Hyungmin Park

- *Write CLEARLY and describe ALL details of your work (answers can be written in English or Korean).*

1. (10 points) Explain the differences between the 'forced vortex' and 'free vortex'.

2. (10 points) Explain the Kelvin's (circulation) Theorem.

3. (15 points) From the momentum equation given below; derive the vorticity equation. You may use the vector identities provided in the APPENDIX.

$$\frac{D\overline{u}}{Dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\overline{u} \qquad \text{(Momentum equation)}$$

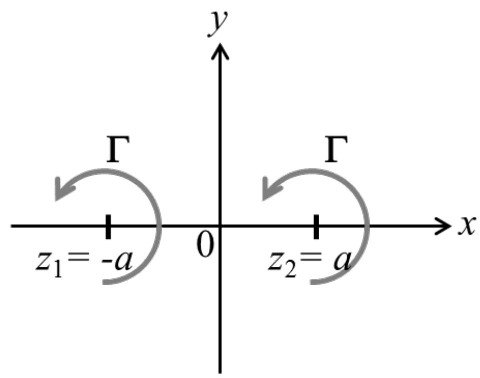
4. (15 points) A long circular cylinder of diameter D [m] is set horizontally in a free stream of velocity U [m/s] and caused to rotate clockwise at ω [rad/s]. Obtain an expression, in terms of ω and U, for the ratio of the pressure difference between the top and bottom of the cylinder to the dynamic pressure of the free stream.

5. (20 points) Consider a two-dimensional, incompressible, irrotational fluid flow. In this case, we can describe the flow with a complex potential as $F(z) = \phi(z) + i\psi(z)$, where $z = x + iy$ (or $Re^{i\theta}$). Here, $\phi(z)$ and $\psi(z)$ are the velocity potential and stream function, respectively.

   (a) (10 points) Describe the flow given by $F(z) = z^2$, both mathematically and with a graphical sketch.

   (b) (10 points) Can this flow represent the flow around an object? If so, derive the pressure coefficient defined as $C_p = (p - p_\infty)/(0.5\rho U^2)$ along the wall, where p_∞ and U are the reference pressure and velocity, respectively.

6. (15 points) Let's consider a two-dimensional potential flow where we have two point vortices which are located at $x = -a$ and $x = a$, respectively, and have a same sign and magnitude of circulation Γ. They are not fixed; i.e., they can freely move in the fluid flow.

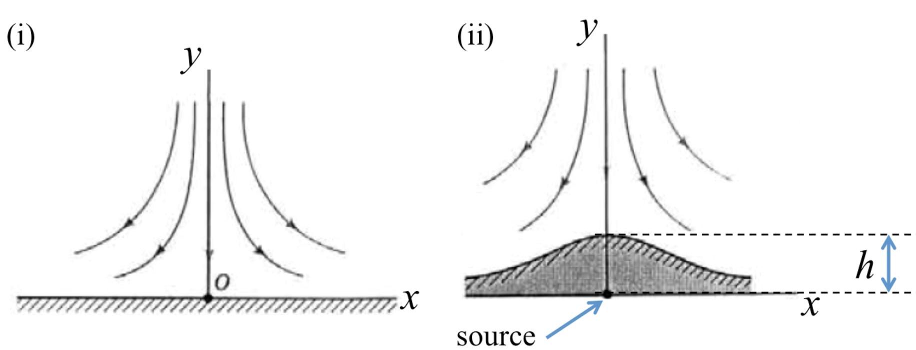


(a) (5 points) Calculate the velocity induced on each vortex by its counterpart.

(b) (5 points) Sketch the trajectory of two vortices, based on your physical intuition and result of (a), and explain their motion.

(c) (5 points) We want to place a third vortex of strength Γ_2 such that all three vortices are fixed at their locations without an induced motion (i.e., zero induced velocity). Where should we put the third vortex and what is the necessary circulation Γ_2?

7. (15 points) Potential flow against a flat plate (figure (i)) can be described with the stream function of $\psi = Axy$, where A is a constant. This type of flow is commonly called a 'stagnation point flow' since it can be used to describe the flow in the vicinity of the stagnation point ***O***. By adding a source of strength m at O ($\psi = m\theta$), stagnation point flow against a flat plate with a 'bump' is obtained as shown in figure (ii). Determine the bump height, h, as a function of the constant A and the source strength m.



APPENDIX

In the following formulas, ϕ is any scalar and a, b, and c are any vectors.

$$\nabla \times \nabla\phi = 0$$

$$\nabla \cdot (\phi \mathbf{a}) = \phi \nabla \cdot \mathbf{a} + \mathbf{a} \cdot \nabla\phi$$

$$\nabla \times (\phi \mathbf{a}) = \nabla\phi \times \mathbf{a} + \phi(\nabla \times \mathbf{a})$$

$$\nabla \cdot (\nabla \times \mathbf{a}) = 0$$

$$(\mathbf{a} \cdot \nabla)\mathbf{a} = \tfrac{1}{2}\nabla(\mathbf{a} \cdot \mathbf{a}) - \mathbf{a} \times (\nabla \times \mathbf{a})$$

$$\nabla \times (\nabla \times \mathbf{a}) = \nabla(\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a}$$

$$\nabla \times (\mathbf{a} \times \mathbf{b}) = \mathbf{a}(\nabla \cdot \mathbf{b}) - \mathbf{b}(\nabla \cdot \mathbf{a}) - (\mathbf{a} \cdot \nabla)\mathbf{b} + (\mathbf{b} \cdot \nabla)\mathbf{a}$$

$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b})$$

$$\nabla \cdot \overline{u} = \frac{\partial u_i}{\partial x_i}$$

$$\nabla \times \overline{u} = \varepsilon_{ijk} \frac{\partial u_k}{\partial x_j} \overline{e}_i$$

$$\nabla^2 \phi = \nabla \cdot \nabla \phi = \frac{\partial^2 \phi}{\partial x_i \partial x_i}$$

$$\varepsilon_{ijk}\varepsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}$$

Complex potential for a vortex located at $z = z_0$: $F(z) = -\dfrac{i\Gamma}{2\pi}\ln(z - z_0)$