Fuzzy cluster algorithm for the automatic identification of joint sets

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-Point: The fuzzy K-means clustering algorithm was applied to joint set identification.

-Types of clustering:

① Hard clustering - Each measurement is set to belong to a certain cluster.

② Soft clustering - Fuzzy concept is adopted. Each measurement has the degree of membership for each cluster.

-K-means algorithm: One of clustering algorithms adopting the fuzzy theory. Major application fields are computer vision and medical imaging.

-Fuzzy set theory: Suggested by Zadeh (1965). Relation between an element and sets is defined by degree of membership expressed with a real number from 0 to 1. Harrison (1992) was the first who applied it to discontinuity analysis

-Set identification by fuzzy K-means algorithm

① Determination of the number of joint sets (K) and initial values of set orientation

1) The number of sets and initial set orientations are determined by man.

2) The number of sets is given by men. The initial orientation of each cluster is set by the mean orientation of all joints (vector sum). The final set orientation is obtained through an optimization process.

3) After determining the number of sets, the initial orientation of each set is randomly given in range of $\pm 3\sigma$ from total mean orientation for each axis component ( ).

※ Different number of joint sets gives different result of set identification.

② Minimization of a fuzzy objective function for the optimum joint set orientation

$$\leq \qquad (1)$$

where, : mean orientation of the i^{th} cluster

: Orientation vector of the j^{th} pole (joint)

: Degree of membership

(Euclidean norm) (2)

≡ ( ) (3)

Degree of membership of joint pole j for cluster i (4)

where is apt to have an extreme value such as 0 or 1 as m approaches 1. m = 2 is preferred in many cases even though there is no theoretical background for this.

To obtain the minimum of equation (1), it is needed to define the cartesian coordinates of joint pole vector and set mean orientation vector and put them to equation (1) as follows.

, (5)

(6)

Because only in equation (5) and (6) is an independent variable, the second term of right side in equation (6) should have the maximum value. Let the i th term of the second term of right side F_i as below.

(7)

Now the problem is to obtain maximizing equation (7).
Becauuse , Lagrange multiplier method can be applied as follow.

(8)

The partial derivative of equation (8) and equalizing it with 0 makes

$$0$$

$$0$$

$$0 \qquad (9)$$

Equation (9) can be expressed in a matrix form as below.

(10)

Equation (10) becomes an eigenvector/eigenvalue problem.

. (11)

where is an eigenvalue and is an eigenvector.

Because , substitution of the square matrix in equation (10) to A gives following result.

(12)

This equation equals the equation (7) which means that we should calculate eigenvector making the eigenvalue maximum. This eigenvector, , is the mean orientation of the set we want to obtain.

The eigenvectors corresponding to the intermediate and minimum eigenvalues means the major and minor axis orientation, respectively, of an pole ellipse projected in a plane whose normal vector is the maximum eigenvector. The eigenvalue of each eigenvector indicates a length of the major and minor axis. When both eigenvaules are similar to each other, the set has a distribution close to a circle.

③ With the joint set orientation from above process, we calculate the membership function using equation (3) and (4). Expressing the newly obtained membership function with , the repetition stops when below relation is satisfied.

(13)

where is a value between 0 and 1 (author recommends 0.001).

Attention should be paid to: The fuzzy object function (equation (1)) may have several stationary points and the above minimization process is just to provide a local minimum. Of course, proper K and well separated clusters can make a better result of set identification.