Various solutions of partial differential equations.

a general solution : a solution which contains at least one arbitrary constant

a particular solution : a solution obtained from any general solution by assigning particular values to the arbitrary constants

singular solution (particular integral) : solutions which cannot be obtained from any general solution by assigning specific values to its arbitrary constants

a complete solution : If a general solution has the property that every solution of the differential equation can be obtained from it by assigned suitable values to its arbitrary constants, it is said to be a complete solution.

- If Y is any solution of the nonhomogeneous equation

$$y'' + P(x)y' + Q(x)y = R(x) \quad \dots. \text{①}$$

and if $c_1 y_1 + c_2 y_2$ is a complete solution of the homogenous equation

$$y'' + P(x)y' + Q(x)y = 0$$

Then $y = c_1 y_1 + c_2 y_2 + Y$ is a complete solution of Eq. ①

Here, a particular integral of Eq. ① : the term Y

the complementary function of Eq. ① : $c_1 y_1 + c_2 y_2$

- Governing Eq. : $m\ddot{z}+kz=F_0\sin wt$

  The complementary function : $A\cos w_n t+B\sin w_n t$

  A particular integral : $z=\dfrac{F_0}{k-mw^2}\sin wt=\dfrac{{w_n}^2\delta_{st}}{{w_n}^2-w}\sin wt$

  A complete solution : $z=A\cos w_n t+B\sin w_n t+\dfrac{{w_n}^2\delta_{st}}{{w_n}^2-w}\sin wt$

  From I.C.

  At t=0 z=0 v=0 ; $z=\dfrac{{w_n}^2\delta_{st}}{w^2-{w_n}^2}(w\sin w_n t-w_n\sin wt)$

- Summary

  In the case of a forced vibrating system, the complete motion of the system consists of two parts : the first described by the complementary function, decaying exponentially, known of the transient, the second part described by the particular integral, representing a harmonic displacement of the same frequency as the excitation, known as the steady state.

  Ex) A system containing a negligible amount of damping is disturbed from its equilibrium position by the sudden application at t=0 of a force equal to $F_0\sin wt$