Engineering Mathematics

- 1st Exam -

Saturday, March 31th, 2012

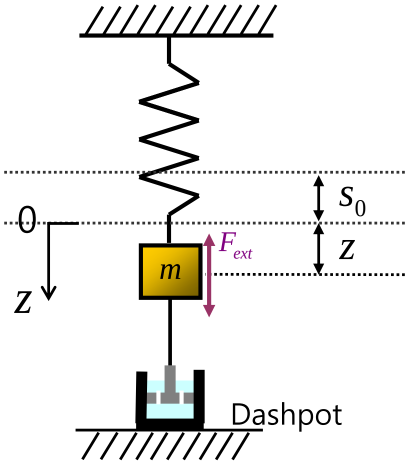
Time: 13:00-16:00 (3 hours)

Name	
SNU ID #	

Note: Budget your time wisely. Some parts of this exam could take you much longer time than others. Move on if you are stuck and return to the problem later.

Problem Number		1		2			3		4				5			6			
		1	2	3	1	2	1	2	1	2	3	4	1	2	3	1	2	3	Total
Grader	max	5	15	3	3	4	5	10	5	5	10	5	2	8	5	5	5	5	100

1. Consider the following figure about a mass-spring-damping system and answer the questions.



Mass: m

Damping coefficient of the dashpot: c

Spring coefficient: k

Gravitational force mg is exerted on the mass.

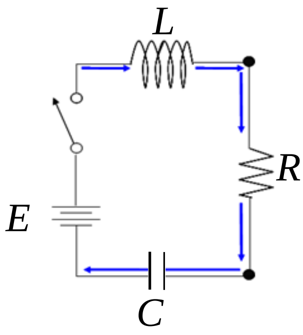
The mass is in static equilibrium at $z = 0$.

s_0 is an elongation of spring in static equilibrium state.

1.1. Model the equation of motion for the mass-spring-damping system by drawing a free-body diagram. **[5 points]**

1.2. Find the motion of the mass $z(t)$ with $m = 1kg$, $c = 6N \cdot s/m$, $k = 18N/m$, when the external force and the initial values are given as $F_{ext} = \cos 3t - 3\sin 3t \ N$, $z(0) = 4m$, and $dz(0)/dt = -8m/s$. **[15 points]**

2. Consider the following figure about a LRC-circuit and answer the questions.



Name	Symbol	Notation	Unit
Ohm's Resister		R	Ohm's(Ω)
Inductor		L	Henrys(H)
Capacitor		C	Farads(F)
Derivative of $E_0 \omega cost$ electromotive force		E	Volts(V)

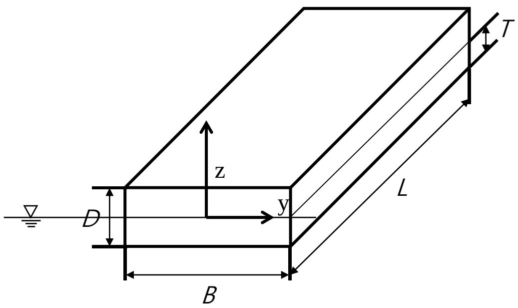
2.1. When we check the analogy of electrical and mechanical quantities, the inductance L corresponds to the mass m and, indeed, an inductor opposes a change in current, having an "inertia effect" similar to that of a mass. The resistance R corresponds to the damping constant c, and a resistor causes loss of energy, just as a damping dashpot does. Likewise, the reciprocal of capacitance C, which is $1/C$, corresponds to the spring-modulus k and derivative of E corresponds to the external force F. Model a differential equation for the RLC-circuit.

[3 points]

2.2. Find a general solution (transient current) $I_G(t)$ in the RLC-circuit with $R = 10\Omega$, $L = 0.1\text{H}$, $C = \frac{1}{340}\text{F}$, which is connected to a source of voltage $E(t) = e^{-t}(160.1\sin t - 169.9\cos t)\text{V}$. **[3 points]**

2.3. When the initial values are given that the current and charge are zero when $t = 0$, find the current $I(t)$. (here, current means the quantity of electricity per unit time and charge means the quantity of electricity of capacitor) **[4 points]**

