Chapter 7

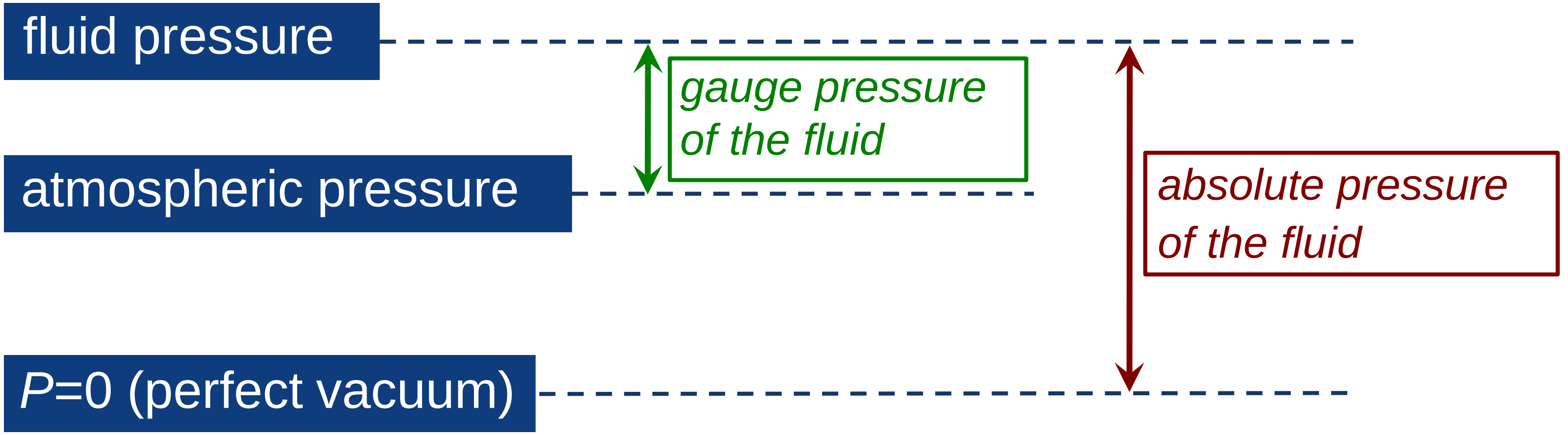
Fluid Flow

What is a fluid?

- Gas
  - loosely associated molecules that are not close together and that travel through space for long distances (many times larger than the molecular diameter) before colliding with each other
- Liquid
  - Molecules that are very close together (on the same order as their molecular diameter) and that are in collision with each other very frequently as they move around each other

The Concept of Pressure

- Absolute pressure
- Gauge pressure
  - Absolute pressure – Atmospheric pressure



Example 7.1

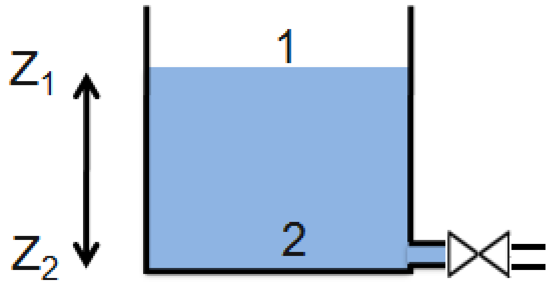
- Absolute pressure

  = Gauge pressure + Atmospheric pressure

  = 34.0 *psig* + 14.2 *psia* (usually 14.7 *psia*)

  = 48.2 *psia*
- *pounds* (lb_f) *per square inch* (*psi*)
  - *psia*
  - *psig*
- 1 *atm* = 14.7 *psi* = 760 *mmHg* = 101,300 *Pa*

