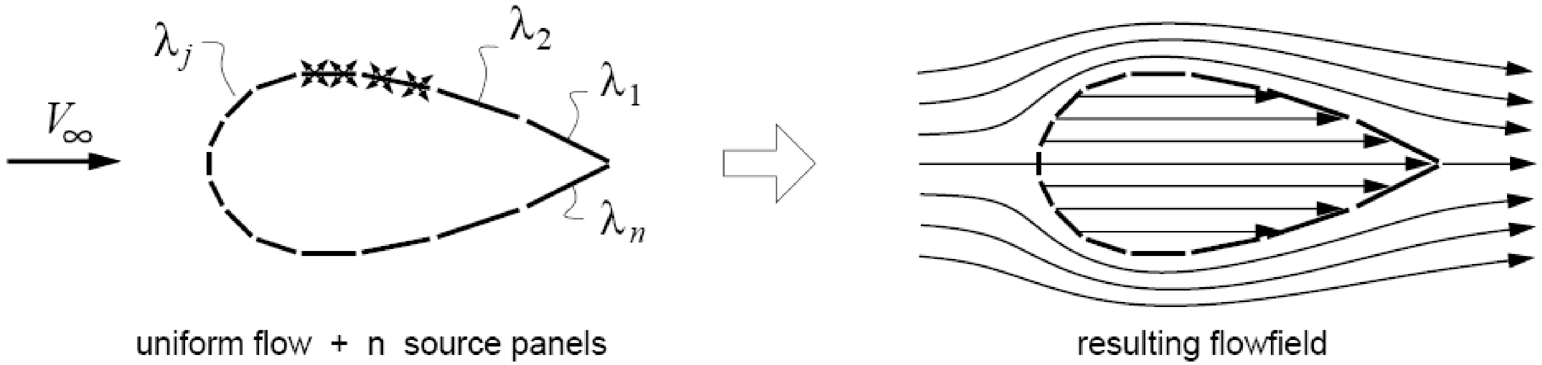
< Panel Method >

❖ Flows over arbitrary bodies

: *Superposition of suitable elementary flows*(uniform flow, sources, vortices, etc.) which produce the velocity field $\boldsymbol{V}(x, y)$ about the body

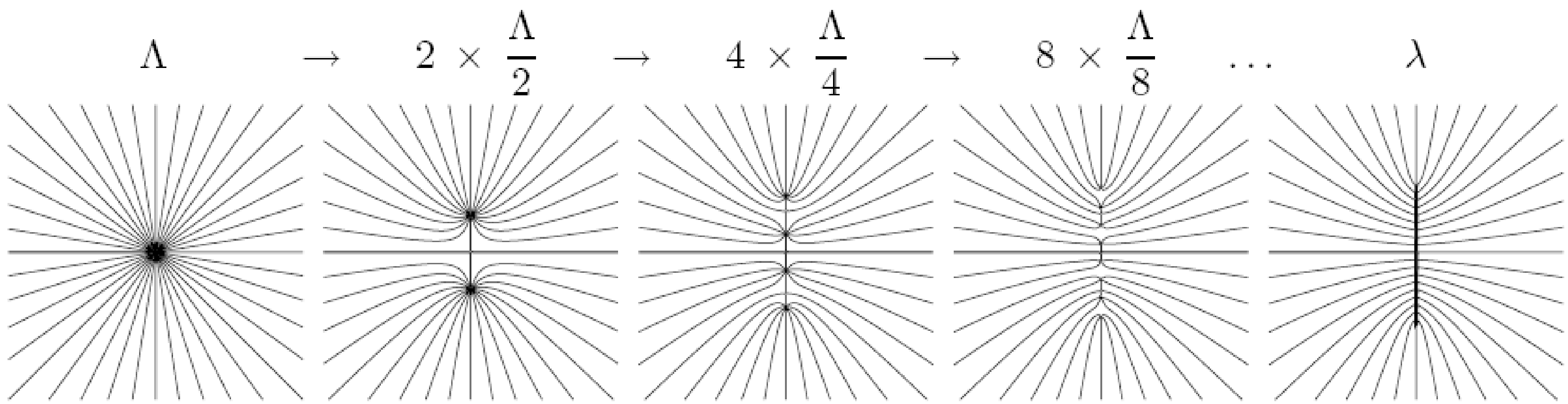
- Source Panel Method
- Vortex Panel Method



< Source Panel Method >

❖ Source panel

- Consider a sequence of flows where a single source of *strength Λ* is *repeatedly subdivided into smaller sources* which are evenly distributed along a line segment of length ℓ. The limit of this subdivision process is a source sheet of strength $\lambda = \Lambda/\ell$.



