1. Given

$m_A = 40\ kg$, $m_B = 30\ kg$

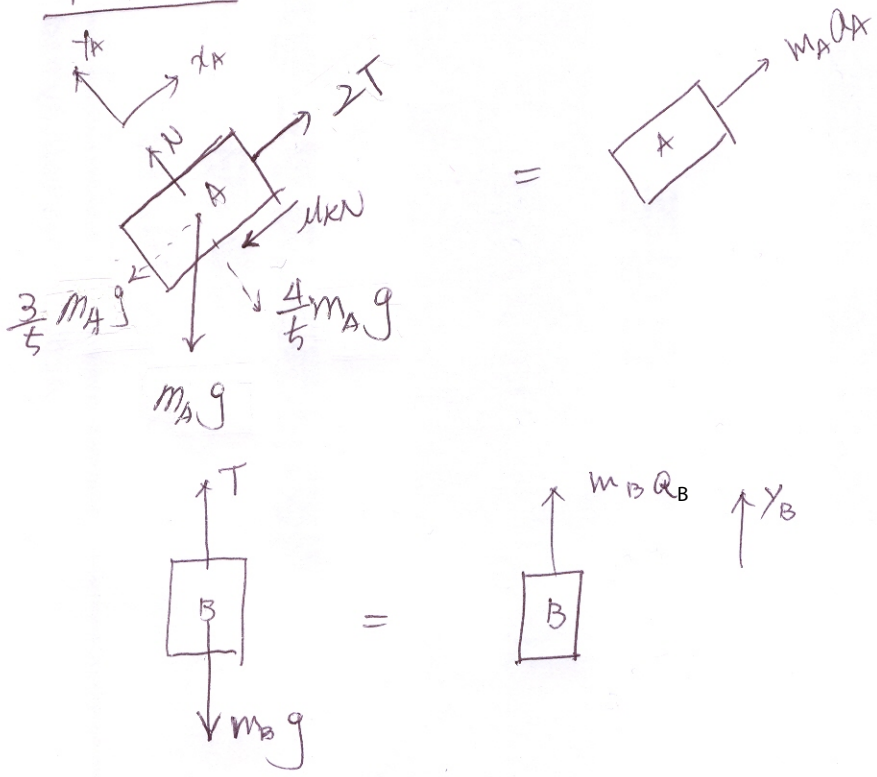
$\mu_k = 0.2$

Constraints

$$2x_A + x_B = const$$
$$2v_A + v_B = 0$$
$$2a_A + a_B = 0$$

(a)

F.B.D (5 points)



Governing Eq. (5 points)

$$\Sigma F_{y_A} = N - \frac{4}{5} m_A g = 0 \Rightarrow N = \frac{4}{5} m_A g$$

$$\Sigma F_{x_A} = 2T - \frac{3}{5} m_A g - \mu_k N$$
$$= 2T - \frac{3}{5} m_A g - \frac{4}{5} m_A g \mu_k = m_A a_A \quad \cdots ①$$

$$\Sigma F_{y_B} = \quad T - m_B g = m_B a_B$$

$$T = m_B (g + a_B) \quad \cdots ②$$

② → ①

eq ① becomes,

$$2 m_B (g + a_B) - \frac{3}{5} m_A g - \frac{4}{5} m_A g \mu_K = m_A a_A \quad (*)$$

By the constraints.

(*) becomes,

$$2 m_B (g - 2a_A) - \frac{3}{5} m_A g - \frac{4}{5} m_A g \mu_K = m_A a_A$$

$$\Rightarrow \quad 2 m_B g - \frac{3}{5} m_A g - \frac{4}{5} m_A g \mu_K = (4 m_B + m_A) a_A$$

$$\therefore \quad a_A = 1.72 \ (m/s^2) \quad \begin{matrix} 3 \\ 4 \end{matrix} \quad (5 \text{ points})$$

(b)

By the constraints

$$a_B = -2a_A$$

$$= -3.43 \ m/s^2$$

$$= 3.43 (m/s^2)(\downarrow)$$ (5 points)

By the eq. ②

$$T = m_B(g + a_B)$$

$$= 20(9.81 + (-3.43))$$

$$= 191.14 \ (N)$$ (5 points)

(c)

By the equation of motion.

$$v_B - \cancel{v_{B0}}^{0} = a_B t$$ (5 points)

$$\therefore v_B = 17.15 \ (m/s)(\downarrow)$$ (5 points)

- Remark

Missing or incorrect unit (-1 point)

Write correct magnitude & direction of your solution.

a) Determine the altitude h of the satellite.