Non-flowing (stagnant) Fluids



$$P_2 - P_1 = \rho\, g\, (z_1 - z_2)$$

ρ is fluid density

z is distance ***UPWARD***

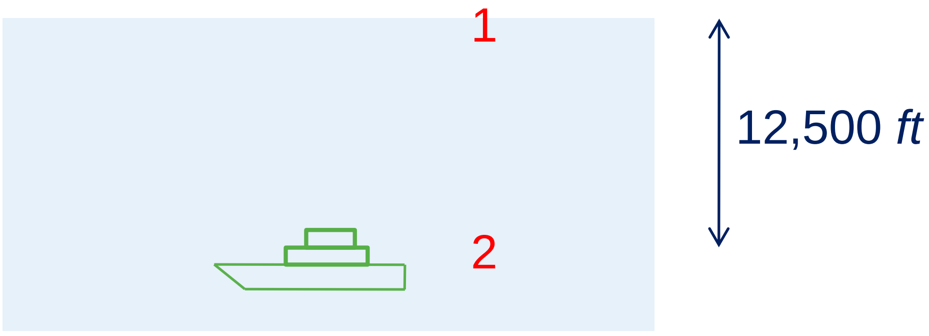
1 and *2* are locations in the liquid

Example

$$P_2 - P_1 = \rho g (z_1 - z_2)$$

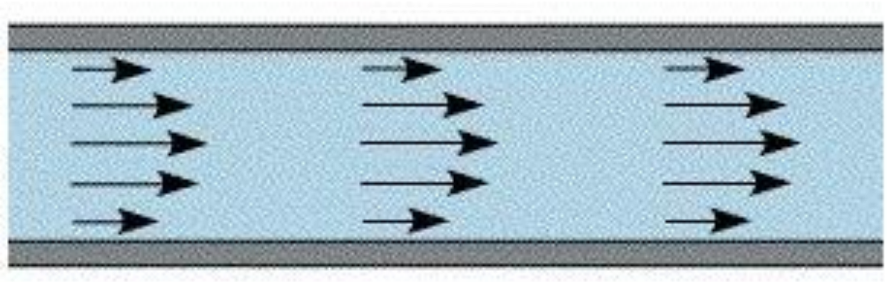
Example: The Titanic sank in 12,500 *ft*. What is the pressure (in *psi*) where she lies?

Note: the density of sea water is ~64.3 lb_m/ft^3

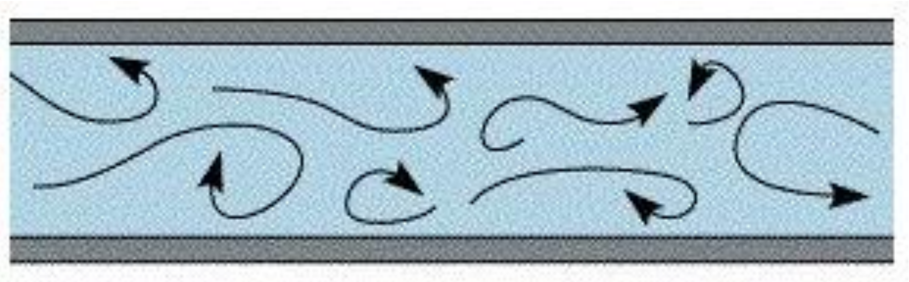


Principles of Fluid Flow

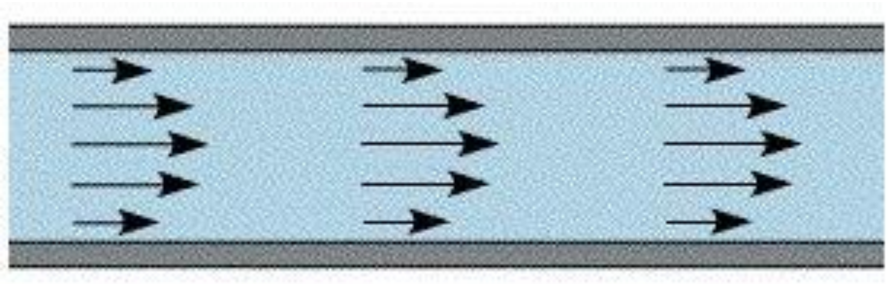
- Laminar flow



- Turbulent flow



Principles of Fluid Flow



- Average velocity (v_{avg})
  - Volumetric flow rate

$$\dot{V} = v_{avg} A_{cs}$$

A_{cs} : cross-sectional area

  - Mass flow rate

$$\rho \dot{V} = \dot{m} = \rho v_{avg} A_{cs}$$

Mechanical Energy Equation

For steady-state incompressible flow
(in the unit of energy per mass of fluid)

$$\left(\frac{P}{\rho}+\frac{1}{2}\alpha v_{ave}^{2}+gz\right)_{out}-\left(\frac{P}{\rho}+\frac{1}{2}\alpha v_{ave}^{2}+gz\right)_{in}=w_s-w_f$$

