Chapter 8

Mass Transfer

Mass Transfer

- Molecular Diffusion
  - Concentration difference
- Mass Convection
  - By bulk fluid flow

Molecular Diffusion

- Random movement (in liquids, called Brownian motion)
- Molecules of one species (A) moving through a stationary medium of another species(B)

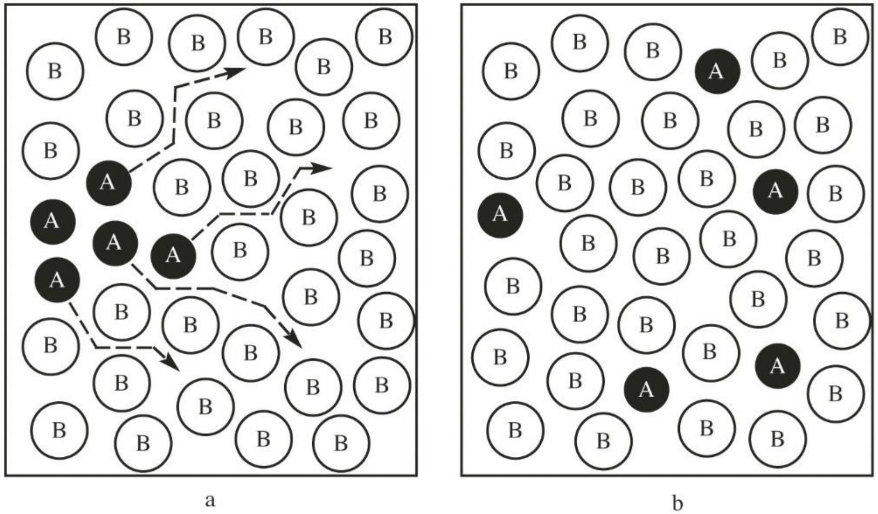
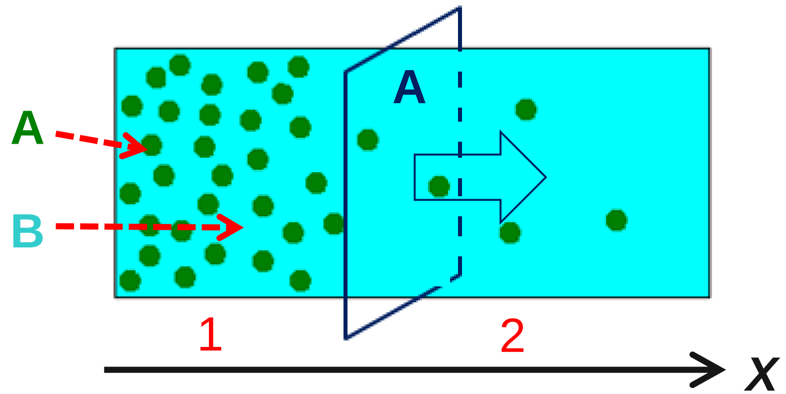


Figure 8.1

Fick's Law



(“A” diffusing through “B”)

$$\dot{N}_A = -D_{AB}\,A\,\frac{c_{A,2} - c_{A,1}}{x_2 - x_1}$$

$\dot{N}_A$ = moles of “A” transferred per time from “1” to “2”

D_{AB} = “diffusivity” of “A” diffusing through “B”

A = area through which diffusion occurs (cross-section)

Fick's Law

$$\dot{N}_A = -D_{AB}A\frac{c_{A,2}-c_{A,1}}{x_2-x_1}$$

Transfer rate = Driving force / Resistance

Analogy with Ohm's Law

$$I = \frac{V}{R} \qquad \dot{N}_A = \frac{c_{A,1}-c_{A,2}}{R} = \frac{c_{A,1}-c_{A,2}}{\left(\frac{x_2-x_1}{D_{AB}A}\right)}$$

What molecular variables affect D_{AB}?

molecular size, shape, charge, temperature

Diffusion in Contact Lens

- "Hard lenses" (polymethylmethacrylate)
  - physically uncomfortable
  - inadequate oxygen diffusion (irritation, inflammation)
- "Soft lenses" (hydrocarbon hydrogels)
  - physically more comfortable
  - inadequate oxygen diffusion
- "Oxygen permeable" (siloxane)
  - physically uncomfortable
  - better oxygen diffusion
- Latest (siloxane hydrogels)
  - physically more comfortable
  - better oxygen diffusion

Mass Convection

- Flow-enhanced transfer of one species moving through another species.