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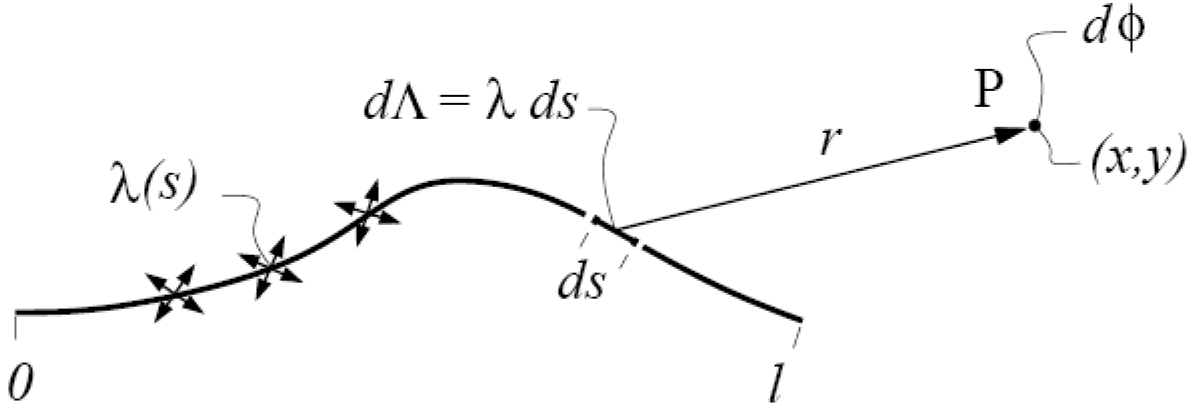
❖ Properties

- Consider an infinitesimal length *ds* of the sheet. The infinitesimal source strength of that piece is $d\Lambda = \lambda ds$, and the *corresponding potential* at some field point P at (x, y) is

$$d\phi = \frac{d\Lambda}{2\pi} \ln r = \frac{\lambda}{2\pi} \ln r \, ds$$



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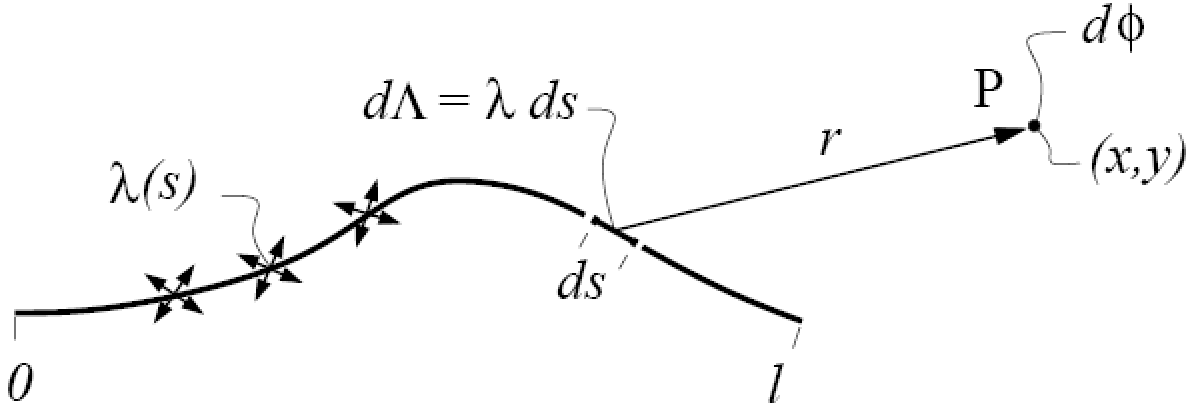
❖ Properties