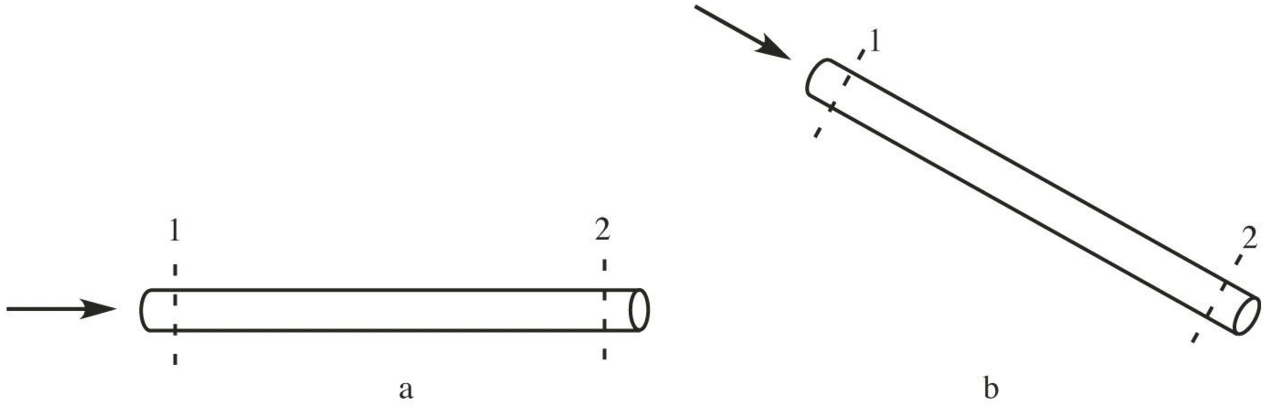


Figure 7.5

Mechanical Energy

- Kinetic energy
  - K.E.: $\frac{1}{2}\, m(v^2)_{avg}$
  - K.E. per mass: $\frac{1}{2}\,(v^2)_{avg} = \frac{1}{2}\,\alpha\,(v_{avg})^2$
  - α : a conversion factor from $(v_{avg})^2$ to $(v^2)_{avg}$
    - Can be assumed to equal 1.0
- Potential energy
  - P.E.: mgz
  - P.E. per mass: gz
- Energy associated with pressure
  - P : force/area, ρ : mass/volume
  - P/ρ : energy/mass

Work and Friction

- Work (w_s)
  - This kind of work is called “shaft work”.
  - Positive when work is done on the fluid (e.g., by a pump)
  - Negative when the fluid does work on its environment (e.g., in a turbine)
- Friction (w_f)
  - Always positive

Mechanical Energy Equation

$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\alpha(v_2^{2} - v_1^{2}) + g(z_2 - z_1) = w_s - w_f$$

Increase in fluid mechanical energy
(pressure + kinetic energy + potential energy)

Positive when work is done on the fluid

Always positive

Note 1: each grouping of variables has units of energy per mass of fluid. To cast the equation in terms of "**Power**" (energy/time), multiply all terms by mass flow rate

energy/mass x mass/time = energy/time

Special Case: No Friction or Shaft Work

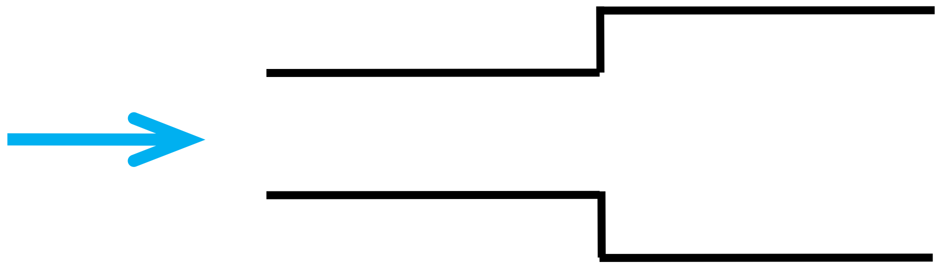
$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\alpha(v_2^2 - v_1^2) + g(z_2 - z_1) = 0$$

Called the "Bernoulli Equation" after Daniel Bernoulli, a 19th Century fluid mechanics expert

For no work or friction

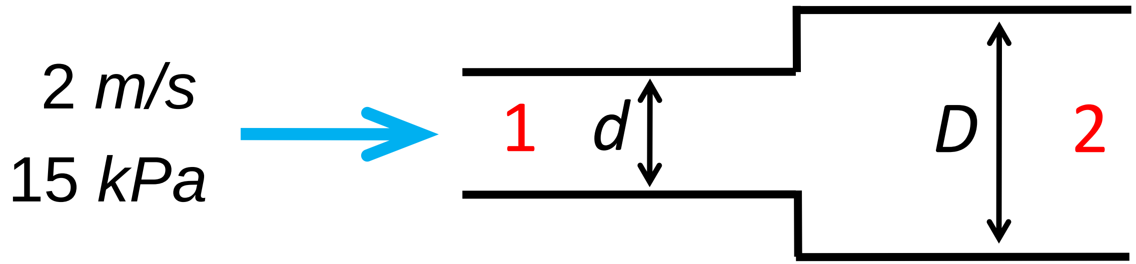
$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\alpha(v_2^2 - v_1^2) + g(z_2 - z_1) = 0$$

What happens to the pressure in a horizontal pipe when it expands to a larger diameter?



