4.6 Compute the maximum fusion power density of a magnetically confined d-h fusion plasma ($N_d = N_h$, $T_d = T_h = T_e$, no impurities) as limited by an assumed upper value of attainable field strength, B=15 Tesla, dependent on the plasma temperature. Superimpose the result for $\beta_{\max} = 2\ \%$ in Fig.4.3.

Solution)

$$N_d = N_h = \frac{N_i}{2}, \quad N_e = \frac{3}{2} N_i$$

$$\beta_{\max} \frac{B^2}{2\mu_0} \geq N_i k T_i + N_e k T_e \qquad \left(T_{\mathrm{i}} = T_e = T\right)$$

$$\beta_{\max} \frac{B^2}{2\mu_0} = N_i kT + \frac{3}{2} N_i kT$$

$$N_i = \frac{\beta_{\max} B^2}{5\mu_0} \frac{1}{kT}$$

$$\begin{aligned} P_{fu} &= N_d N_h < \sigma v >_{dh} Q_{dh} \\ &= \frac{N_i^2}{4} < \sigma v >_{dh} Q_{dh} \\ &= \frac{1}{4} \frac{\beta_{\max}^2 B^4}{25\mu_0^2} \frac{< \sigma v >_{dh}}{\left(kT\right)^2} Q_{dh} \end{aligned}$$

$$\therefore P_{fu,\max} = 3.21 \times 10^5 \quad [\mathrm{w/m^3}]$$

8.5 Derive-analogously to Eq. (8.28)-a more realistic MCF reactor criterion accounting also for cyclotron radiation losses Display graphically its temperature dependence, $N_e \tau_{E^*}\left(T\right)$ for the two cases

Note: For the derivation of the criterion, use fractions of the ion density such that $N_j = \kappa_j N_i$ with j denoting the considered ion species. You should finally obtain:

$$\left(N_e \tau_{E^*}\right)_{ab} > \frac{\frac{3}{2}\left(\frac{T_i}{\sum \kappa_j Z_j} + T_e\right)}{\frac{\eta_{in}\eta_{out}}{1-\eta_{in}\eta_{out}}\left[\frac{\kappa_a \kappa_b}{\left(\sum \kappa_j Z_j\right)^2}\frac{<\sigma v>_{ab}}{(1+\delta_{ab})}Q_{ab}\right] - \frac{\sum \kappa_j Z_j^2}{\sum \kappa_j Z_j} A_{br}\sqrt{T_e} - A_{cyc}B^2 \frac{\psi}{N_e} T_e}$$

for j=a, b, impurities.

Solution)

(Required input power) ≤ (Output electric power)

$$P_{th} + P_{rad} + P_{cyc} \le \eta_{in}\eta_{out}\left(P_f + P_{rad} + P_{th} + P_{cyc}\right)$$
$$\rightarrow P_{th} + P_{rad} + P_{cyc} = \frac{\eta_{in}\eta_{out}}{1-\eta_{in}\eta_{out}} P_f$$

$$\sum Z_j \kappa_j N_i = N_e \quad \rightarrow \quad N_i = \frac{N_e}{\sum Z_j \kappa_j}$$

$$\frac{3}{2}\frac{\frac{\sum \kappa_j}{\sum Z_j \kappa_j} N_e T_i + N_e T_e}{\tau_{E^*}} + A_{br} N_e^2 \frac{\sum Z_j^2 \kappa_j}{\sum Z_j \kappa_j}\sqrt{T_e} + A_{cyc} B^2 N_e T_e \psi$$
$$= \frac{\eta_{in}\eta_{out}}{1-\eta_{in}\eta_{out}} \frac{\kappa_a \kappa_b N_e^2 <\sigma v>_{ab}}{\left(\sum Z_j \kappa_j\right)^2 (1+\delta_{ab})} Q_{ab}$$

$$\therefore \left(N_e \tau_{E^*}\right)_{ab} > \frac{\frac{3}{2}\left(\frac{T_i}{\sum \kappa_j Z_j} + T_e\right)}{\frac{\eta_{in}\eta_{out}}{1-\eta_{in}\eta_{out}}\left[\frac{\kappa_a \kappa_b}{\left(\sum \kappa_j Z_j\right)^2}\frac{<\sigma v>_{ab}}{(1+\delta_{ab})}Q_{abe}\right] - \frac{\sum \kappa_j Z_j^2}{\sum \kappa_j Z_j} A_{br}\sqrt{T_e} - A_{cyc}B^2 \frac{\psi}{N_e} T}$$