- The *potential of the entire sheet* at point P is then obtained by integrating the infinitesimal contributions all along the sheet

$$\phi(x,y) = \int_0^l \frac{\lambda}{2\pi} \ln r \, ds$$



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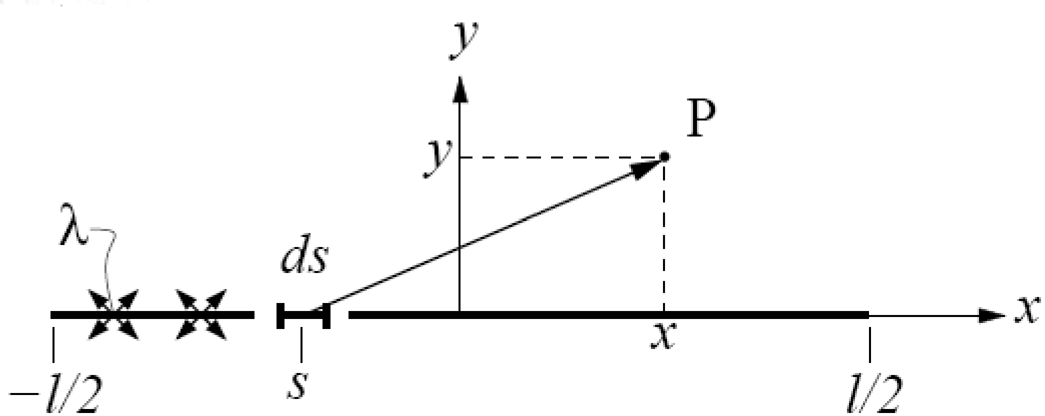
❖ Example

- Simple straight source panel

$$\phi(x, y) = \frac{\lambda}{2\pi} \int_{-l/2}^{l/2} \ln \sqrt{(x-s)^2 + y^2}\, ds$$



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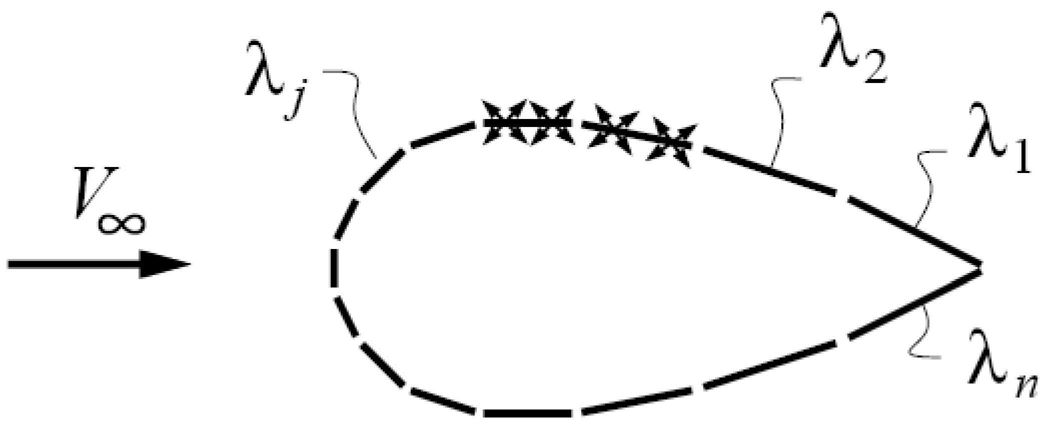
❖ Modeling approach

- Place some number of such panels end to end on the surface of the body.



