l^2}{\varepsilon_0} \left(1 - e^{-\frac{1}{R_e}} \right) = 0.$$

[3]

(a) $\phi = \frac{V_0}{a^2}x_1 x_2$, $E_1 = -\frac{V_0}{a^2}x_2$, $E_2 = -\frac{V_0}{a^2}x_1$, $E_3 = 0$, $\frac{\partial}{\partial x_3} = 0$

$$T_{11} = \frac{\varepsilon_0}{2}\left(\frac{V_0^2}{a^4}x_2^2 - \frac{V_0^2}{a^4}x_1^2\right) = \frac{\varepsilon_0}{2}\frac{V_0^2}{a^4}(x_2^2 - x_1^2)$$

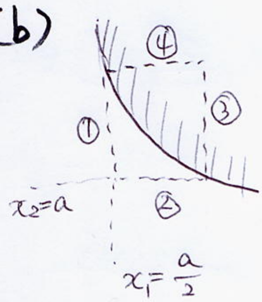
$$T_{22} = -T_{11}$$

$$T_{12} = T_{21} = \varepsilon_0 \frac{V_0^2}{a^4}x_1 x_2$$

$$T_{33} = -\frac{\varepsilon_0}{2}\left(\frac{V_0^2}{a^4}\right)(x_1^2 + x_2^2)$$

$$T_{13} = T_{31} = T_{23} = T_{32} = 0.$$

(b)



$$f_1 = -\int_{①} T_{11}\,da + \int_{②} -T_{12}\,da$$

$$= -\int \frac{\varepsilon_0 V_0^2}{2a^4}\left(x_2^2 - \frac{a^2}{4}\right)da - \int \varepsilon_0 \frac{V_0^2}{a^4}(a)\cdot x_1\,da$$

$$= -D\left[\frac{\varepsilon_0 V_0^2}{2a^4}\left[\frac{1}{3}x_2^3 - \frac{a^2}{4}x_2\right]_a^{2a} + \varepsilon_0 \frac{V_0^2}{a^3}\left[\frac{1}{2}x_1^2\right]_{\frac{a}{2}}^{a}\right]$$

$$= -D\frac{\varepsilon_0 V_0^2}{2a^3}\left[\frac{1}{a}\left(\frac{8a^3}{3} - \frac{a^3}{2} - \frac{a^3}{3} + \frac{a^3}{4}\right) + \left(a^2 - \frac{a^2}{4}\right)\right]$$

$$= -\frac{17}{12}\frac{D}{a}\varepsilon_0 V_0^2$$

$$f_2 = -\int_{①} T_{21}\,da - \int_{②} T_{22}\,da$$

$$T_{21}\left(x_1 = \frac{a}{2}\right) = \frac{\varepsilon_0 V_0^2}{2a^3}x_2$$

$$T_{22}(x_2 = a) = -\frac{\varepsilon_0 V_0^2}{2a^4}(a^2 - x_1^2) = \frac{\varepsilon_0 V_0^2}{2a^4}(x_1^2 - a^2)$$

$$f_2 = -D\left\{\frac{\varepsilon_0 V_0^2}{2a^3}\int_a^{2a} x_2\,dx_2 + \frac{\varepsilon_0 V_0^2}{2a^4}\int_{\frac{a}{2}}^{a}(x_1^2 - a^2)\,dx_1\right\}$$

$$= -D\frac{\varepsilon_0 V_0^2}{2a^3}\left\{\frac{1}{2}\{(2a)^2 - a^2\} + \frac{1}{a}\left[\frac{1}{3}\left(a^3 - \frac{a^3}{8}\right) - a^2\left(a - \frac{a}{2}\right)\right]\right\}$$

$$= -D\frac{\varepsilon_0 V_0^2}{2a^3}a^2\left\{\frac{3}{2} + \frac{7}{24} - \frac{1}{2}\right\}$$

$$= -\frac{31}{48}\frac{D}{a}\varepsilon_0 V_0^2$$

$f_3 = 0.$

i) $V_0 = 10^5 V,\ a = 0.1\,m,\ D = 0.1\,m.$

$f_1 = -0.125\,N,\ f_2 = -0.057\,N$

ii) $V_0 = 1\,MV,\ a = 0.1\,m,\ D = 0.1\,m.$

$f_1 = -12.5\,N,\ f_2 = -5.7\,N.$

[4]