- For gravitational force and a circular orbit.

$$|F_r| = \frac{GMm}{r^2} = \frac{mv^2}{r} \quad \text{or} \quad v = \sqrt{\frac{GM}{r}} \cdots ①$$) 4점

- Let T be the periodic time to complete one orbit.

$$v\,T = 2\pi r \quad \text{or} \quad T\cdot\sqrt{\frac{GM}{r}} = 2\pi r \cdots ②$$) 4점

$$\Rightarrow \quad r = \left(\frac{GM\,T^2}{4\pi^2}\right)^{1/3}$$

- $GM = gR^2 \cdots ③$) 2점

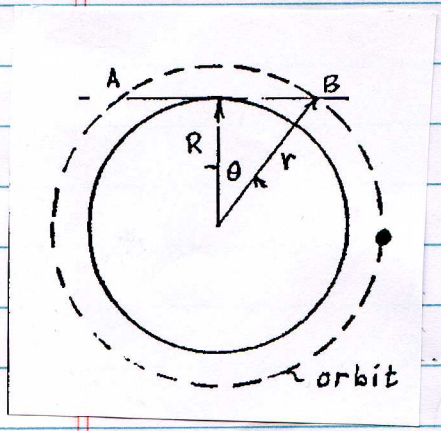
$$\Rightarrow \quad r = 8055.33\ km$$) 5점

$$\therefore\ h = r - R = 8055.33 - 6370 = 1685.33\ km$$

1. 답이 맞을 경우 15점
2. 답이 틀렸을 경우

(식 ①을 구하면 4점
식 ②를 구하면 4점
식 ③을 구하면 2점

b) Determine the time during which the satellite is above the horizon for a observer located at the North Pole.



