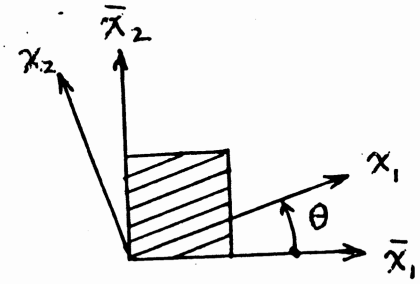
General Laminate Transformations



$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \underbrace{\begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & (c^2 - s^2) \end{bmatrix}}_{T_\sigma} \begin{Bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\sigma}_6 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} = \underbrace{\begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & (c^2 - s^2) \end{bmatrix}}_{T_\varepsilon} \begin{Bmatrix} \bar{\varepsilon}_1 \\ \bar{\varepsilon}_2 \\ \bar{\varepsilon}_6 \end{Bmatrix}$$

$T_\sigma^{-1} = T_\varepsilon^T \qquad T_\varepsilon^{-1} = T_\sigma^T$

$\sigma = Q\ \varepsilon \qquad \bar{\sigma} = \bar{Q}\ \bar{\varepsilon} \qquad \bar{Q} = T_\varepsilon^T\ Q\ T_\varepsilon$

$$\bar{Q}_{11} = c^4 Q_{11} + s^4 Q_{22} + 2c^2s^2 (Q_{12} + 2Q_{66}) = I_1 + I_2 + R_1 \cos 2\theta + R_2 \cos 4\theta$$

$$\bar{Q}_{12} = c^2s^2 (Q_{11} + Q_{22} - 4Q_{66}) + (c^4 + s^4) Q_{12} = I_1 - I_2 - R_2 \cos 4\theta$$

$$\bar{Q}_{16} = c^3s\, Q_{11} - cs^3 Q_{22} - cs(c^2 - s^2)(Q_{12} + 2Q_{66}) = \frac{1}{2} R_1 \sin 2\theta + R_2 \sin 4\theta$$

$$\bar{Q}_{22} = s^4 Q_{11} + c^4 Q_{22} + 2c^2s^2 (Q_{12} + 2Q_{66}) = I_1 + I_2 - R_1 \cos 2\theta + R_2 \cos 4\theta$$

$$\bar{Q}_{26} = cs^3 Q_{11} - c^3s\, Q_{22} + cs(c^2 - s^2)(Q_{12} + 2Q_{66}) = \frac{1}{2} R_1 \sin 2\theta - R_2 \sin 4\theta$$

$$\bar{Q}_{66} = c^2s^2 (Q_{11} + Q_{22} - 2Q_{12}) + (c^2 - s^2)^2 Q_{66} = I_2 - R_2 \cos 4\theta$$

$$I_1 = \frac{1}{4}(Q_{11} + Q_{22} + 2Q_{12})$$

$$I_2 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66})$$

$$R_1 = \frac{1}{2}(Q_{11} - Q_{22})$$

$$R_2 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})$$