3. A barge ship is floating on the sea as shown in following figure.



L=40m, B=10m, D=10m, T=5m
Density of sea water: $\rho \approx 1\mathrm{Mg/m^3}$
Damping coefficient: c =4.8MN·s/m
Added mass m_{added} is same as the mass of the ship.
The ship is in static equilibrium state at z=0

The equation of heave motion of the barge ship is as follows:

$$
\begin{aligned}
\mathbf{M}\ddot{\mathbf{z}} = \sum \mathbf{F} &= (Body\ \ Force) + (Surface\ \ Force) \\
&= \mathbf{F}_{Gravity}(\mathbf{z}) + \mathbf{F}_{Fluid}(\mathbf{z}) \\
&= \underbrace{\mathbf{F}_{Gravity}(\mathbf{z}) + \mathbf{F}_{Buoyancy}(\mathbf{z})}_{\mathbf{F}_{\mathrm{Re}storing}(\mathbf{z})} + \underbrace{\mathbf{F}_{F.K}(\mathbf{z}) + \mathbf{F}_{D}(\mathbf{z})}_{\mathbf{F}_{Exciting}} + \underbrace{\mathbf{F}_{R,Mass}(\mathbf{z},\ddot{\mathbf{z}})}_{-m_{added}\ddot{\mathbf{z}}} + \underbrace{\mathbf{F}_{R,Damping}(\mathbf{z},\dot{\mathbf{z}})}_{-c\dot{\mathbf{z}}} \\
&= \mathbf{F}_{\mathrm{Re}storing}(\mathbf{z}) + \mathbf{F}_{Exciting} - m_{added}\ddot{\mathbf{z}} - c\dot{\mathbf{z}}
\end{aligned}
$$

Gravitational force acting on the ship is considered as a body force.

$$(Body\ \ Force) = \mathbf{F}_{Gravity}(\mathbf{z})$$

Hydromechanical force acting on the ship is considered as a surface force.

$$(Surface\ \ Force) = \mathbf{F}_{Fluid}(\mathbf{z},\dot{\mathbf{z}},\ddot{\mathbf{z}})$$

The Hydromechanical force acting on the ship is expressed as follows:

$$\mathbf{F}_{Fluid}(\mathbf{z},\dot{\mathbf{z}},\ddot{\mathbf{z}}) = \mathbf{F}_{Buoyancy}(\mathbf{z}) + \mathbf{F}_{F.K}(\mathbf{z}) + \mathbf{F}_{D}(\mathbf{z}) + \mathbf{F}_{R,Mass}(\mathbf{z},\ddot{\mathbf{z}}) + \mathbf{F}_{R,Damping}(\mathbf{z},\dot{\mathbf{z}})$$

$\mathbf{F}_{Gravity}(\mathbf{z})$ and $\mathbf{F}_{Buoyancy}(\mathbf{z})$ are restoring force $\mathbf{F}_{\mathrm{Re}storing}(\mathbf{z})$.

$\mathbf{F}_{F.K}(\mathbf{z})$ and $\mathbf{F}_{D}(\mathbf{z})$ are external exciting force $\mathbf{F}_{Exciting}(\mathbf{z})$ which is generated by ocean wave.

$\mathbf{F}_{R,Damping}(\mathbf{z},\dot{\mathbf{z}})$ and $\mathbf{F}_{R,Mass}(\mathbf{z},\ddot{\mathbf{z}})$ are radiation force which are generated by motion of a ship. $\mathbf{F}_{R,Damping}(\mathbf{z},\dot{\mathbf{z}})$ is proportional to the velocity of a ship, and $\mathbf{F}_{R,Mass}(\mathbf{z},\ddot{\mathbf{z}})$ is proportional to the acceleration of a ship.

$\mathbf{F}_{R,Damping}(\mathbf{z},\dot{\mathbf{z}}) = -c\dot{\mathbf{z}}$.

$\mathbf{F}_{R,Mass}(\mathbf{z},\ddot{\mathbf{z}}) = -m_{added}\ddot{\mathbf{z}}$.