uniform flow + n source panels

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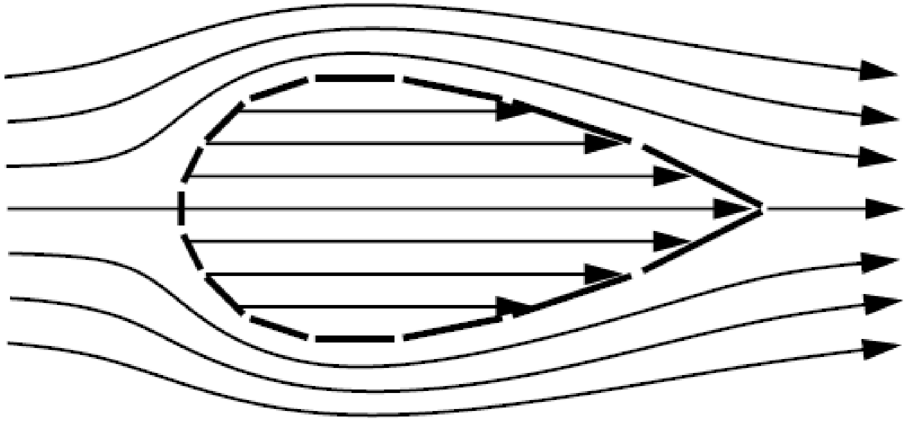
❖ Modeling approach

- We then determine the strengths λ_j of all the panels such that *the flow is tangent everywhere on the surface of the body*. The superposition also incidentally produces some flow inside the body, but this is not physical and is simply *ignored*.



resulting flowfield

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❖ Suitable case

- The approach presented here(source panel method) is actually *suitable only for non-lifting bodies* such as fuselages.

- For airfoils, wings, and other lifting bodies, *vortices* must be added in some form to enable circulation to be represented.

< Source Panel Method >

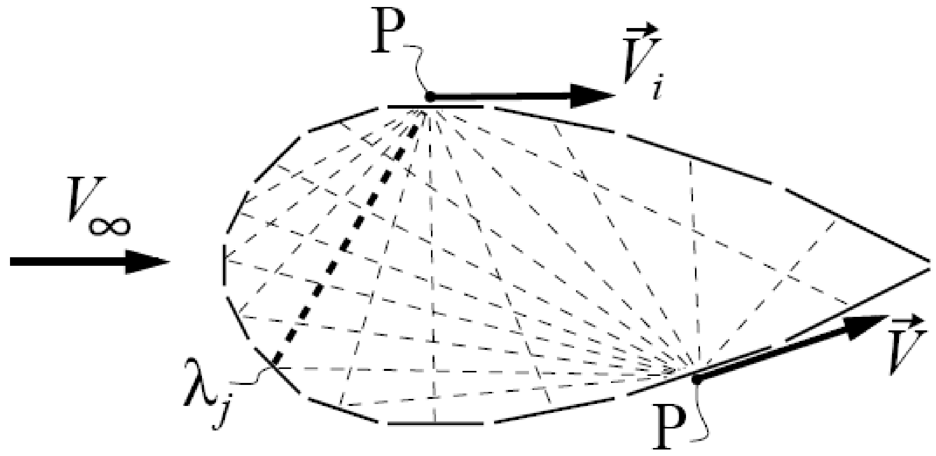
❖ Solution technique

- With more than one panel present, the velocity ***V*** and hence the flow tangency condition ***V·n*=0** at any point *i* on the surface is influenced not only by that panel's λ_i, but also by the strengths λ_j of all the other panels.

$$\left(\vec{V}\cdot\hat{n}\right)_i = A_{ij}\lambda_j + \vec{V}_\infty \cdot \hat{n}_i$$

$$\downarrow$$

$$A_{ij}\lambda_j = -\vec{V}_\infty \cdot \hat{n}_i$$



< Source Panel Method >

❖ Solution technique

- With the λ_j determined, the *velocity* and *pressure* (via Bernoulli) can then be computed at any point in the flowfield and on the surface of the body.

- *Forces* are then computed by integrating the surface pressures. This completes the aerodynamic analysis problem.