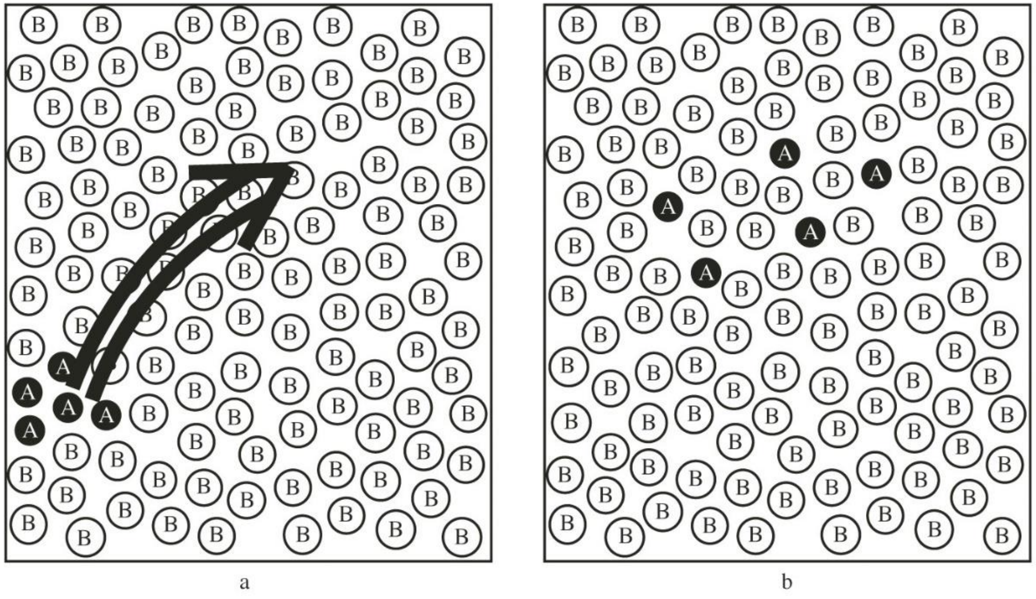


Figure 8.3

Mass Transfer across Phase Boundaries

- Mass convection + Molecular diffusion
  - Mass convection >> Molecular diffusion
- Phase boundaries
  - Liquid/Gas, Solid/Liquid, Liquid/Liquid

