# Engineering Mathematics I

December 12, 2007

(Instructor : In-Joong Ha, Professor)

## FINAL EXAM

**[Problem 1](10 pts.)** Find a general solution and the particular solution (Show the details of your work)

$$y''' - y'' - y' + y = 0, \qquad y(0) = 2,\ y'(0) = 1,\ y''(0) = 0 \tag{1.1}$$

**[Problem 2](10 pts.)** Find a general solution. Show the details of your work

$$\begin{cases} y_1' = y_1 + 4y_2 - 2\cos t \\ y_2' = y_1 + y_2 - \cos t + \sin t \end{cases} \tag{2.1}$$

**[Problem 3](10 pts.)** Consider the following time-invariant linear system of ODEs :

$$y'(t) = A(t)y(t) + g(t), \quad y(t_0) = y_0 \tag{3.1}$$

where $y(t) \in R^n$ and $g(t) \in R^n$. Assume that the components of $A(t)$ and $g(t)$ are continuous in $t$. Now, suppose that we have found $n$ linearly independent solutions, say, $y^{(k)}(t), k = 1, 2, \cdots, n$, of (1.1) with $g(t) = 0$. Then, describe the solution of **(3.1)** explicitly by the method of variation of parameters.

**[Problem 4](10 pts.)** Find all critical points of the following nonlinear system and also discuss their stability.

$$\begin{cases} \dot{x}_1 = -ax_1 + b \\ \dot{x}_2 = -cx_2 + x_1(\alpha - \beta x_1 x_2) \end{cases} \tag{4.1}$$

where all coefficients are positive.

**[Problem 5](10 pts.)** Consider the 2nd order linear ODE

$$x^2 y'' - 2xy' + (2 + x)y = 0 \tag{5.1}$$

Using Frobenius method, we want to find series solutions near the origin

(a) Find two solutions $r_1 \geq r_2$ of the indicial equation for the above ODE.

(b) Using $r_1$, find one solution of the ODE(you must explicitly write down the recurrence formula and at least three non zero terms of the series solution).

(c) Explain why you cannot generate the second independent series solution using $r_2$.

**[Problem 6] (10 pts.)** Find a general solution using a power series method. Show the details of your work.

$$y'' - 4xy' + (4x^2 - 2)y = 0 \qquad \textbf{(6.1)}$$

**[Problem 7] (10 pts.)** Calculate the inverse Laplace transform

$$L^{-1}\left\{\left(1 - e^{-s}/s\right)^2\right\} \qquad \textbf{(7.1)}$$

**[Problem 8] (10 pts.)** Suppose that $g$ is piecewise continuous on $[0, b]$ for every $b \rangle 0$, and that there are real numbers $M, k,$ and $a$ such that $|g(t)| \le Me^{kt}$ for $t \ge a$. Then, show that

$$L\{\int_a^t g(\tau)d\tau\} = \frac{1}{s}L\{g(t)\} - \frac{1}{s}\int_0^a g(\tau)d\tau \qquad \textbf{(8.1)}$$

**[Problem 9] (10 pts.)** Solve the following integral equation (Hint : Use the Laplace transform).

$$\int_0^t y(t-x)e^{-x}dx - \int_0^t y(x)dx = t^{10}e^{-t} \qquad \textbf{(9.1)}$$

**[Problem 10] (10 pts.)** Solve the following problems using the Laplace transform. Show the details of your work.

$$\begin{cases} y_1' + y_2' = 2\sinh t \\ y_2' + y_3' = e^t \\ y_3' + y_1' = 2e^t + e^{-t} \end{cases} \qquad \textbf{(10.1)}$$

where $y_1(0) = 1,\ y_2(0) = 1,\ y_3(0) = 0$

-- E N D --