Computer Aided Ship Design

- 3rd Exam -

Thursday, December 22th, 2011

Time: 10:00-13:00 (3 hours)

Name	
SNU ID #	

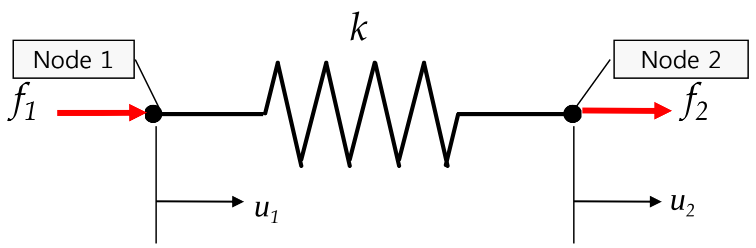
Note: Budget your time wisely. Some parts of this exam could take you much longer time than others. Move on if you are stuck and return to the problem later.

FEM and Grillage

Problem Number		1			2	3		4				Total
		1	2	3	-	1	2	1	2	3	4	
Grader	Max	5	10	10	20	10	10	5	10	15	5	100
	Score											

1. Derivation of the stiffness equation of a bar element.

1.1. Consider a bar element as a linear spring element, and derive the stiffness equation $\mathbf{Kd} = \mathbf{F}$ for a bar element by using the **direct equilibrium approach. [5 points]**

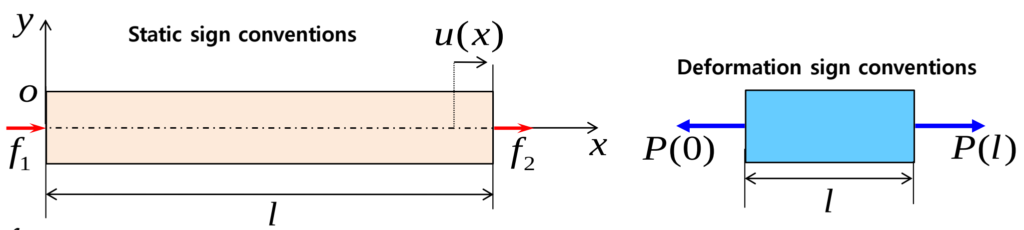


f_1, f_2 : Nodal forces acting on the ends of the bar element

u_1, u_2 : Nodal displacement at the ends of the bar element

Figure 1 Linear spring element with nodal displacements and forces

1.2 Derive the differential equation governing the following linear-elastic bar element. **[10 points]**



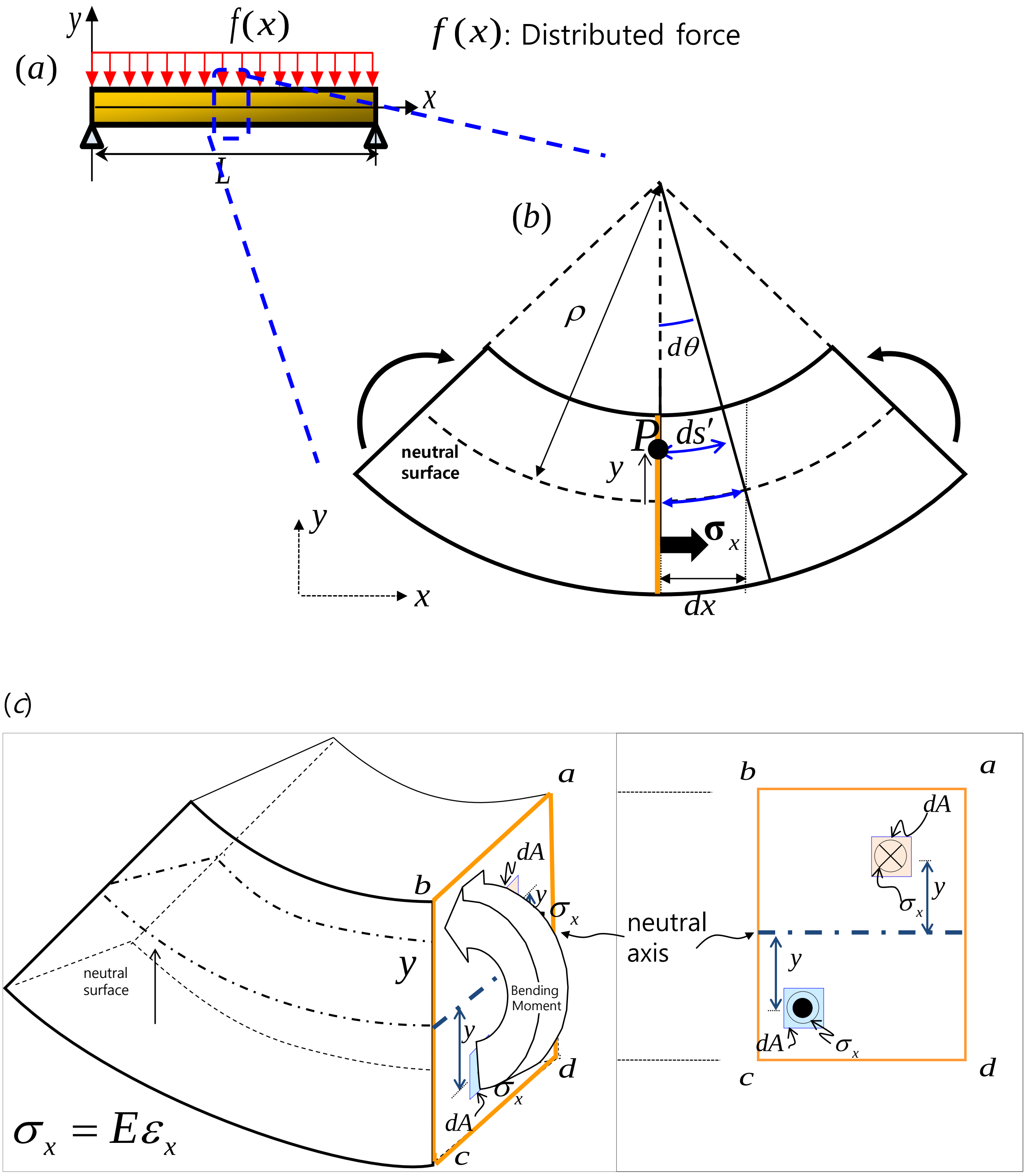
f_1, f_2 : Nodal forces acting on the ends of the bar element