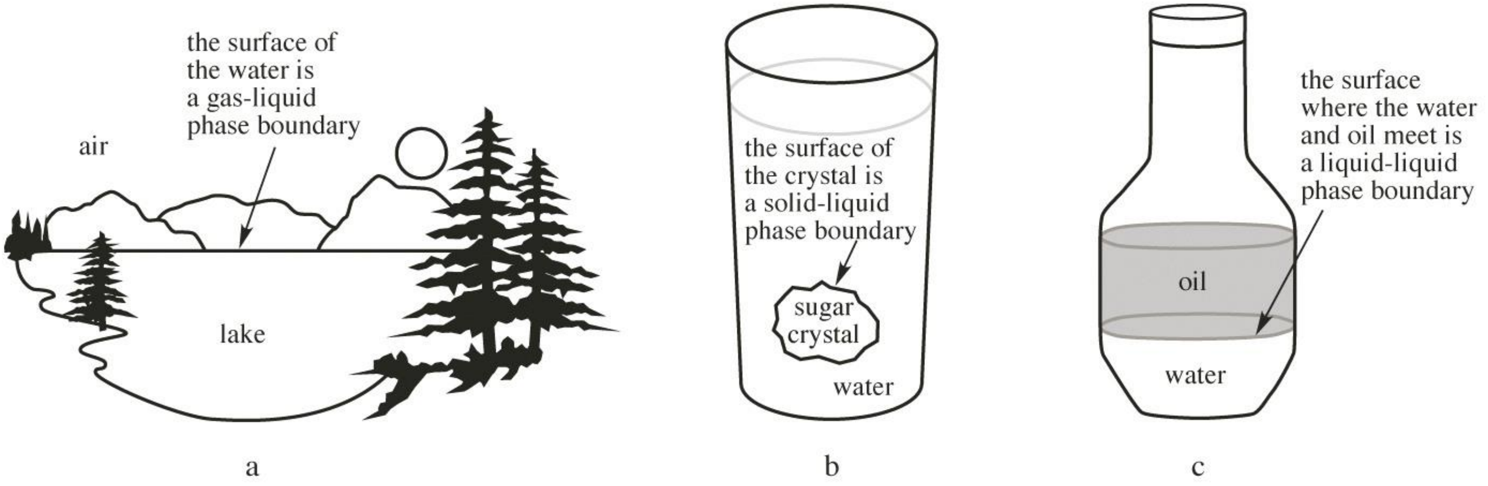


Figure 8.4

Mass Transfer across Phase Boundaries

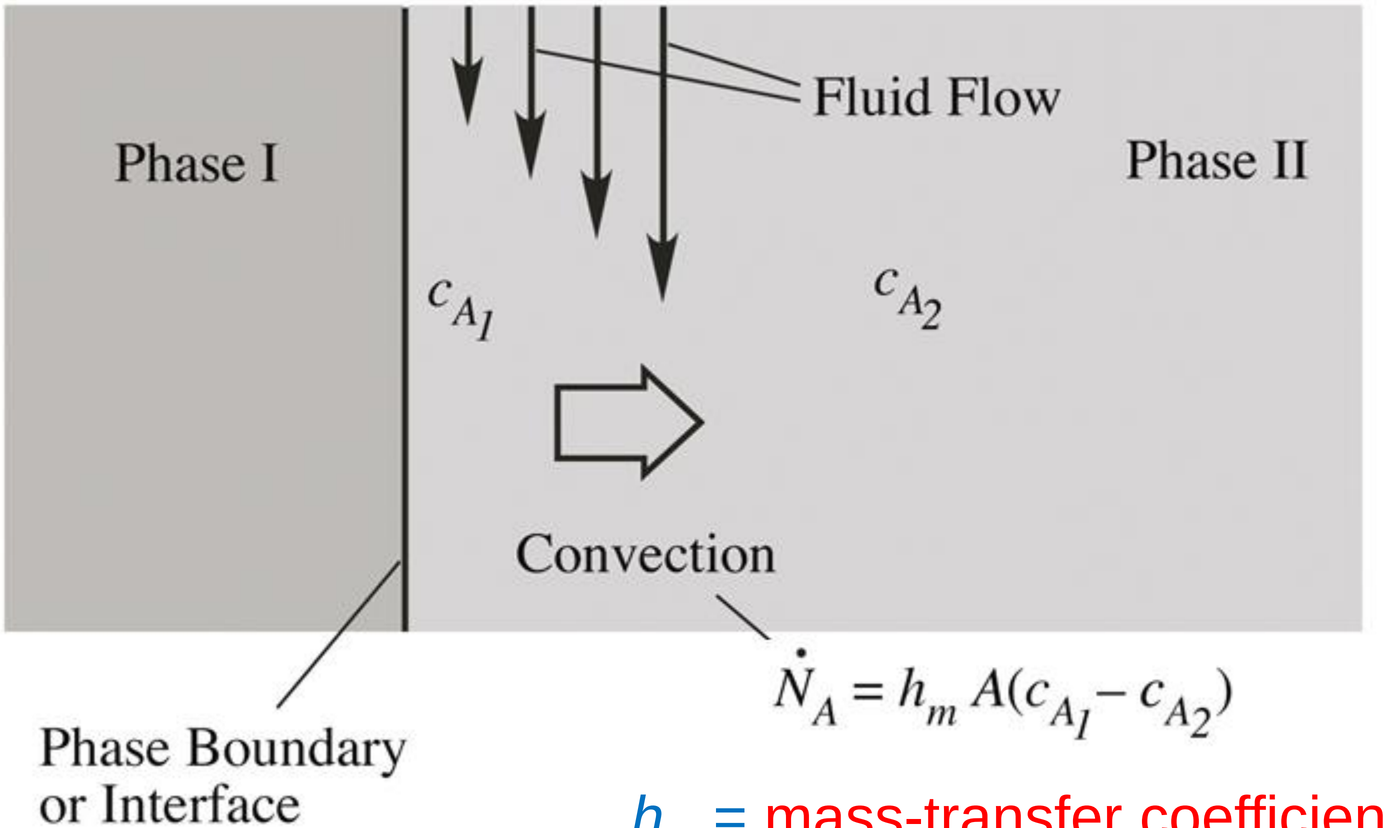


Figure 8.5

h_m = mass-transfer coefficient

c_{A1} = concentration of “A” at the phase boundary in phase II

Mass Transfer across Phase Boundaries

$$\dot{N}_A = h_m A(c_{A,1} - c_{A,2})$$

Analogy with Ohm's Law

$$I = \frac{V}{R} \qquad \dot{N}_A = \frac{c_{A,1} - c_{A,2}}{R} = \frac{c_{A,1} - c_{A,2}}{\left(\frac{1}{h_m A}\right)}$$

What variables affect h_m?