(a) $q\delta v = \delta W' - f\delta\xi$

$\Rightarrow q\delta v = \int_V \delta w' dV - \int_V \bar{F}\cdot\delta\xi \, dV$

here $q\delta v = -\int_S \varepsilon\bar{\nabla}\phi\cdot n \, \delta\phi \, da$

$= \int_V \bar{\nabla}\cdot[(\varepsilon\bar{\nabla}\phi)\cdot(\delta\phi)] dV$

$= \int_V \frac{1}{2}\varepsilon\delta(\bar{\nabla}\phi)^2 dV$

$\Rightarrow \int_V \frac{1}{2}\varepsilon\delta(\bar{\nabla}\phi)^2 dV = \int_V \delta w' dV - \int_V \bar{F}\cdot\delta\xi \, dV$

여기서 $\int_V \delta\left[\frac{1}{2}\varepsilon(\bar{\nabla}\phi)^2\right] dV = \int_V \delta w' dV$

$\Rightarrow \delta w' = \delta\left[\frac{1}{2}\varepsilon(\bar{\nabla}\phi)^2\right] = \frac{1}{2}(\bar{\nabla}\phi)^2\delta\varepsilon + \frac{1}{2}\varepsilon\delta(\bar{\nabla}\phi)^2$

$\varepsilon\bar{\nabla}\phi\cdot\bar{\nabla}(\delta\phi) = \bar{\nabla}\cdot(\delta\phi\varepsilon\cdot\bar{\nabla}\phi) - \delta\phi\bar{\nabla}\cdot(\varepsilon\bar{\nabla}\phi) = 0$

$\Rightarrow \delta w' = \frac{1}{2}(\bar{\nabla}\phi)^2\delta\varepsilon$

즉, $\int_V \left(\frac{1}{2}\vec{E}\cdot\vec{E}\,\delta\varepsilon - \vec{F}\cdot\delta\xi\right) dV = 0$ — ①

$\delta\varepsilon = \frac{\partial\varepsilon}{\partial\rho}\delta\rho, \quad -\delta\rho = \rho\bar{\nabla}\cdot\delta\xi$

따라서 ①식의 첫 번째 term은,

$\int_V \frac{1}{2}\vec{E}\cdot\vec{E}\,\delta\varepsilon \, dV = \int_V -\frac{1}{2}\vec{E}\cdot\vec{E}\frac{\partial\varepsilon}{\partial\rho}\rho\bar{\nabla}\cdot\delta\xi \, dV$

$$= -\int_V \nabla\cdot\left(\frac{1}{2}\vec{E}\cdot\vec{E}\,\frac{\partial\varepsilon}{\partial\rho}\rho\,\delta\xi\right)dV + \int_V \nabla\left(\frac{1}{2}\vec{E}\cdot\vec{E}\,\frac{\partial\varepsilon}{\partial\rho}\rho\right)\cdot\delta\xi\,dV$$

정리하여 ① 식을 다시 정리하면

$$\int_V\left[-\frac{1}{2}\vec{E}\cdot\vec{E}\,\nabla\varepsilon + \nabla\left(\frac{1}{2}\vec{E}\cdot\vec{E}\,\frac{\partial\varepsilon}{\partial\rho}\rho\right) - \vec{F}\right]\cdot\delta\xi\,dV = 0.$$

$$\Rightarrow \vec{F} = -\frac{1}{2}\vec{E}\cdot\vec{E}\,\nabla\varepsilon + \nabla\left(\frac{1}{2}\vec{E}\cdot\vec{E}\,\frac{\partial\varepsilon}{\partial\rho}\rho\right)$$

$$\therefore \vec{F} = \rho_f\vec{E} - \frac{1}{2}\vec{E}\cdot\vec{E}\,\nabla\varepsilon + \nabla\left(\frac{1}{2}\vec{E}\cdot\vec{E}\,\frac{\partial\varepsilon}{\partial\rho}\rho\right)$$

