Formulation of a Laminated Shell Theory Incorporating Embedded Distributed Actuators

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❖ Shape Memory Alloys

An object in the low-temperature martensitic condition,

when plastically deformed and the external stress removed, will regain its

original(memory) shape when heated ← martensitic transformation during heating

- Regaining the original shape is associated with a reverse transformation of the deformed martensitic phase to the higher temperature austenite phase
- Nickel – Titanium alloys(Nitinol NiTi) ··· Ni(Nickel) – Ti(Titanium)
  - · NOL (Naval Ordnance Laboratory)
  - · Plastic strains of typically 6~8% may be completely recovered by heating it so as to transform it to its austenite phase
  - · Restraining the material can yield stresses of up to 100,000 psi

    (yield strength of martensitic Nitinol = 12,000 psi)

❖ Shape Memory Alloy Reinforced Composite

❖ Active Control Concepts

" Active Model Modification "

" Active Strain Energy Tuning "

❖ Behavior of Shape Memory Alloy Fibers

- total strain ε_a = initial plastic strain ε_0 + induced global strain ε
- resulting stress in the SMA fibers $\sigma_a = f(T, \varepsilon_0)$

Nonlinear function(activation temperature initial plastic strain)

- quasi–linear expression relating the actuator stress and strain

$$\sigma_a = E_a^* \varepsilon + \sigma_r^* \quad (3)$$

where $\sigma_a = E_a^* = E(T)$, $\sigma_r^* = \sigma_r(T,\ \varepsilon_0 + \ \varepsilon)$ (4)~(5)

"recovery stress"

* : variables that are activation temperature dependent and typically highly nonlinear

- Rewriting Eq.(3) to solve for the resulting elastic strain in the actuator

$$\varepsilon = \frac{\sigma_a}{E_a^*} - \frac{\sigma_r^*}{E_a^*} \quad (6)$$

- since σ_r^* is a function of strain $(\varepsilon_0 + \varepsilon)$, an iterative method must be used to solve for the strain ε

$$\varepsilon_t = \frac{\sigma_a}{E_a^*} - \frac{\sigma_r^*(t, \varepsilon_{t-\Delta t})}{E_a^*} \quad (7)$$

- 2-D transversely isotropic lamina relationship in the principal coordinates

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}^* \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} - \begin{Bmatrix} v_a \sigma_2^* \\ 0 \\ 0 \end{Bmatrix} \quad (8)$$

- in the general coordinate

$$\begin{Bmatrix} \sigma_x^* \\ \sigma_y^* \\ \sigma_{xy}^* \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} v_a \sigma_r^* \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} c^2 & v_a & \sigma_r \\ s^2 & v_a & \sigma_r \\ 0 & 0 & 0 \end{bmatrix} \qquad (9)$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = [C]^* \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} + \begin{Bmatrix} \sigma_x^* \\ \sigma_y^* \\ \sigma_{xy}^* \end{Bmatrix} \qquad (10)$$

··· stress-strain relationship for the single layer, or lamina, of

· SMA reinforced composite

❖ Theory of Multi-layered thin orthotropic SMA Shell

- Assumptions

  i) Shell has a thickness considerably less than either of the other two dimensions

  ii) A line originally normal to the shell reference will remain normal to the deformed reference surface

  iii) deflections are assumed to be small

  iv) SMA composite shell is composed of a number of thin, orthotropic laminate. the radius of curvature, R_1 and R_2 are much greater then the thickness of each layer. i.e. $R_j \gg \delta_j$

  v) strain in the shell are smaller than the initial plastic strains of SMA

- Shell coordinates

  · Middle surface of the shell → third coordinate direction

  · in this surface, an orthogonal curvilinear coordinate system (α_1, α_2) is established and which coincides with the orthogonal lines of the principal curvature of the surface

- Shell coordinates(cont')

  · thickness direction → third coordinate direction

  · position vector to an arbitrary point within the shell

$$\vec{R}(\alpha_1,\alpha_2,z)=\vec{r}(\alpha_1,\alpha_2)+z\vec{n}(\alpha_1,\alpha_2) \qquad (12)$$

$\vec{r}$: Position vector to a point on the reference surface

$\vec{n}$: unit vector normal to the reference surface

z : coordinate measure from the reference surface, through the thickness along $\vec{n}(\alpha_1,\alpha_2)$