- *flow patterns (depends on geometry, etc.)*
- *molecular size*
- *molecular shape*
- *molecular charge*
- *temperature*

Ex. 8.1. The level of a lake drops throughout the summer due to water evaporation.

(a) How much volume will the lake lose per day to to evaporation?

(b) How long will it take for the water level to drop 1m?

conc. of water at the water surface	$1.0\times10^{-3} kgmol/m^3$
conc. of water in the wind	$0.4\times10^{-3} kgmol/m^3$
area of the lake	$1.7 mi^2$
mass transfer coefficient	$0.012 m/s$
density of the lake water	$1000 kg/m^3$

Multi-Step Mass Transfer

- Membrane Separation

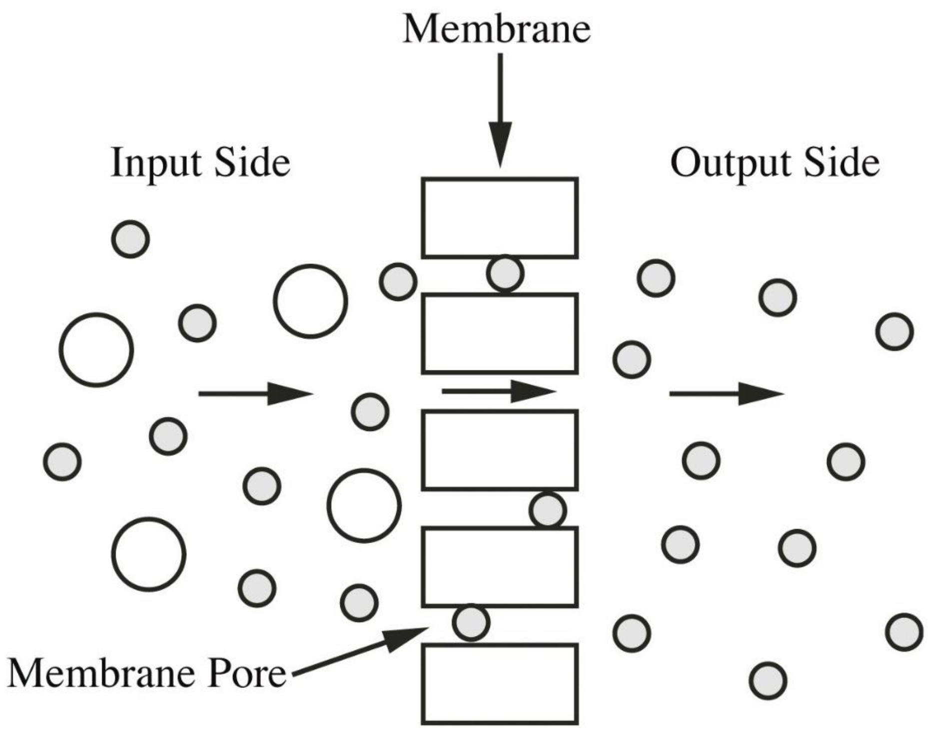


Figure 8.6

Membrane Separation

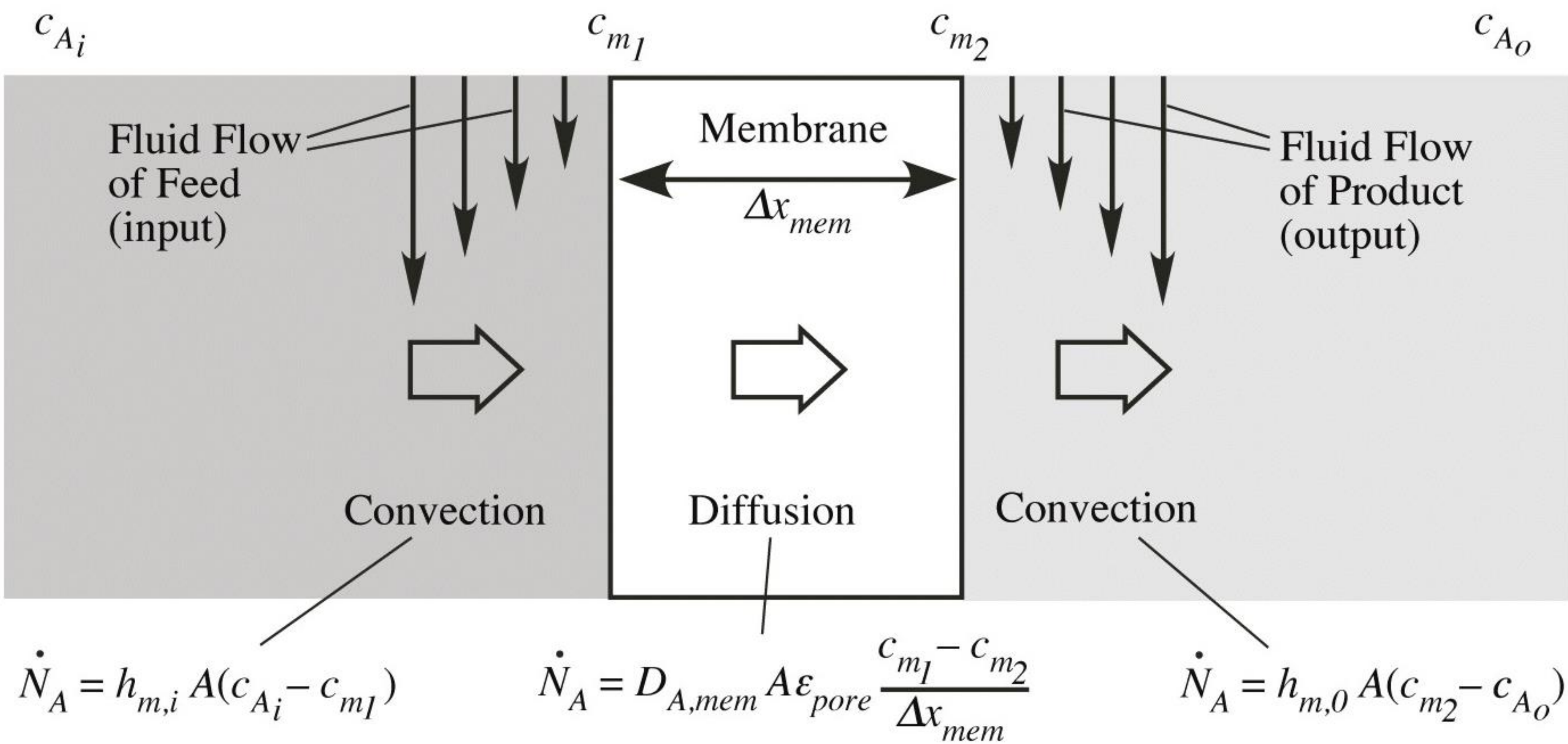
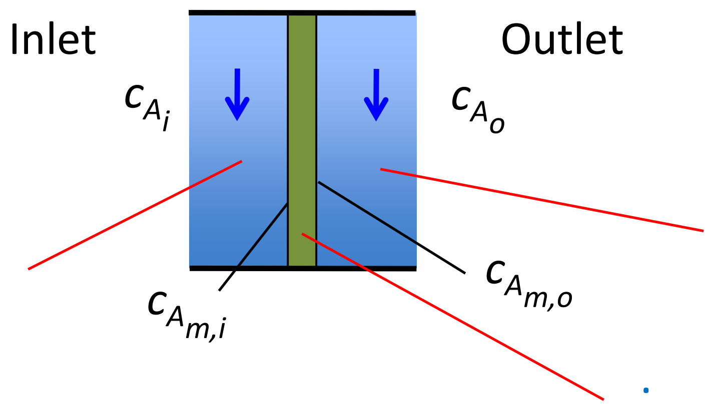


Figure 8.7

Membrane Separation



$$\dot{N}_A = h_{m_i} A(c_{A_i} - c_{A_{m,i}})$$

$$\dot{N}_A = h_{m_o} A(c_{A_{m,o}} - c_{A_o})$$

$$\dot{N}_A = D_{A,m} A \varepsilon_{pore} \frac{c_{A_{m,i}} - c_{A_{m,o}}}{\Delta x_m}$$