Answer the following questions

3.1 Model the equation of heave motion for the barge ship by drawing a free-body diagram. **[5 points]**

3.2. Find the motion of the ship $z(t)$ when the exciting force and the initial values are given as $F_{exciting} = \sin(0.25\pi t)MN$, $z(0) = 0m$, and $dz(0)/dt = 0m/s$. **[10 points]**

4. In the same manner with the heave motion of the ship, the equation of roll motion is as follows:

$$
\begin{aligned}
I\ddot{\phi} = \sum M &= (Body\ \ Force) + (Surface\ \ Force) \\
&= \overbrace{\mathbf{M}_{Gravity}(\phi)} + \overbrace{\mathbf{M}_{Fluid}(\phi)} \\
&= \underbrace{\mathbf{M}_{Gravity}(\phi) + \mathbf{M}_{Buoyancy}(\phi)} + \underbrace{\mathbf{M}_{F.K}(\phi) + \mathbf{M}_{D}(\phi)} + \underbrace{\mathbf{M}_{R,Mass}(\phi,\ddot{\phi})} + \underbrace{\mathbf{M}_{R,Damping}(\phi,\dot{\phi})} \\
&= \mathbf{M}_{Restoring}(\phi) + \mathbf{M}_{Exciting} - I_{added}\ddot{\phi} - B\dot{\phi}
\end{aligned}
$$

$\mathbf{M}_{Gravity}(\phi)$ and $\mathbf{M}_{Buoyancy}(\phi)$ are restoring moment $\mathbf{M}_{Restoring}(\phi)$.

$\mathbf{M}_{F.K}(\phi)$ and $\mathbf{M}_{D}(\phi)$ are exciting moment $\mathbf{M}_{Ecxiting}(\phi)$.

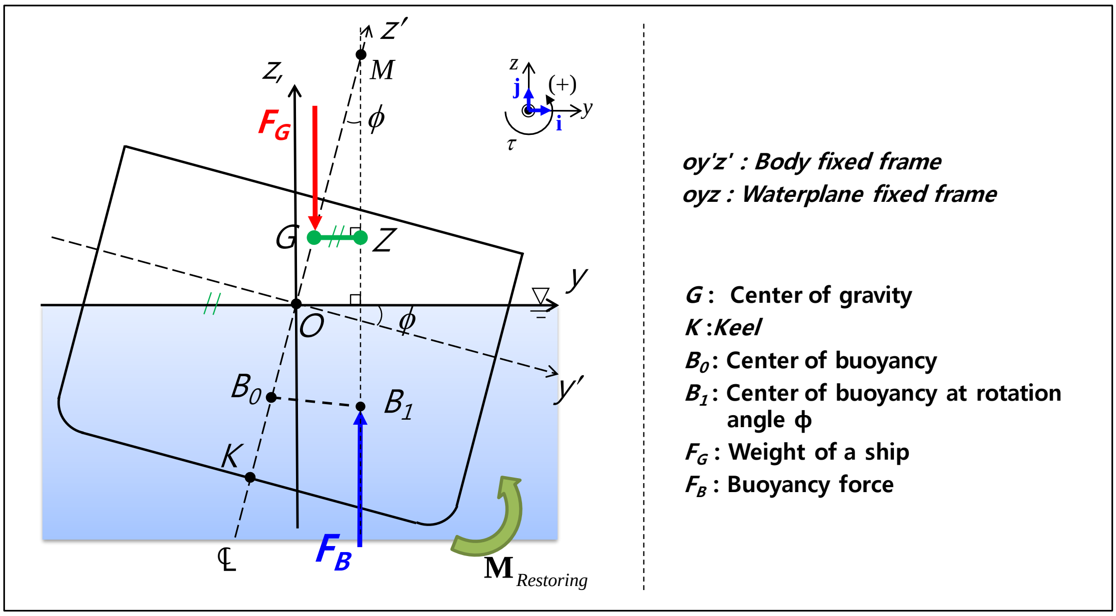
$\mathbf{M}_{R,Damping}(\phi,\dot{\phi})$ and $\mathbf{M}_{R,Mass}(\phi,\ddot{\phi})$ are radiation moment which are generated by the motion of a ship.

$\mathbf{M}_{R,Damping}(\phi,\dot{\phi})$ is proportional to the angular velocity of roll motion.

