Chapter 10

Heat Transfer

Energy Balance for a Closed System

- *First Law of Thermodynamics for a closed system* (a fixed volume or space with no streams entering or leaving the system)

$$\Delta E = Q + W \qquad (10.1)$$

E = total energy of a system

Q = heat transferred into the system
(e.g. through the boundaries)

W = work done on the system

Energy Balance for an Open System

- Open system
  - a system with streams entering and leaving
- For a steady-state open system

$$\sum_{\substack{\text{output}\\ \text{streams}}} \left\{\dot{m}\,\hat{E}\right\}_{out} - \sum_{\substack{\text{input}\\ \text{streams}}} \left\{\dot{m}\,\hat{E}\right\}_{in} = \dot{Q} + \dot{W} \qquad (10.2)$$

$\dot{m}$ = mass flow rate of a stream (units of mass per time)

$\hat{E}$ = energy per mass of a stream of flowing material

$\dot{Q}$ = rate of transfer of energy across the boundaries of a stream into the system (units of energy per time)

$\dot{W}$ = rate that work is done on a system (units of energy per time)

Energy Balance for an Open System

$$\hat{E}_{total} = \hat{E}_{internal} + \hat{E}_{kinetic} + \hat{E}_{potential} = \hat{U} + \frac{1}{2}\alpha v^2 + gz \quad (10.3)$$

$$\sum_{\substack{output\\streams}} \left\{\dot{m}\,\hat{E}\right\}_{out} - \sum_{\substack{input\\streams}} \left\{\dot{m}\,\hat{E}\right\}_{in} = \dot{Q} + \dot{W} \quad (10.2)$$

$$\sum_{\substack{output\\streams}} \dot{m}\left[\hat{U} + \frac{1}{2}\alpha v^2 + gz\right]_{out} - \sum_{\substack{input\\streams}} \dot{m}\left[\hat{U} + \frac{1}{2}\alpha v^2 + gz\right]_{in} = \dot{Q} + \dot{W}$$

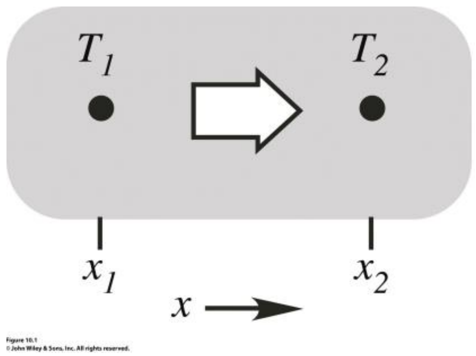
$$(10.4)$$

Heat Transfer

- Conduction
  - through a stationary medium
  - by temperature difference
- Convection
  - through a moving medium
- Radiation
  - As electromagnetic waves
  - without a medium (even through a vacuum)
  - e.g. sun radiating its heat to the earth

Conduction

- Fourier's Law of Heat Conduction



$$\dot{Q}_{cond,x} = -\,k\,A\,\frac{T_2 - T_1}{x_2 - x_1}$$

k = thermal conductivity

A = cross-sectional area through which the heat conducts

- cf. Fick's Law of Diffusion

$$\dot{N}_A = -\,D_{AB}\,A\,\frac{c_{A,2} - c_{A,1}}{x_2 - x_1}$$

Conductivity (k)

k (W/m℃) @25℃	
air	*.026*
water	*.61*
glass	*1.4*
aluminum	*237*

Which has the lowest k, gas or solid?

What are the best insulations?

Why is a double-pane window
an effective thermal barrier?