- $\cos\theta = \frac{R}{r} = 0.791 \qquad \theta = 37.72° \cdots ①$) 5점

- $t_{AB} = \frac{2\theta}{360°}\,T = 1508.8\ s = 25.15\ min$) 5점

① 5점

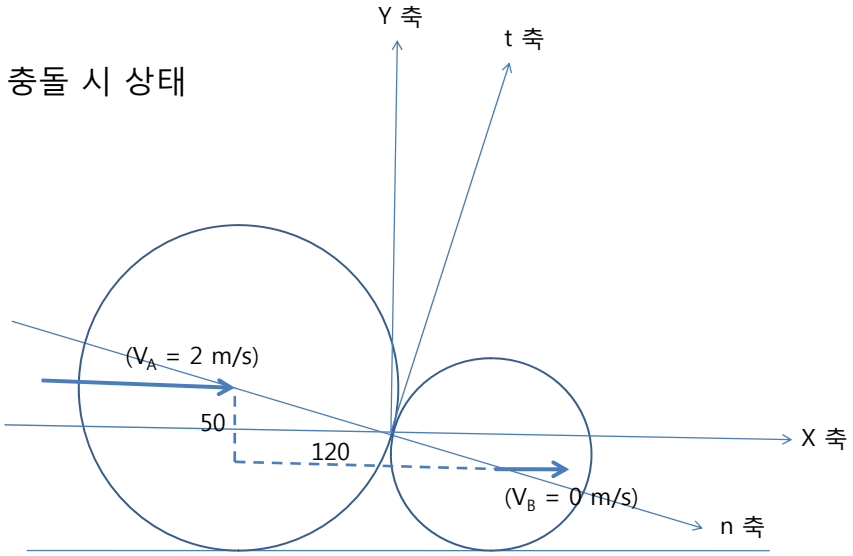
답이 맞을 경우 15점

답이 틀렸을 경우

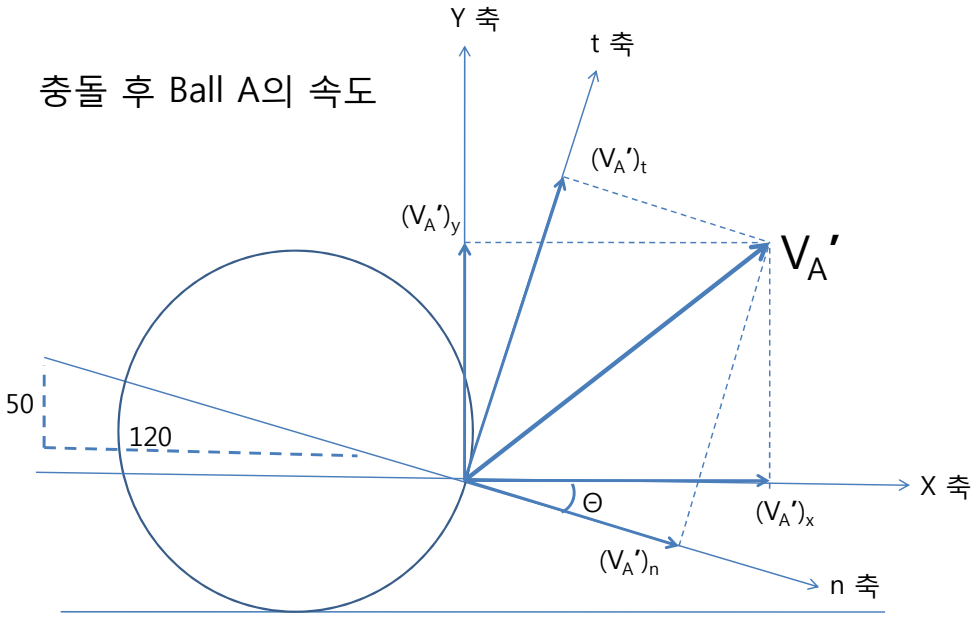
(θ를 구하면 5점
t_{AB}에 관한 식을 제대로 세우면 5점
θ에 대한 시간만 구한경우 2점

3. Total 35 points; 15 points for (a) and 10 points for (b)

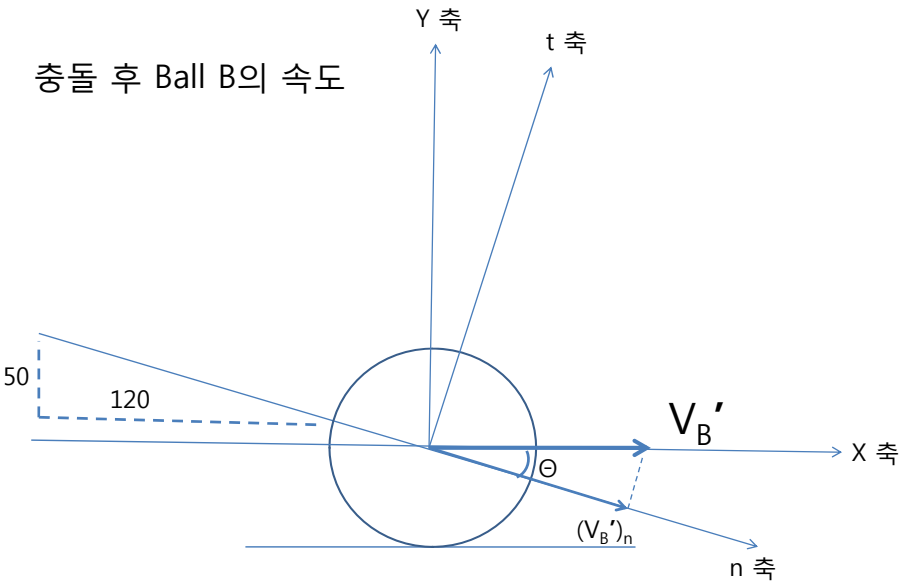
Draw FBDs



------ **(1)**



----- **(2)**



------ **(3)**

Given Conditions

m_A = 23.1 kg, m_B = 23.1 kg

V_o = 2 m/s, h = 0.25 m

Since, the impact occurs obliquely, we will use three conditions

1. $(V_A)_t = (V'_A)_t$ and $(V_B)_t = (V'_B)_t$ ------ **(4)**
2. $m_A(V_A)_n + m_B(V_B)_n = m_A(V'_A)_n + m_B(V'_B)_n$ ------ **(5)**
3. $(V'_B)_n - (V'_A)_n = \boldsymbol{e}\,[\,(V_A)_n - (V_B)_n\,]$ ------ **(6)**

From FBD (1),

$$\boldsymbol{\cos\theta = \left(\frac{120}{130}\right),\ \theta = 22.62°}$$

Using the 3rd condition,

$$\boldsymbol{e} = \frac{(V'_B)n - (V'_A)n}{(V_A)n - (V_B)n} \qquad \text{------ (7)}$$