  · Lame's constants

$$A_1^2=\frac{\partial\vec{r}}{\partial\alpha_1}\cdot\frac{\partial\vec{r}}{\partial\alpha_1} \qquad A_2^2=\frac{\partial\vec{r}}{\partial\alpha_2}\cdot\frac{\partial\vec{r}}{\partial\alpha_2} \qquad (13)\sim(14)$$

- Shell coordinates(cont')

  - Magnitude of a differential length element

$$(ds)^2 = d\vec{R}\cdot d\vec{R} = (d\vec{r} + zd\vec{n} + dz\vec{n})\cdot(d\vec{r} + zd\vec{n} + dz\vec{n}) \qquad (15)$$

$$= A_1^2(1+\frac{z}{R_1})^2(d\alpha_1)^2 + A_2^2(1+\frac{z}{R_1})^2(d\alpha_2)^2 + (dz)^2$$

  - If a shell element of thickness dz at an altitude z from the middle surface is isolated, the length : of the edge, of the element

$$ds_1(\mathrm{z}) = \mathrm{A}_1(1+\frac{Z}{R_1})(d\alpha_1) \qquad (16)\sim(17)$$

$$ds_2(\mathrm{z}) = \mathrm{A}_2(1+\frac{Z}{R_2})(d\alpha_2)$$

  - corresponding area elements of the forces

$$d\sum\nolimits_1(\mathrm{z}) = \mathrm{A}_1(1+\frac{Z}{R_2})(d\alpha_1)(\mathrm{dz}) \qquad (18)\sim(19)$$

$$d\sum\nolimits_2(\mathrm{z}) = \mathrm{A}_2(1+\frac{Z}{R_2})(d\alpha_2)(\mathrm{dz})$$

- Constitutive Relation in Shell Coordinates

· Eq.(10) : stress – strain relation in cartesian coordinate system

→ orthogonal curvilinear coordinates, (α_1, α_2)

$$\begin{Bmatrix} \sigma_{\alpha_{11}} \\ \sigma_{\alpha_{22}} \\ \sigma_{\alpha_{12}} \end{Bmatrix} = [C]^* \begin{Bmatrix} \varepsilon_{\alpha_{11}} \\ \varepsilon_{\alpha_{22}} \\ \varepsilon_{\alpha_{12}} \end{Bmatrix} + \begin{Bmatrix} \sigma_{\alpha_{11}} \\ \sigma_{\alpha_{22}} \\ \sigma_{\alpha_{12}} \end{Bmatrix}^* \qquad (20)$$

- Strain – displacement Relations

· displacement vector in the shell coordinate system

$$\vec{u}(\alpha_1,\alpha_2,z) = v_1(\alpha_1,\alpha_2,z)\vec{t}_1 + v_2(\alpha_1,\alpha_2,z)\vec{t}_2 + w_2(\alpha_1,\alpha_2,z)\vec{n} \qquad (21)$$

v_1, v_2 : magnitude of the projections on the unit tangent vector of the displacement of a point P

3. Ply Elasticity

- Strain – displacement Relations(cont')

· normal and shearing strain component in an orthogonal curvilinear system

$$\varepsilon_{a_{ii}} = \frac{\partial}{2\alpha_i}\left(\frac{v_1}{\sqrt{g_i}}\right) + \frac{1}{2g_i}\sum_{k=1}^{3}\frac{\partial g_i}{g\alpha_k}\frac{U_k}{\sqrt{g_k}} \tag{22}$$

$$\varepsilon_{a_{ij}} = \frac{1}{\sqrt{g_i g_j}}\left[g_i\frac{\partial}{\partial\alpha_j}\left(\frac{v_i}{\sqrt{g_i}}\right) + g_j\frac{\partial}{\partial\alpha_i}\left(\frac{v_j}{\sqrt{g_j}}\right)\right] \tag{23}$$

· Entities are correspondent to the quantities in the shell theory

$$\begin{gathered}\alpha_1 = \alpha_1, \alpha_2 = \alpha_2, \alpha_3 = z, U_1 = U_1, U_2 = U_2, U_3 = W \\ g_1 = A_1^2(1+\frac{z}{R_1})^2, g_2 = A_2^2(1+\frac{z}{R_2})^2, g_3 = 1\end{gathered} \tag{24}$$

→ Exact strain-displacement relationship (25) within the framework of infinitesimal elasticity