1) The Nodal forces f_1 and f_2 are acting on the ends of the bar element.
2) There is no distributed force.
3) P(0) and P(l) are the stress resultant(tensile forces) at the end of the bar.

Figure 2 Linear-elastic bar element with the nodal displacements and forces

1.3 Derive the stiffness equation $\mathbf{Kd} = \mathbf{F}$ of the bar element by applying the **Galerkin's residual method** to the differential equation derived from equation 1.2, and show that the stiffness equations derived by using the Galerkin's residual method and the direct equilibrium approach are the same. **[10 points]**

2. Derivation of deflection curve of beam **[20 points]**



E : Young's modulus, $I = \int_A y^2 dA$, σ_x : stress in x-direction, ε_x: strain in x-direction

Figure 3 Beam element under distributed load and differential beam element

1) Derive the strain in x-direction at the point P in figure 3-(b).

2) Derive the stress in x-direction at the point P in figure 3-(b).

3) Derive the force in x-direction exerted on the area dA in figure 3-(c).

4) Derive the bending moment about neutral axis in figure 3-(c).

5) Derive the deflection curve of beam $M / EI = d^2 y / dx^2$

6) Mark the shear force and bending moment caused by the distributed load $f(x)$ in the following figure and derive the equations relating the bending moment and the shear force, and relating the shear force and the distributed load.