From FBD (2) and (3)

$(V_A)_n = V_A \cos\Theta$ ------ **(8)**

$(V'_A)_n = (V'_A)_x \cos\Theta - (V'_A)_y \sin\Theta$ ------ **(9)**

$(V'_B)_n = V'_B \cos\Theta$ ------ **(10)**

$(V_B)_n = 0$ (at rest) ------ **(11)**

Plug in conditions with respect to normal direction into ***e***

$$\boldsymbol{e} = \frac{V'_B \cos\theta - (V'_A)n}{V_A \cos\theta} \qquad \text{------ (7 *)}$$

$$\boldsymbol{e} = \frac{(V'_B)\cos\theta - (V'_A)_x \cos\theta + (V'_A)_y \sin\theta}{(V_A)\cos\theta}$$

$$\boldsymbol{e} = \frac{(V'_B) - (V'_A)_x + (V'_A)_y \tan\theta}{(V_A)} \qquad \text{------ (12)} \qquad \textbf{(15 Points)}$$

From FBD (3) with Conservation of Momentum

$$\frac{1}{2} m_B (V'_B)^2 = m_B g h, \quad V'_B = \sqrt{2gh}$$

$$\mathbf{V'_B = \sqrt{2(9.81)(0.25)} = 2.215\ m/s} \qquad \text{------ (13)}$$

Also, using Conservation of Momentum in X-direction

$m_A V_A = m_A (V'_A)_x + m_B V'_B$

$(23.1)(2) = (23.1)(V'_A)_x + (2.1)(2.215)$

$$\mathbf{(V'_A)_x = 1.7986\ m/s} \qquad \text{------ (14)}$$

We need numbers for (8), (9), (10), and (11)

(8) : $(V_A)_n$ = 2 cos(22.62) = **1.8462 m/s** ------ **(15)**

(9) : $(V'_A)_n = (V'_A)_x \cos\Theta - (V'_A)_y \sin\Theta$

Since we don't know $(V'_A)_y$. Use FBD (2), **(4)** and **(14)**.

$(V_A)_t = (V'_A)_t$

$V_O \sin\Theta = (V'_A)_x \sin\Theta + (V'_A)_y \cos\Theta$

$\mathbf{(V'_A)_y = 0.08392\ m/s}$

Therefore, $(V'_A)_n$ = (1.7986)cos22.62 – (0.08392)sin22.62 = **1.628 m/s** ------ **(16)**

(10) : $(V'_B)_n$ = (2.215)cos22.62 = **2.0446 m/s** ------ **(17)**

(11) : $(V_B)_n$ = **0 m/s** ------ **(18)**

Plug in numbers (15), (16), (17), and (18) into **(7) or (12)**

$$e = \frac{(V'_B)n - (V'_A)n}{(V_A)n - (V_B)n} = \frac{2.0446 - 1.628}{1.8462} = \mathbf{0.22565}$$

$$e = \frac{(V'_B) - (V'_A)_x + (V'_A)_y \tan\theta}{(V_A)} = \frac{2.215 - 1.7986 + (0.08392)\tan(22.62)}{2} = \mathbf{0.22568}$$

채점기준

1. **(a)(b):** (b)의 ***e*** 값이 맞으면, (a)의 ***e*** 에 관한 수식과 $(V'_A)_n$, $(V'_A)_t$, $(V'_A)_x$, and $(V'_A)_y$ 의 값을 비교 후 필요한 속도 성분들이 정확히 구해 졌으면 답으로 인정. **(35점)**

2. **(a):** (b)의 ***e*** 값이 틀리고, (a)의 ***e*** 에 관한 수식을 **(7)**까지 세우면 **5점**

3. **(a):** (b)의 ***e*** 값이 틀리고, (a)의 ***e*** 에 관한 수식을 (8)~(11) 이용하여 **(7*)**까지 세우면 **10점**
   (단 FBD를 그리고, 그에 따른 충돌 전, 후 Notation을 명확히 표시한 경우)

4. **(b):** V'_B 을 구하면 2점
   (b): $(V'_A)_x$ 를 구하면 2점
   (b): $(V'_A)_n$ 을 구하면 3점

5. **$(V'_A)_n$은 단순히 $(V'_A)_x \cos\Theta$가 절대 아님**. **FBD (2) 참고**