< Source Panel Method >

❖ In Practice

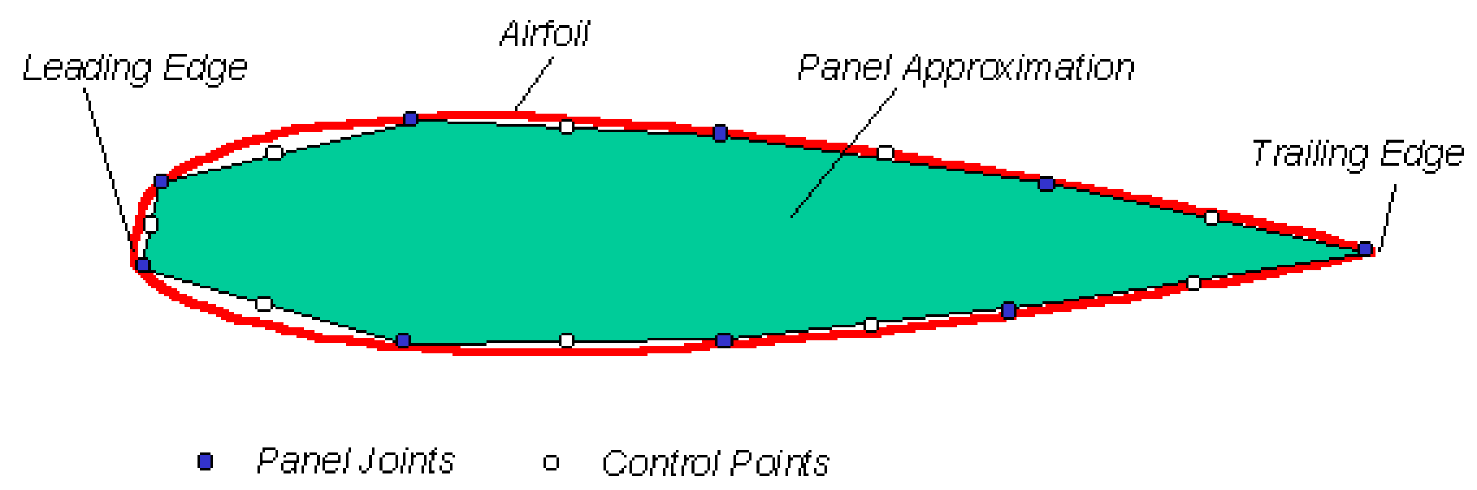


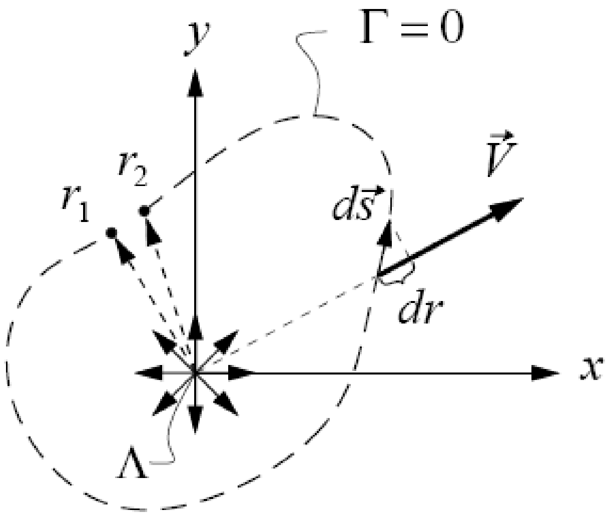
Figure 1. Vortex panel approximation to an airfoil.

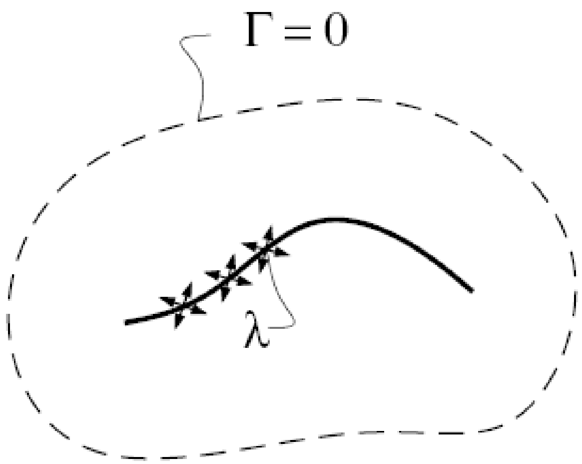
< Vortex Panel Method >

❖ Limitation of Source panel

- A source sheet, which effectively consists of infinitesimal sources, must have zero circulation as well.