Convection

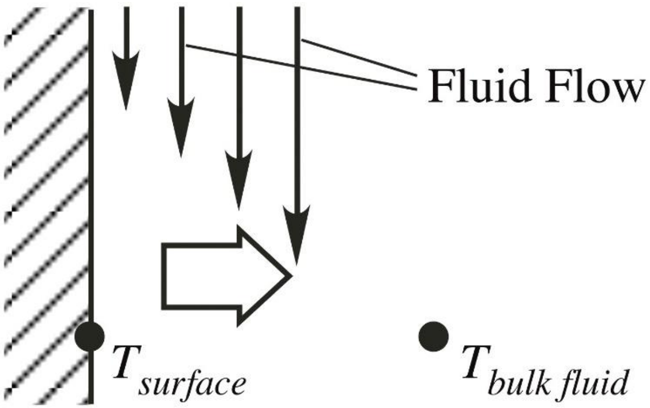


Figure 10.2

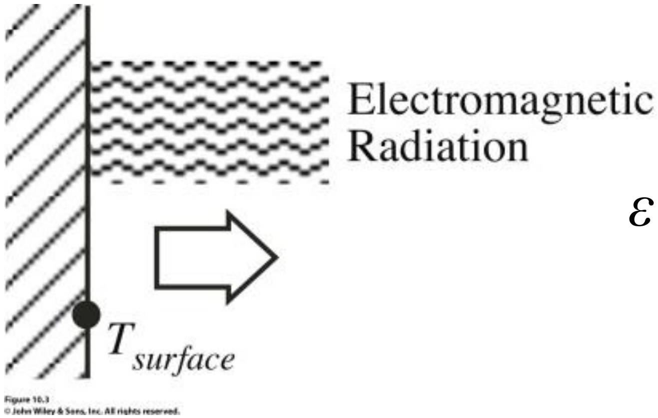
$$\dot{Q}_{conv} = h\,A\,(T_{surface} - T_{bulk\ fluid})$$

h = heat-transfer coefficient (depends on geometry and flow)

A = cross-sectional area

cf. Mass Transfer Rate: $\dot{N}_A = h_m A\,(c_{A,1} - c_{A,2})$

Radiation



$$\dot{Q}_{rad} = \varepsilon\, \sigma\, A\, (T_{surface})^4$$

ε = emissivity, which indicates how well the surface emits radiation compared with a "perfect" radiator (unit-less)

σ = Stefan-Boltzmann constant

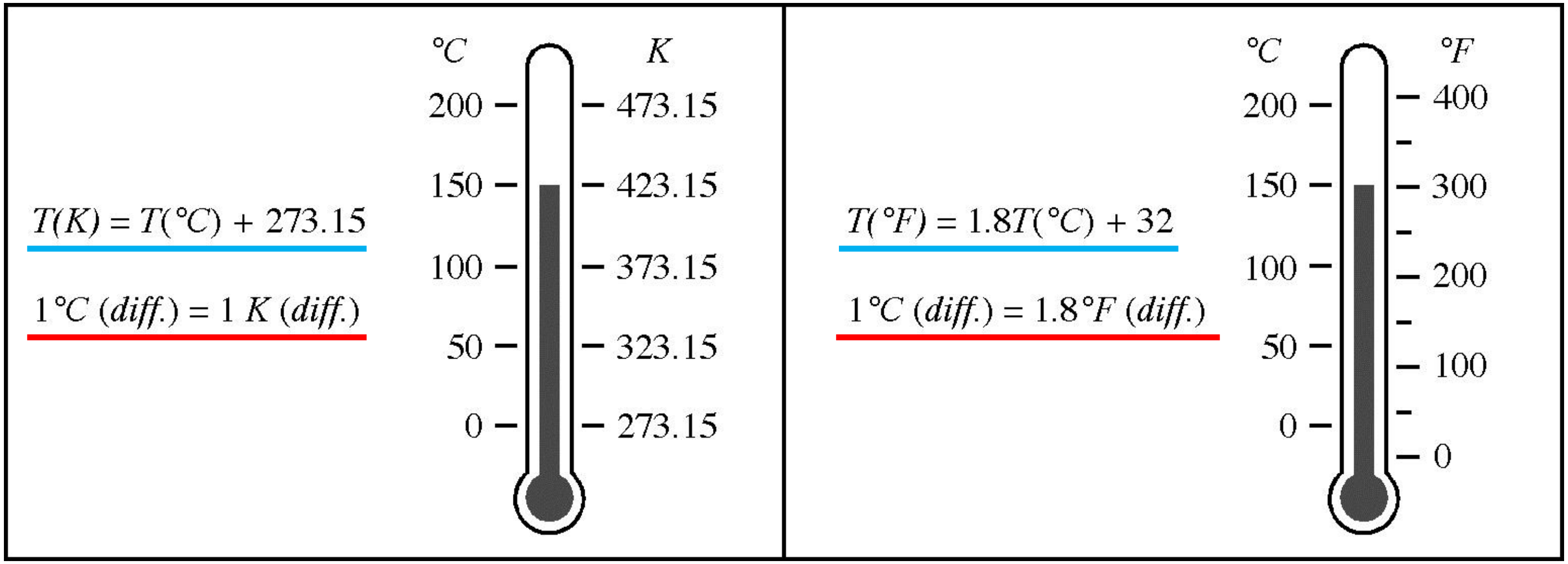
(5.67×10^{-8} $W / m^2 K^4$)