이를 Maxwell stress tensor를 이용하여 정리하면,

$$F_m = \frac{\partial}{\partial x_n}\left(\varepsilon E_m E_n\right) - \frac{\varepsilon}{2}\frac{\partial}{\partial x_m}\left(E_k E_k\right)$$
$$- \frac{1}{2}E_k E_k\,\frac{\partial\varepsilon}{\partial x_m} + \frac{\partial}{\partial x_m}\left(\frac{1}{2}E_k E_k\,\frac{\partial\varepsilon}{\partial\rho}\rho\right)$$

정리하면,

$$F_m = \frac{\partial T_{mn}}{\partial x_n} = \frac{\partial}{\partial x_n}\Big[\underbrace{\varepsilon E_m E_n - \frac{1}{2}\delta_{mn}E_k E_k\left(\varepsilon - \frac{\partial\varepsilon}{\partial\rho}\rho\right)}_{T_{mn}}\Big]$$

(b) $q\delta v = \delta W' - f_q \xi$.

$W' = \int q \delta v$, $\quad W' = \frac{1}{2} v^2 C(\xi)$

$\Rightarrow \left(\frac{1}{2} v^2 \frac{\partial C(\xi)}{\partial \xi} - f \right) q\xi = 0$

$\Rightarrow f = \frac{1}{2} v^2 \frac{\partial C(\xi)}{\partial \xi}$

$$C(\xi) = \varepsilon_1(\rho) \frac{(b+\xi) \cdot D}{a} + \varepsilon_0 \frac{(c-(b+\xi))D}{a}$$

$$\Rightarrow \frac{\partial C(\xi)}{\partial \xi} = \frac{b \cdot D}{a} \cdot \frac{\partial \varepsilon_1(\rho)}{\partial \xi} + \frac{D}{a}\varepsilon_1(\rho) + \frac{\xi \cdot D}{a} \cdot \frac{\partial \varepsilon_1(\rho)}{\partial \xi} - \frac{D}{a}\varepsilon_0.$$

$$= \frac{D}{a}\left[(\varepsilon_1(\rho) - \varepsilon_0) + (b+\xi) \frac{\partial \varepsilon_1(\rho)}{\partial \xi} \right]$$

$\frac{\partial \rho}{\partial \xi} = -\frac{\rho}{\xi}$ 이므로.

$$\Rightarrow \frac{D}{a}\left[(\varepsilon_1(\rho) - \varepsilon_0) + (b+\xi) \frac{\partial \varepsilon_1(\rho)}{\partial \rho} \left(-\frac{\rho}{\xi} \right) \right]$$

$$= \frac{D}{a}\left[(\varepsilon_1(\rho) - \varepsilon_0) - \frac{\rho}{\xi}(b+\xi) \frac{\partial \varepsilon_1(\rho)}{\partial \rho} \right]$$

$$\therefore f = \frac{1}{2} v^2 \cdot \frac{D}{a}\left[(\varepsilon_1(\rho) - \varepsilon_0) - \frac{\rho}{\xi}(b+\xi) \frac{\partial \varepsilon_1(\rho)}{\partial \rho} \right]$$

$$= a \cdot D \cdot \frac{1}{2} \vec{E}^2 \left[(\varepsilon_1(\rho) - \varepsilon_0) - \frac{\rho}{\xi}(b+\xi) \frac{\partial \varepsilon_1(\rho)}{\partial \rho} \right]$$