$$\Gamma \equiv -\oint \vec{V} \cdot ds = -\oint V_r dr = -\int_{r_1}^{r_2} \frac{\Lambda}{2\pi r} dr = -\frac{\Lambda}{2\pi}\left(\ln r_2 - \ln r_1\right) = 0$$

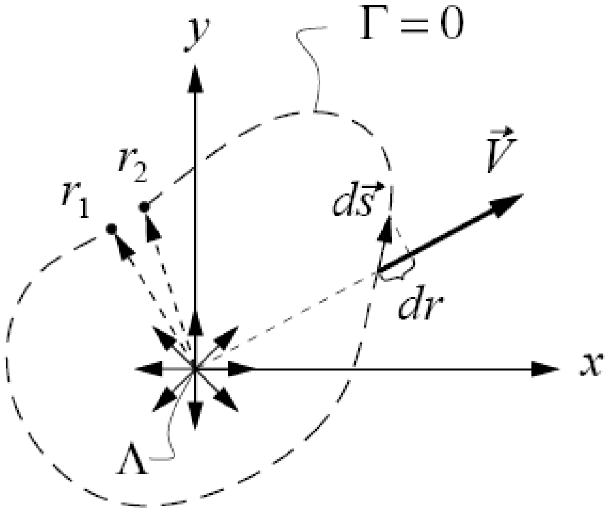


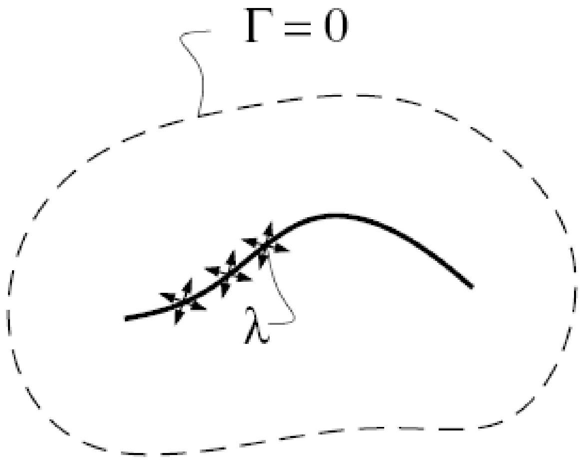


< Vortex Panel Method >

❖ Limitation of Source panel

- Any aerodynamic model consisting only of a freestream and superimposed source panels will have $\Gamma=0$, and hence $L'=0$ as well. Hence, *lifting flows can not be represented by source panels alone*.