A = area of the radiating surface

T = absolute surface temperature (K)

$$\dot{Q}_{rad,net} = \varepsilon_1\, \sigma\, A_1\, (T_{surface,1})^4 - \varepsilon_2\, \sigma\, A_2\, (T_{surface,2})^4$$

Temperature & Temp. Difference



Ex. 10.1. A typical value for the thermal conductivity of steel is 53 *W/m K*. What is the corresponding value in unit of *Btu/hr ft °F*?

ΔT

Rate of Work

- Rate of shaft work ($\dot{W}_s$)
  - Energy per time
  - Positive when work is done on the system (such as in a pump, "push" the fluid)
  - Negative when work is done by the fluid (such as in turbine)
- Rate of flow work ($\dot{W}_{PV}$)
  - Work resulting from the displacement of fluid during flow
  - Similar to the pressure-volume work associated with the compression or expansion of a closed system

Energy Balance

$$\dot{W}_{PV} = \sum_{\substack{input\\streams}} (P\dot{V})_{in} - \sum_{\substack{output\\streams}} (P\dot{V})_{out} = \sum_{\substack{input\\streams}} (\dot{m}P\hat{V})_{in} - \sum_{\substack{output\\streams}} (\dot{m}P\hat{V})_{out}$$

(10.8)

$$\dot{W} = \dot{W}_s + \dot{W}_{PV} \qquad (10.9)$$

$$\sum_{\substack{output\\streams}} \dot{m}\left[\hat{U} + \frac{1}{2}\alpha v^2 + gz\right]_{out} - \sum_{\substack{input\\streams}} \dot{m}\left[\hat{U} + \frac{1}{2}\alpha v^2 + gz\right]_{in} = \dot{Q} + \dot{W}$$

(10.4)

$$\sum_{\substack{outpu\\streams}} \left\{\dot{m}\left[\hat{U} + \frac{1}{2}\alpha v^2 + gz\right]\right\}_{out} - \sum_{\substack{input\\streams}} \left\{\dot{m}\left[\hat{U} + \frac{1}{2}\alpha v^2 + gz\right]\right\}_{in} = \dot{Q} + \dot{W}_s + \dot{W}_{PV}$$

(10.10)

Energy Balance

$$\sum_{\substack{outpu\\streams}}\left\{\dot{m}\left[\hat{U}+P\hat{V}+\frac{1}{2}\alpha v^2+gz\right]\right\}_{out}-\sum_{\substack{input\\streams}}\left\{\dot{m}\left[\hat{U}+P\hat{V}+\frac{1}{2}\alpha v^2+gz\right]\right\}_{in}$$

$$=\dot{Q}+\dot{W}_s \qquad (10.11)$$

Compare Eq. 10.11 with mechanical energy balance.