Solving for the concentration differences

$$c_{A_i} - c_{A_{m,i}} = \frac{\dot{N}_A}{h_{m_i} A}$$

$$c_{A_{m,i}} - c_{A_{m,o}} = \frac{\dot{N}_A \Delta x_m}{D_{A,m} A \varepsilon_{pore}}$$

$$c_{A_{m,o}} - c_{A_o} = \frac{\dot{N}_A}{h_{m_o} A}$$

Summing these…

$$c_{A_i} - c_{A_o} = \dot{N}_A \left(\frac{1}{h_{m_i} A} + \frac{\Delta x_m}{D_{A,m} A \varepsilon_{pore}} + \frac{1}{h_{m_o} A} \right)$$

Membrane Separation

$$\dot{N}_A = \frac{c_{A_i} - c_{A_o}}{\dfrac{1}{h_{m_i} A} + \dfrac{\Delta x_m}{D_{A,m} A\, \varepsilon_{pore}} + \dfrac{1}{h_{m_o} A}} = \frac{\text{overall driving force}}{\sum \text{ resistances}}$$

Concept: Limiting Resistance

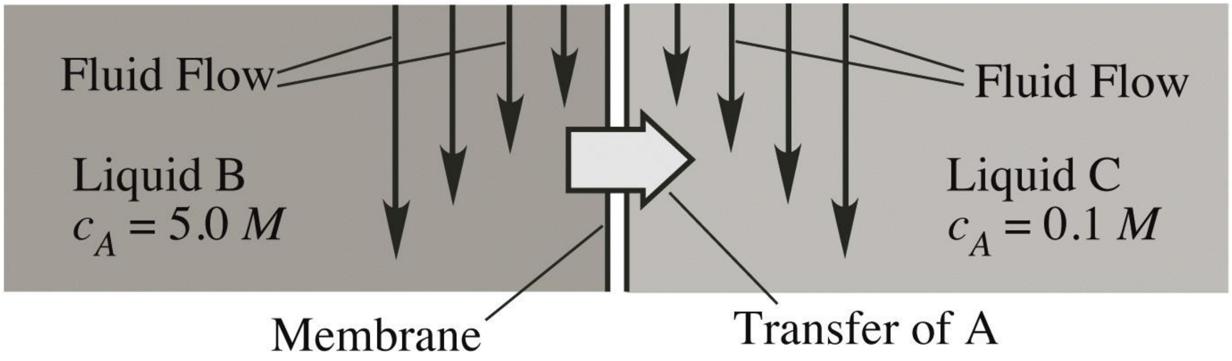
$$\text{Total Resistance} = \frac{1}{h_{m_i} A} + \frac{\Delta x_m}{D_{A,m} A\, \varepsilon_{pore}} + \frac{1}{h_{m_o} A}$$

= convection resistance on the inlet side + diffusion resistance in the membrane + convection resistance on the outlet side

If one resistance >> the others, changing the others will not change the total resistance significantly

Ex. 8.2. Liquid B flows on one side of a membrane, and liquid C flows along the other side. Species A present in both liquids transfers from liquid B into liquid C.
(a) What is the transfer rate of A from B to C?
(b) Calculate the limiting resistance.

conc. of A in liquid B	$5.0M$
conc. of A in liquid C	$0.1M$
thickness of the membrane	$200\mu m$
diffusivity of A in the membrane	$1.0\times10^{-9} m^2/s$
area of membrane	$1m^2$
porosity of membrane	70%
mass transfer coefficient on side B	$7.0\times10^{-4} m/s$
mass transfer coefficient on side C	$3.0\times10^{-4} m/s$



Example 8.2

Ex. 8.3. In patient with severe kidney disease, urea must be removed from the blood with a hemodialyzer. In that device, the blood passes by special membranes through which urea can pass. A salt solution (dialysate) flows on the other side of the membrane to collect the urea and to maintain the desired concentration of vital salts in the blood.

(a) What is the initial removal rate of urea? (Note. This rate will decrease as the urea concentration in the blood decrease.)

(b) One might be tempted to try to increase the removal rate of urea by developing better hemodialyzer membrane. Is such an effort justified?

Blood side	
mass transfer coeff. for the urea	$0.0019 cm/s$
urea conc. within the dialyzer	$0.020 gmol/l$
Dialysate side	
mass transfer coeff. for the urea	$0.0011 cm/s$
urea conc. within the dialyzer	$0.003 gmol/l$
Membrane	
thickness	$0.0016 cm$
diffusivity of urea in the membrane	$1.8\times10^{-5} cm^2/s$
total membrane area	$1.2 m^2$
porosity	20%