Optoelectronics EE 430.423.001 Final Exam 2016. 12. 15.

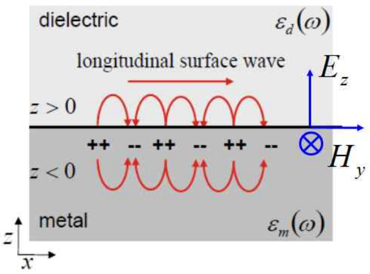
1. (20 point) Consider a nonconducting, isotropic dielectric material where the electrons are bound to the atoms and the number of electrons per unit volume is N.

(1) Calculate the complex refractive index $\widetilde{N}(\omega)=n+i\kappa$ by considering the bound electrons as classical damped harmonic oscillators under the applied electric field with the harmonic time dependence $e^{-i\omega t}$. Assume each electron with a mass m and charge $-e$ is elastically bound to its equilibrium position with a force constant K and the damping constant γ.

(2) Plot the index of refraction n and extinction coefficient κ as a function of the frequency ω.

2. (10 point) Assume the metal is a lossless (*i.e.*, no damping $\gamma=0$) free electron gas with the electron density N. Calculate the plasma frequency ω_p and the dielectric constant $\epsilon(\omega)$. Explain the behavior of light with frequency $\omega>\omega_p$ and $\omega<\omega_p$.

3. (30 point) Assume that the metal/dielectric interface is defined by z = 0 and that the lower half space (z < 0) is filled with a metal. The dielectric constant of dielectric and metal is $\epsilon_d(\omega)$ and $\epsilon_m(\omega)$, respectively. The TM polarized (p-polarized) wave is propagating at the metal/dielectric interface as shown in the figure. The wavevector is decomposed as $\vec{k}=k_z\hat{z}+k_x\hat{x}$.



(1) Show that the following relation should be satisfied using the boundary conditions at z = 0 for the Maxwell equations.

$$\frac{k_{zd}}{\epsilon_d}+\frac{k_{zm}}{\epsilon_m}=0$$

(2) Show that k_{zd} and k_{zm} are imaginary, *i.e.*, the waves are evanescent with respect to the z axis.

(3) Show that the dispersion relation for the surface plasmon polariton (SPP) is given by

$$k_x=\frac{\omega}{c}\left(\frac{\epsilon_d\,\epsilon_m}{\epsilon_d+\epsilon_m}\right)^{1/2}.$$

(4) Determine the surface plasmon frequency ω_{sp} for which $k_x\rightarrow\infty$. Assume the metal is a lossless free electron gas with plasma frequency ω_p.

(5) Plot the dispersion relation for the SPP mode and the photon in air. Explain why radiation incident from air cannot excite the SPP mode.

(6) Explain the Kretschmann configuration for exciting the SPP mode.

4. (20 point) In the principle coordinate system of a crystal, the dielectric tensor ϵ is given by

$$\epsilon=\begin{pmatrix}\epsilon_x & 0 & 0\\ 0 & \epsilon_y & 0\\ 0 & 0 & \epsilon_z\end{pmatrix}.$$

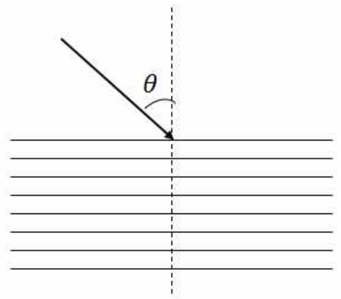
Then the equation for the normal surface can be written as

$$\frac{\omega^4}{c^4}-\frac{\omega^2}{c^2}\left(\frac{k_x^2+k_y^2}{n_z^2}+\frac{k_z^2+k_x^2}{n_y^2}+\frac{k_y^2+k_z^2}{n_x^2}\right)+\left(\frac{k_x^2}{n_y^2n_z^2}+\frac{k_y^2}{n_x^2n_z^2}+\frac{k_z^2}{n_x^2n_y^2}\right)(k_x^2+k_y^2+k_z^2)=0$$

Here $n_i^2=\dfrac{\epsilon_i}{\epsilon_o}$ $(i=x,y,z)$ and $\vec{k}=\dfrac{\omega}{c}n\hat{s}$, where $\hat{s}$ is a unit vector in the direction of propagation.

(1) Calculate the equation for the normal surface in a positive uniaxial crystal $(n_o<n_e)$ where $n_o^2=\dfrac{\epsilon_x}{\epsilon_o}=\dfrac{\epsilon_y}{\epsilon_o}$ (ordinary index) and $n_e^2=\dfrac{\epsilon_z}{\epsilon_o}$ (extraordinary index). And plot the cross section of the normal surfaces with the *yz* plane.

(2) When a wave is incident on the positive uniaxial crystal having the optic axis parallel to the boundary and parallel to the plane of incidence as shown in the figure, draw the direction and polarization of the refracted waves for the case of the ordinary ray and the extraordinary ray, respectively.



Positive Uniaxial Crystal (n_e>n_o)

5. (20 point)

(1) Explain the selection rules for an allowed dipole transition in the case of LS coupling.

(2) When a magnetic field is applied to the atom, explain the spectral lines and polarization for the light emitted for the transition from $2p$ state to the ground state in a direction perpendicular to the magnetic field.