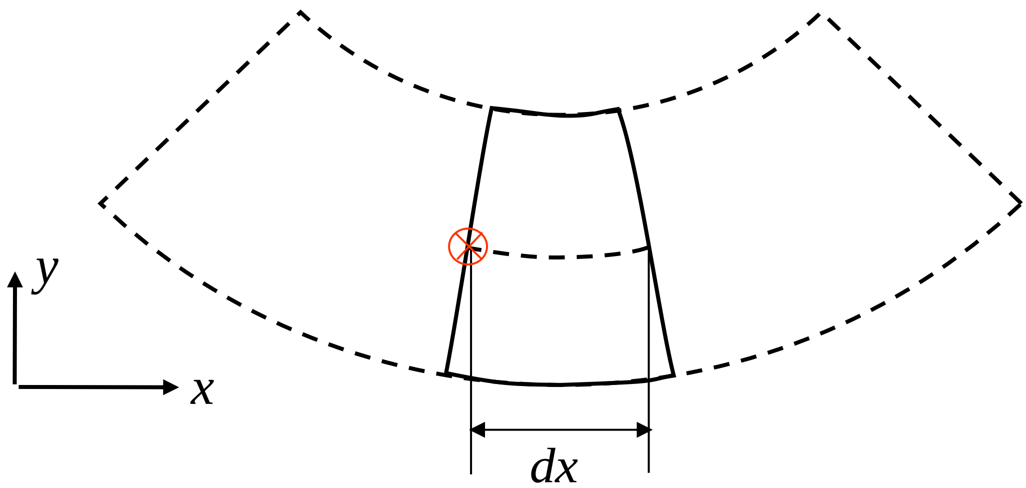


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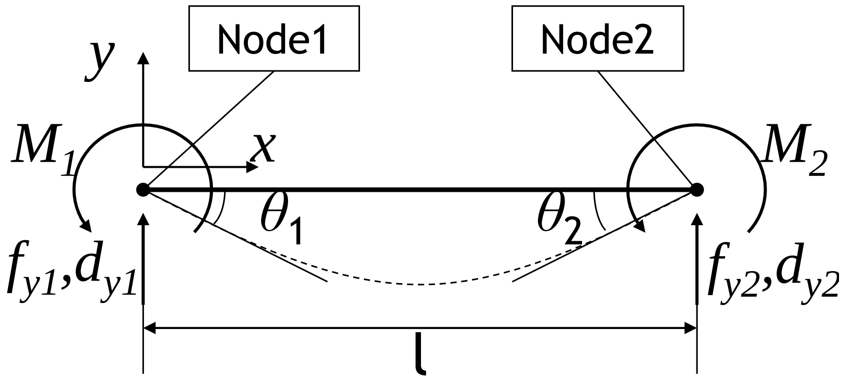
Figure 4 Differential beam element

7) Derive the deflection curve of beam $EI \cdot d^4 y / dx^4 = -f(x)$

3. Derivation of the stiffness equation of a beam element.



M_i **: nodal moments at the node i**

f_{yi} **: nodal forces in y-direction at the node i**

d_{yi} **: nodal displacements at the node i**

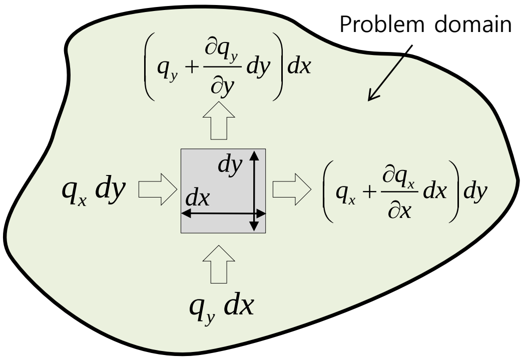
θ_i **: nodal rotations at the node i**

Figure 5 Beam element with nodal displacements, rotations, forces, and moments

3.1 Derive the stiffness equation $\mathbf{Kd} = \mathbf{F}$ of beam element by applying the **direct equilibrium approach** to the differential equations derived from the problem 2. **[10 points]**

3.2 Derive the stiffness equation $\mathbf{Kd} = \mathbf{F}$ of the beam element by applying the **Galerkin's residual method** to the differential equation derived from the problem 2, and show that the stiffness equations derived using the Galerkin's residual method and the equilibrium approach are the same. **[15 points]**

4. Considering the two-dimensional heat conduction problem, the governing equation is derived as following



$$\frac{\partial}{\partial x}\left(k\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial \phi}{\partial y}\right)+Q-\rho c\frac{\partial \phi}{\partial t}=0 \quad \text{-(1)}$$

, where $\phi(x, y, t)$: Temperature distribution.

Q : Quantity of the heat

k : Thermal conductivity

ρ : Density

c : Specific heat

Figure 6 Problem of heat flow in a two-dimensional domain

4.1 Suppose that steady-state conditions are assumed and only one-dimensional variation in x-direction occurs. Simplify the governing equation (1). **[5 points]**

4.2 Suppose that the quantity of the heat Q is equal to $-\phi$ and thermal conductivity k is 1 in the simplified governing equation derived from the problem 4.1. Determine a function $\phi(x)$ which satisfies the simplified governing equation in the region $0<x<1$ with associated boundary conditions $\phi=0$ at $x=0$ and $\phi=2$ at $x=1$ using the finite difference method. **[10 points]**

Assumption:

- Mesh spacing $\Delta x=\frac{1}{3}$ is chosen
- Central difference approximation is used for the derivative of the function.

4.3 Determine a function $\phi(x)$ which satisfies the governing equation and boundary conditions of the problem 4.2 using the Galerkin's residual method. **[15 points]**

Assumption: An approximated function is assumed as $\hat{\phi}=a_1+a_2x+a_3x^2$

4.4 $\phi(x)=\frac{2}{e-1/e}e^{x}+\frac{2}{-e+1/e}e^{-x}$ is an exact solution of the problem 4.2 and 4.3. Compare the function values determined from the problem 4.2 and 4.3 at x=1/3 and x=2/3 with the exact solution. **[5 points]**