$$\left(\frac{P}{\rho}+\frac{1}{2}\alpha v_{ave}^2+gz\right)_{out}-\left(\frac{P}{\rho}+\frac{1}{2}\alpha v_{ave}^2+gz\right)_{in}=w_s-w_f$$

(7.8a)

Energy Balance

$$\hat{H} = \hat{U} + P\hat{V} \qquad (10.12)$$

Most common form of
the steady-state open-system energy balance.

$$\boxed{\sum_{\substack{output\\streams}} \left\{ \dot{m} \left[\hat{H} + \frac{1}{2}\alpha v^2 + gz \right] \right\}_{out} - \sum_{\substack{input\\streams}} \left\{ \dot{m} \left[\hat{H} + \frac{1}{2}\alpha v^2 + gz \right] \right\}_{in} = \dot{Q} + \dot{W}_s}$$

(10.13)

Energy Balance

Steady-state energy balance
with negligible change in kinetic and potential energies
and with no shaft work

$$\sum_{\substack{output\\streams}} \left\{ \dot{m} \left[\hat{H} + \overbrace{\cancel{\frac{1}{2}\alpha v^2 + gz}}^{0} \right] \right\}_{out} - \sum_{\substack{input\\streams}} \left\{ \dot{m} \left[\hat{H} + \overbrace{\cancel{\frac{1}{2}\alpha v^2 + gz}}^{0} \right] \right\}_{in} = \dot{Q} + \overbrace{\cancel{\dot{W}_s}}^{0} \quad (10.13)$$

$$\sum_{\substack{output\\streams}} \left(\dot{m}\ \hat{H} \right)_{out} - \sum_{\substack{input\\streams}} \left(\dot{m}\ \hat{H} \right)_{in} = \dot{Q} \quad (10.14)$$

Energy Balance

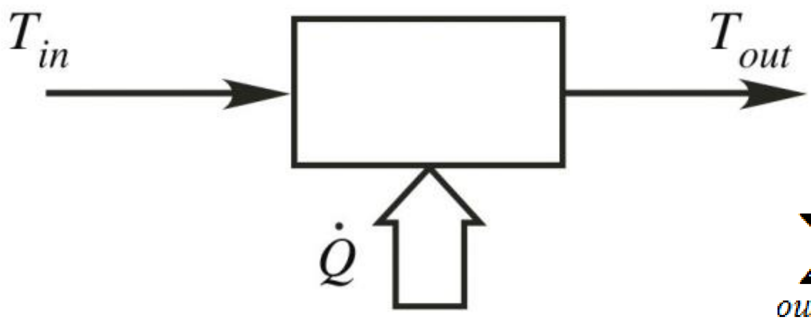


Figure 10.5

$$\sum_{\substack{output\\streams}} (\dot{m}\,\hat{H})_{out} - \sum_{\substack{input\\streams}} (\dot{m}\,\hat{H})_{in} = \dot{Q} \qquad (10.14)$$

(i) Sensible Heating/Cooling: $\sum_{out} \dot{m}\bar{C}_p(T-T_{ref}) - \sum_{in} \dot{m}\bar{C}_p(T-T_{ref}) = \dot{Q}$

$T_{out} > T_{in}$ (No phase change) (10.16)

(ii) Phase Change: $\dot{m}_{phase\ change}\, \Delta\hat{H}_{phase\ change} = \dot{Q}$

$T_{out} = T_{in}$ (Phase change) (10.18)

(iii) Chemical Reaction: $r_{consumption,A}\, \Delta\tilde{H}_{reaction,A} = \dot{Q}$

$T_{out} = T_{in}$ (10.20)