Which form(s) of energy is (are) decreasing, and which is (are) increasing?

Example: Liquid Flow in an Expanding Pipe



a. What is the average velocity in the larger pipe?

$$v_{avg,2} = v_{avg,1}\, A_1/A_2 = v_{avg,1}\, d_1^2/D_2^2 = v_{avg,1}/4 = 0.5\ m/s$$

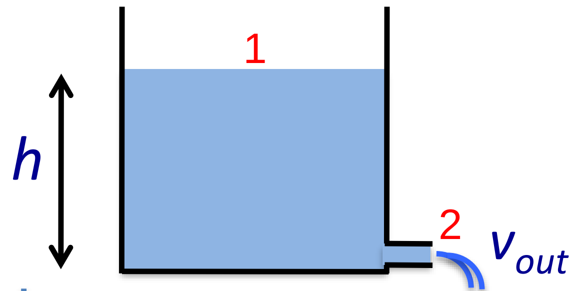
b. What is the pressure in the larger pipe?

$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\alpha\,(v_2^2 - v_1^2) + g(z_2 - z_1) = 0$$

$$P_2 - P_1 = \frac{1}{2}\rho\,\alpha\,(v_1^2 - v_2^2)$$ The Pressure Increases!

Example: An Emptying Tank

Liquid in an open tank flows out through a small outlet near the bottom of the tank. Friction is negligible. What is the outlet velocity as a function of the height of the liquid in the tank?



$$P_1 = P_2 = 0$$

$$v_1 = 0 \qquad v_2 = v_{out}$$

$$z_2 - z_1 = -h$$

$$\alpha \approx 1$$

$$\frac{\overset{0}{\cancel{P_2}} - \overset{0}{\cancel{P_1}}}{\rho} + \frac{1}{2}\overset{1}{\cancel{\alpha}}(v_2^2 - \overset{0}{\cancel{v_1^2}}) + g\overset{-h}{(z_2 - z_1)} = 0$$

$$\frac{1}{2}v_{out}^2 - gh = 0$$

$$\boxed{v_{out} = \sqrt{2gh}}$$

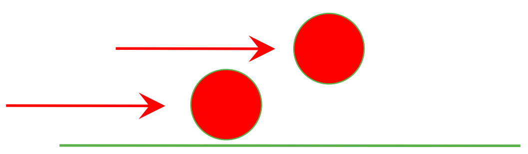
Torricelli's Equation

The Effects of Fluid Friction

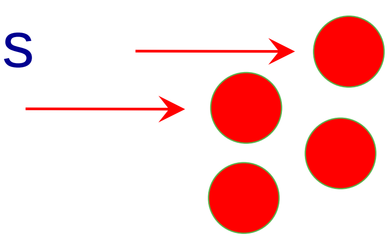
The mechanical energy equation says that friction (w_f) causes mechanical energy to decrease.

Friction is produced in flowing fluid, because fluid molecules...

- Flow past solid boundaries



- Flow past other fluid molecules



Friction in liquid flow through horizontal constant-diameter pipe:

$$v_1 = v_2$$

$$z_2 - z_1 = 0$$

$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\alpha\overset{0}{\cancel{(v_2^2 - v_1^2)}} + g\overset{0}{\cancel{(z_2 - z_1)}} = - w_f$$

$$P_2 = P_1 - \rho w_f$$

Friction in liquid pipe flow *reduces pressure* (not velocity)

Pumps

Example 7.8

$$\frac{P_2 - P_1}{\rho} + \frac{1}{2}\alpha(\cancelto{0}{v_2^2 - v_1^2}) + g(\cancelto{0}{z_2 - z_1}) = w_s - w_f$$

$$P_2 = P_1 + \rho w_{pump} - \rho w_f$$

Pumps

work per mass

$$Power = \frac{work}{time} = \dot{m}\, w_s = \rho \dot{V}\, w_s$$

$$Pump\ Efficiency = \frac{Power\ delivered\ to\ the\ fluid}{Power\ to\ operate\ the\ pump}$$

Turbines

- The calculated power
  - Power extracted from the fluid using a perfect turbine
  - Actual power delivered by the turbine is smaller than that value. (friction loss, mechanical inefficiencies, etc.)

$$Turbine\ Efficiency = \frac{Power\ delivered\ by\ the\ turbine}{Power\ extracted\ from\ the\ fluid}$$