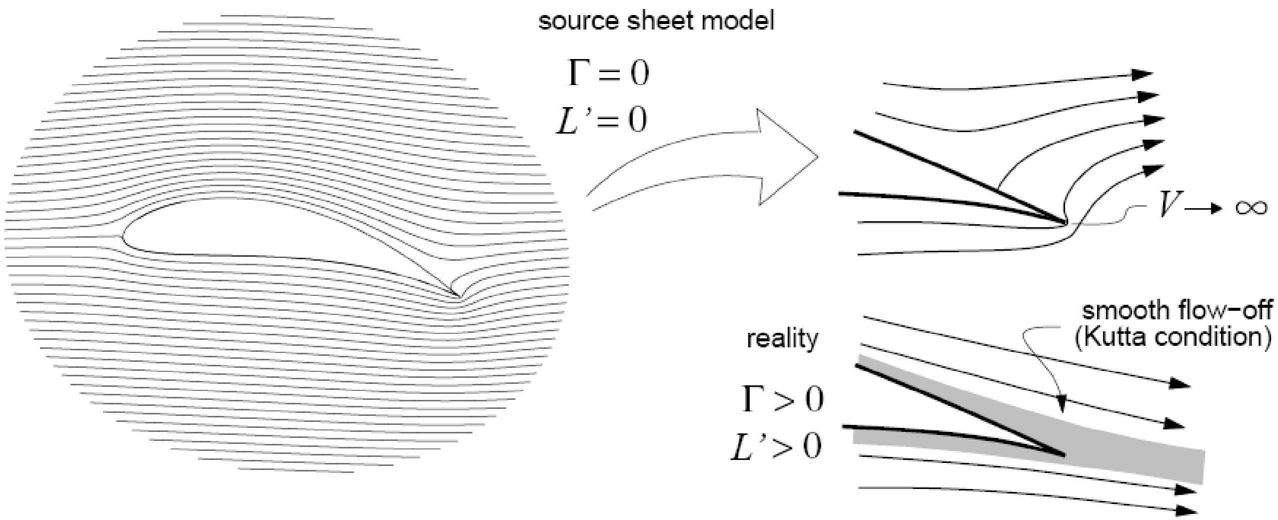




< Vortex Panel Method >

❖ Limitation of Source panel

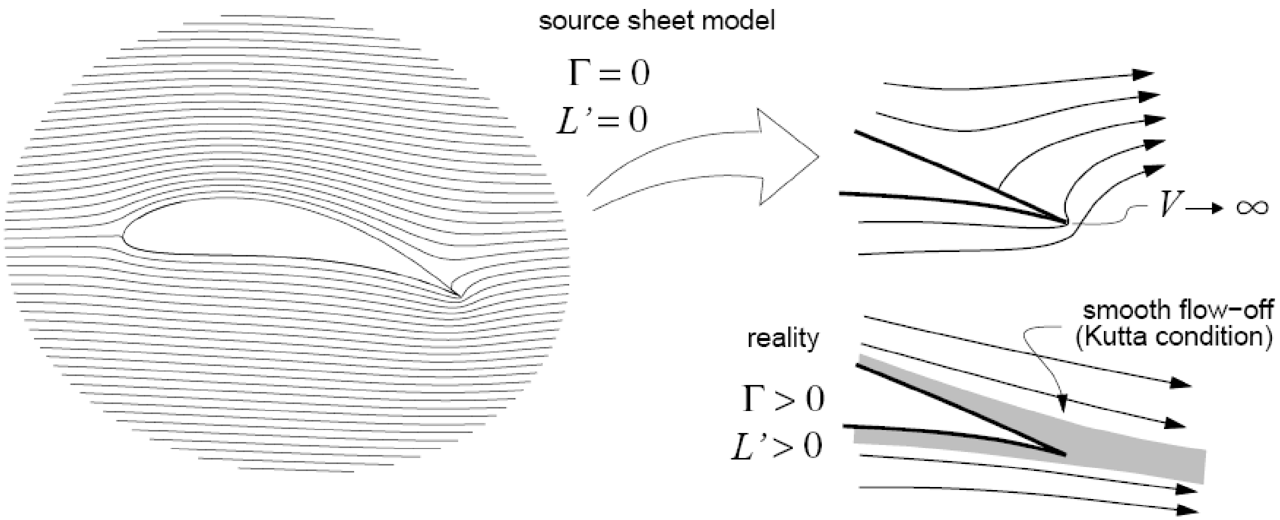
- Examination of the streamlines reveals that the rear dividing streamline leaves the airfoil off one surface as shown in the figure. The model also predicts an infinite velocity going around the sharp trailing edge.



< Vortex Panel Method >

❖ Limitation of Source panel

- On real airfoils the flow always flows smoothly off the sharp trailing edge, with no large local velocities. This smooth flow-off is known as the *Kutta condition*, and it must be faithfully duplicated in any flow model which seeks to *predict the lift correctly*.