Heat-Exchangers

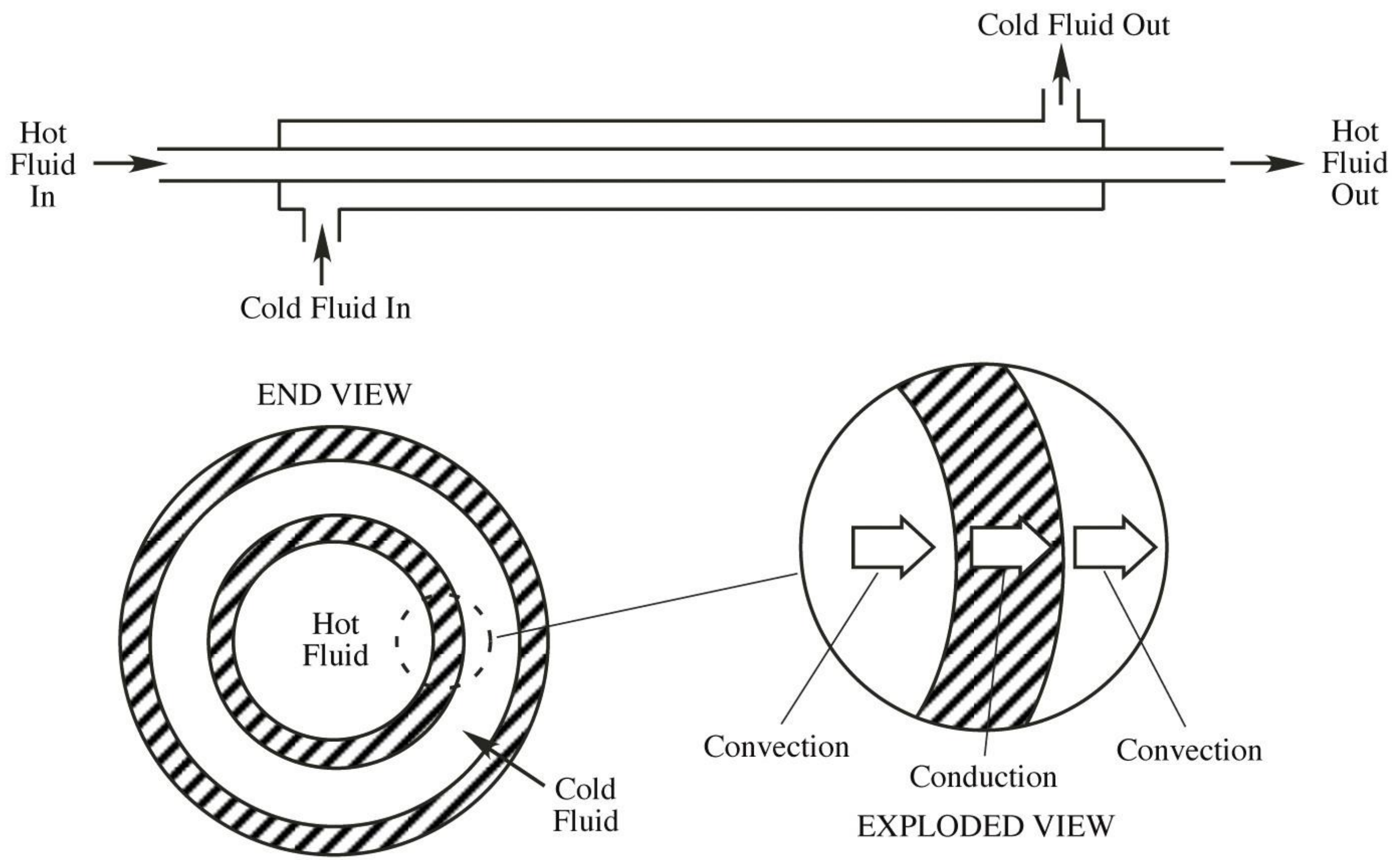


Figure 10.10

Concentric-Cylinder Heat Exchanger

Heat-Exchangers

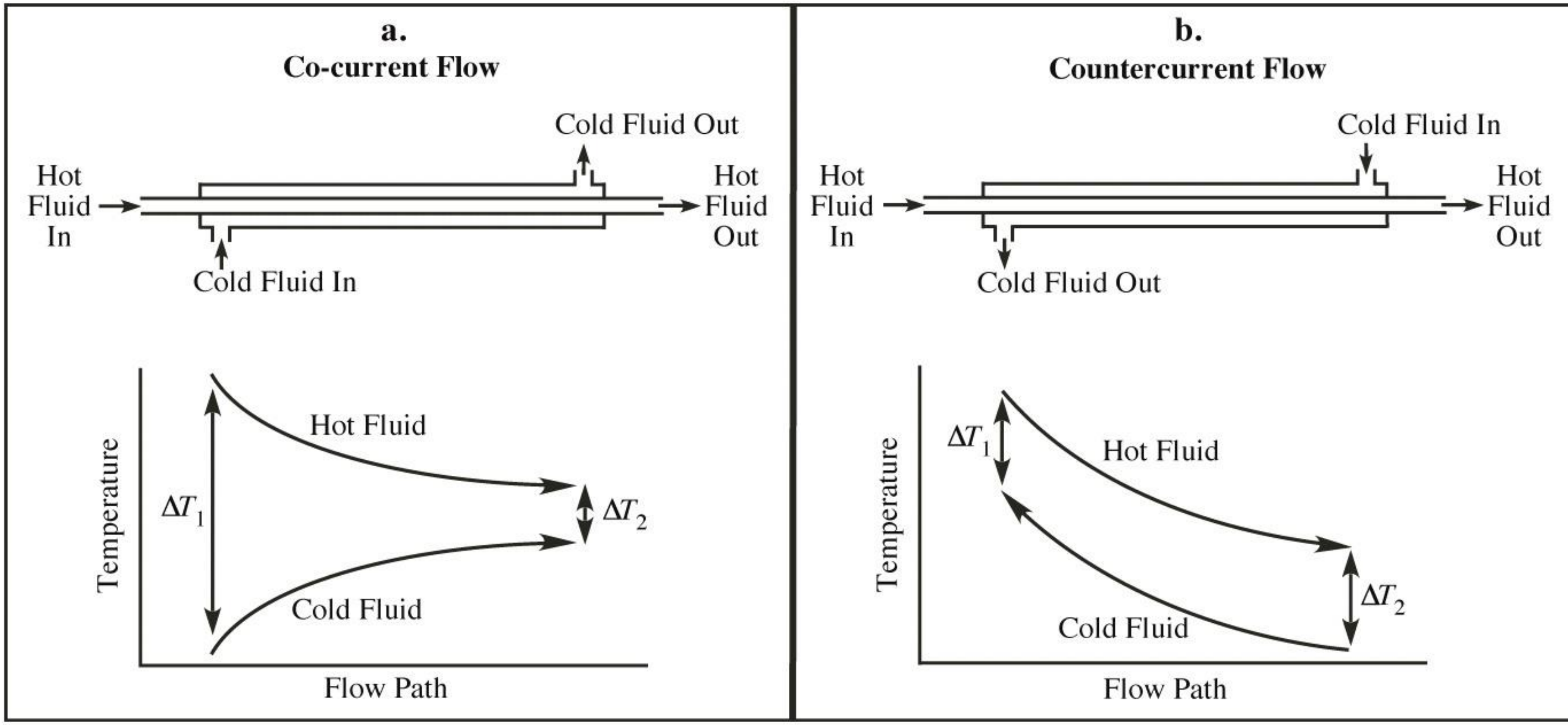


Figure 10.11

Heat-Exchangers

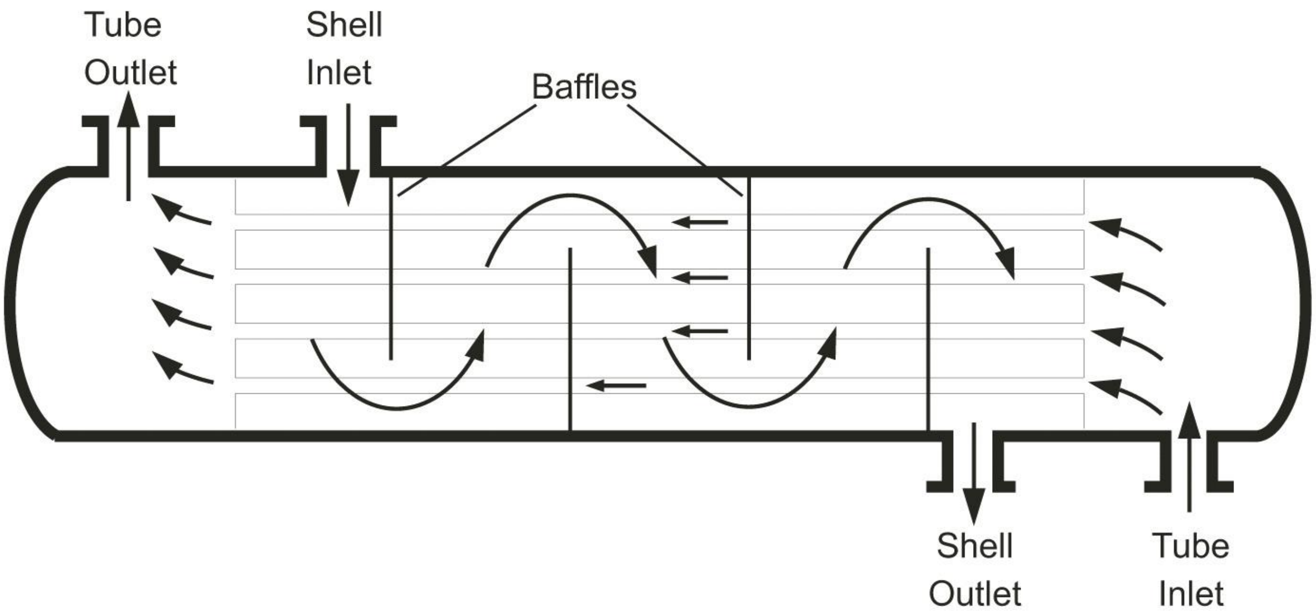


Figure 10.12

Shell-and-Tube Heat Exchanger

Heat-Exchangers

(i) Sensible Heating/Cooling:

$$\left[\dot{m}C_p(T_{out} - T_{in})\right]_{hot} = -\dot{Q}_{duty} \qquad (10.24a)$$

$$\left[\dot{m}C_p(T_{out} - T_{in})\right]_{cold} = \dot{Q}_{duty} \qquad (10.24b)$$

(ii) Phase Change:

$$\left[\dot{m}\Delta\hat{H}_{phase\,change}\right]_{hot} = -\dot{Q}_{duty} \qquad (10.24c)$$

$$\left[\dot{m}\Delta\hat{H}_{phase\,change}\right]_{cold} = \dot{Q}_{duty} \qquad (10.24d)$$

(iii) Chemical Reaction:

$$\left[r_{consumption,A}\Delta\tilde{H}_{reaction,A}\right]_{hot} = -\dot{Q}_{duty} \qquad (10.24e)$$

$$\left[r_{consumption,A}\Delta\tilde{H}_{reaction,A}\right]_{cold} = \dot{Q}_{duty} \qquad (10.24f)$$

