

1. (10 points)

It is a tautology. (2 points)

Proof: truth table (8 points) or symbolic reasoning

p	q	LHS	RHS
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	T

2. (6/6/8 points each)

a) $\forall x (x \neq \text{Joseph} \leftrightarrow C(\text{Sanjay}, x))$

b) $\exists x \forall y (x = y \leftrightarrow I(y))$

c) $\exists x \exists y (x \neq y \wedge \forall z (C(x, z) \vee C(y, z)))$

or $\exists x \exists y (x \neq y \wedge \forall z (C(x, z) \wedge C(y, z)))$

3. a) (10 points)

a proof by contraposition

We must prove the contrapositive: if n is odd, then $3n+2$ is odd.

Assume that if n is odd, then $n=2k+1$ for some integer k .

Then $3n+2 = 6k+5$, which is also odd. And our proof is complete.

b) (10 points)

a proof by contradiction:

Suppose if $3n+2$ is even and n is odd. since $3n+2$ is even, so is $3n$,

If we subtract an odd number n from an even number $3n$, we get an odd number, so $3n-n=2n=\text{odd}$. But this is a contradiction.

Therefore our supposition was wrong, and the proof by contradiction is complete.

4. (5 points each)

a) $n^2 \log n$

b) $n^2 (\log n)^2$

c) n^{2^n}

d) $\log n$

5. a) (5 points)

A propositional operator can be viewed as a function from ordered pairs of truth values to truth values.

Example: OR $((F, T)) = T$.

b) (5 points)

A set operator can be viewed as a function from pairs of sets to sets.

Example: Intersec $((\{1,3\}, \{3,4\})) = \{3\}$

6. a) (5 points)

linear: 1,2,3,4, ..., 32, average #: 16.5

binary: 16 \rightarrow 1, 8/24 \rightarrow 2, 4/12/20/28 \rightarrow 3, 2/6/10/14/18/22/26/30 \rightarrow 4,

1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 \rightarrow 5, average = $144/32 = 4.5$

b) (5 points)

linear search: 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 comparison, average 2.5

binary search: average 4

16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1

16 \rightarrow 8 \rightarrow 4 \rightarrow 2

16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 3

16 \rightarrow 8 \rightarrow 4

7. (15/5 points each)

a) $O(s^3)$ - 5 points, Plausible explanation - 10 points

b) $O(s^2)$ - 5 points