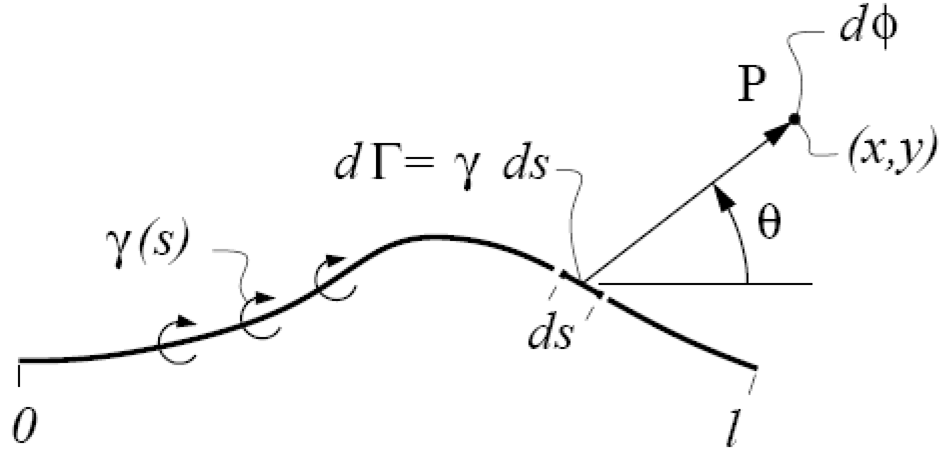


< Vortex Panel Method >

❖ Properties

- The analysis of the vortex panel closely follows that of the source panels. The potential of the vortex panel at point P is

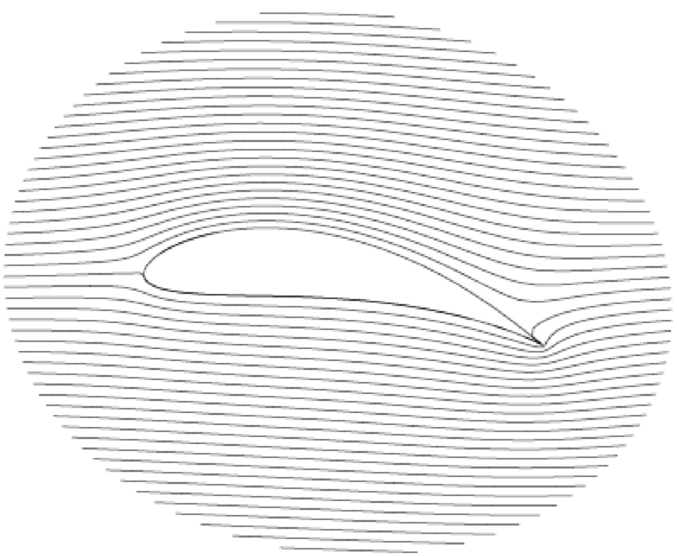
$$\phi(x, y) = -\int_0^l \frac{\gamma}{2\pi} \theta \, ds$$



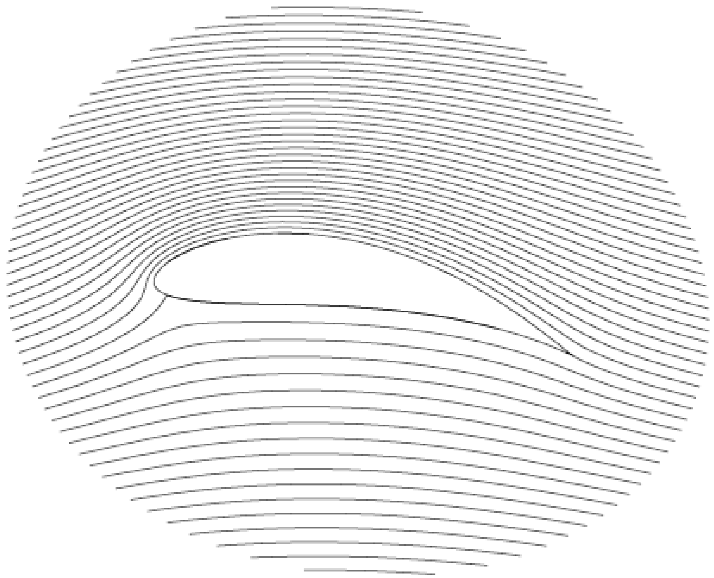
< Vortex Panel Method >

❖ Advantages

- The vortex panels smoothly deforms the flow field in the manner required to impose circulation and lift.



Source panel method



Vortex panel method

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❖ Solution technique

- The solution approach is nearly the *same* as with source panels. The $\boldsymbol{V} \cdot \boldsymbol{n} = 0$ flow tangency condition is imposed for each panel, but now the additional *Kutta condition* at the trailing edge is also imposed. The resulting linear system is then solved for all the panel strengths γ_i.