Heat-Exchangers

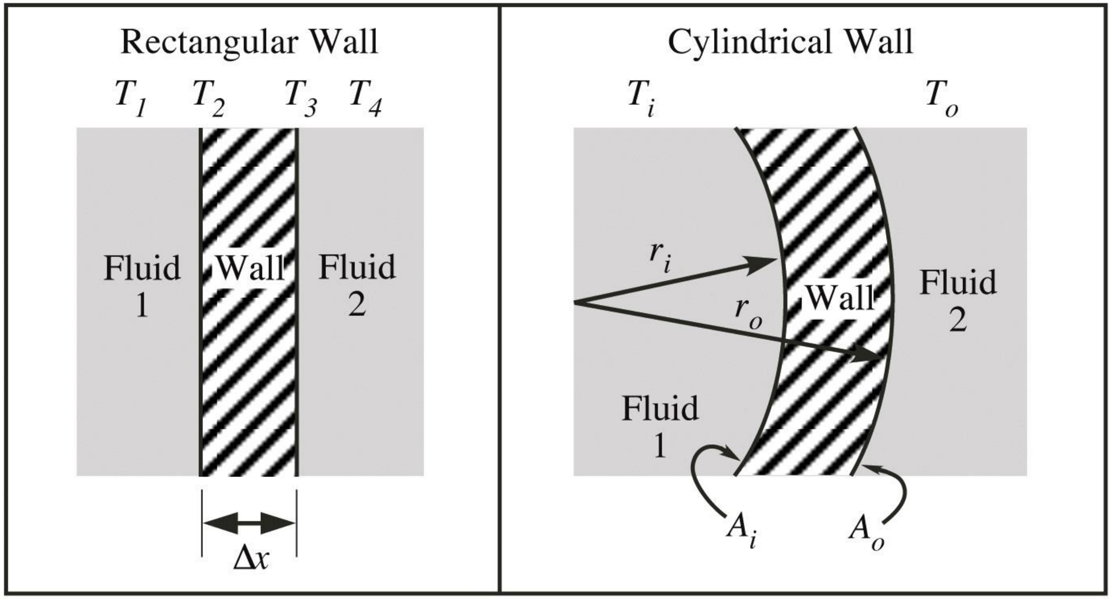


Figure 10.15

$$\dot{Q} = h_1 A(T_1 - T_2) = kA\frac{T_2 - T_3}{\Delta x} = h_2 A(T_3 - T_4) \qquad (10.25)$$

Homework

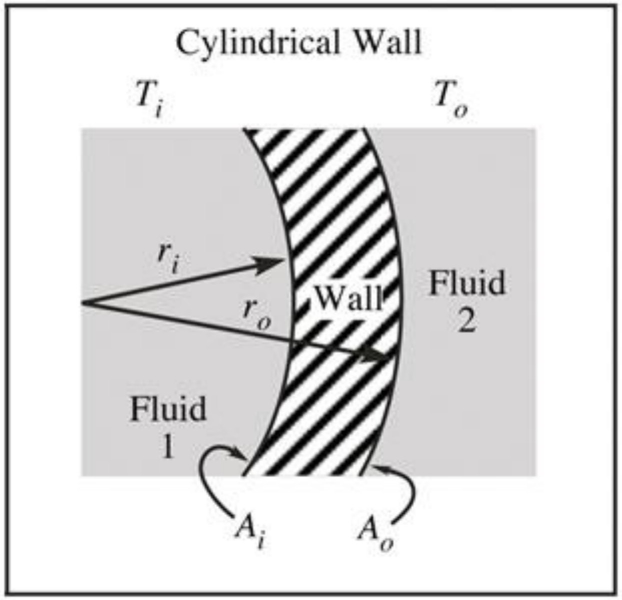
- Derive following equations.

$$\dot{Q} = \frac{T_1 - T_4}{\frac{1}{h_1 A} + \frac{\Delta x}{kA} + \frac{1}{h_2 A}} \qquad (10.26)$$

$$\dot{Q} = \frac{T_i - T_o}{\frac{1}{h_i A_i} + \frac{\ln(r_o / r_i)}{2\pi k L} + \frac{1}{h_o A_o}} \qquad (10.27)$$

Heat-Exchangers

- T_i and T_o change along the length of the device.
- The prediction of the values of h_i and h_o is complex.



$$\dot{Q}_{duty} = U_o A \Delta T_{ave} \qquad (10.28)$$

U_o : overall heat transfer coefficient

$$\Delta T_{\log mean} = \frac{\Delta T_1 - \Delta T_2}{\ln \dfrac{\Delta T_1}{\Delta T_2}} \qquad (10.29)$$

log mean temperature difference