$\mathbf{M}_{R,Mass}(\phi,\ddot{\phi})$ is proportional to the angular acceleration of roll motion.

$$\mathbf{M}_{R,Damping}(\phi,\dot{\phi}) = -B\dot{\phi}$$

$$\mathbf{M}_{R,Mass}(\phi,\ddot{\phi}) = -I_{added}\ddot{\phi}$$



M : For a ship in upright position, the line of action of the buoyant force before inclination is the centerline, which is denoted by dot-point line. The new line of action of the buoyant force after inclination passes through the changed center of buoyancy B_1, and is perpendicular to the waterplane. The two lines intersect in the point M, called metacenter.

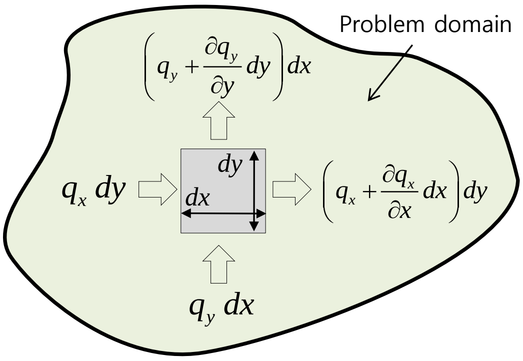
4.1 Derive transverse restoring moment with proper assumption. **[5 points]**

4.2 Model the equation of roll motion for the ship. **[5 points]**

4.3 Find a general solution $\phi(t)$ with assumption that there is no exciting moment and damping moment. **[10 points]**

4.4 Explain the relation of GM and Period of Roll motion. **[5 points]**

5. Considering the two-dimensional heat conduction problem, the governing equation is derived as follows:



$$\frac{\partial}{\partial x}\left(k\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial \phi}{\partial y}\right) + Q - \rho c \frac{\partial \phi}{\partial t} = 0 \quad \text{-(1)}$$

, where $\phi(x, y, t)$: Temperature distribution.
Q : Quantity of the heat
k : Thermal conductivity
ρ : Density
c : Specific heat

5.1. Suppose that steady-state conditions are assumed and only one-dimensional variation in x-direction occurs. Simplify the governing equation (1). **[2 points]**

5.2. Suppose that the quantity of the heat Q is equal to $-\phi$ and thermal conductivity k is 1 in the simplified governing equation derived from the problem 4.1. Determine a function $\phi(x)$ which satisfies the simplified governing equation in the region $0 < x < 1$ with associated boundary conditions $\phi = 0$ at $x = 0$ and $\phi = 2$ at $x = 1$ using the finite difference method. **[8 points]**

Assumption:

- Mesh spacing $\Delta x = \frac{1}{3}$ is chosen
- Central difference approximation is used for the derivative of the function.

5.3. $\phi(x) = \frac{2}{e - 1/e} e^x + \frac{2}{-e + 1/e} e^{-x}$ is an exact solution of the problem 5.2. Compare the function values determined from the problem 5.2 at x=1/3 and x=2/3 with the exact solution. **[5 points]**

6. Considering following Bernoulli equation, answer the questions.

$$\rho \frac{\partial \Phi}{\partial t} + P + \frac{1}{2}\rho \left|\nabla \Phi\right|^2 + \rho g z = F(t)$$

6.1. In case that $F(t)$ of Bernoulli equation is expressed by zero, pressure P means "gauge pressure" as shown in following equation. **[5 points]**

$$\rho \frac{\partial \Phi}{\partial t} + P_{Fluid} + \frac{1}{2}\rho \left|\nabla \Phi\right|^2 + \rho g z = 0$$

Derive the above Bernoulli equation, which is expressed with gauge pressure, and explain what gauge pressure is.

6.2. Determine the hydrostatic force exerted on the floating body from Bernoulli equation. **[5 points]**

Hints:

- P_{Fluid} is classified into hydrodynamic pressure $P_{Dynamic}$ and hydrostatic pressure P_{Static} , and the hydrostatic force is related to the hydrostatic pressure.

- The differential force, which is exerted on the infinitesimal area of the wetted surface, is $d\mathbf{F} = P \cdot d\mathbf{S} = P \cdot \mathbf{n} dS$, where $\mathbf{n}$ is surface normal vector.

- The force exerted on the floating body can be determined by integrating the differential force over the wetted surface area S_B.

6.3. Explain Archimedes' principle using the answer of problem 6.2. **[5 points]**

Hint:- divergence theorem: $\iint_S f \cdot \mathbf{n} dA = \iiint_V \nabla f dV$.