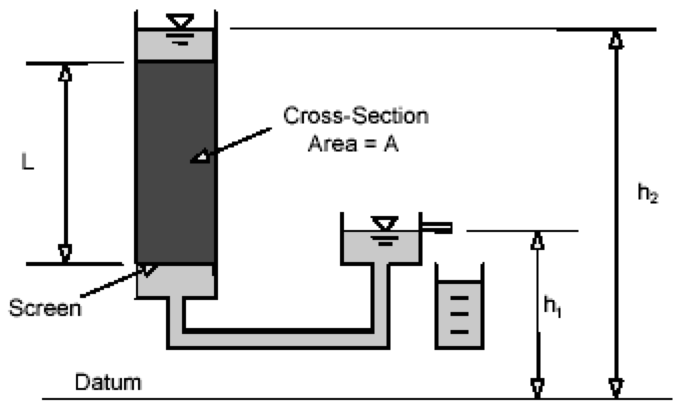
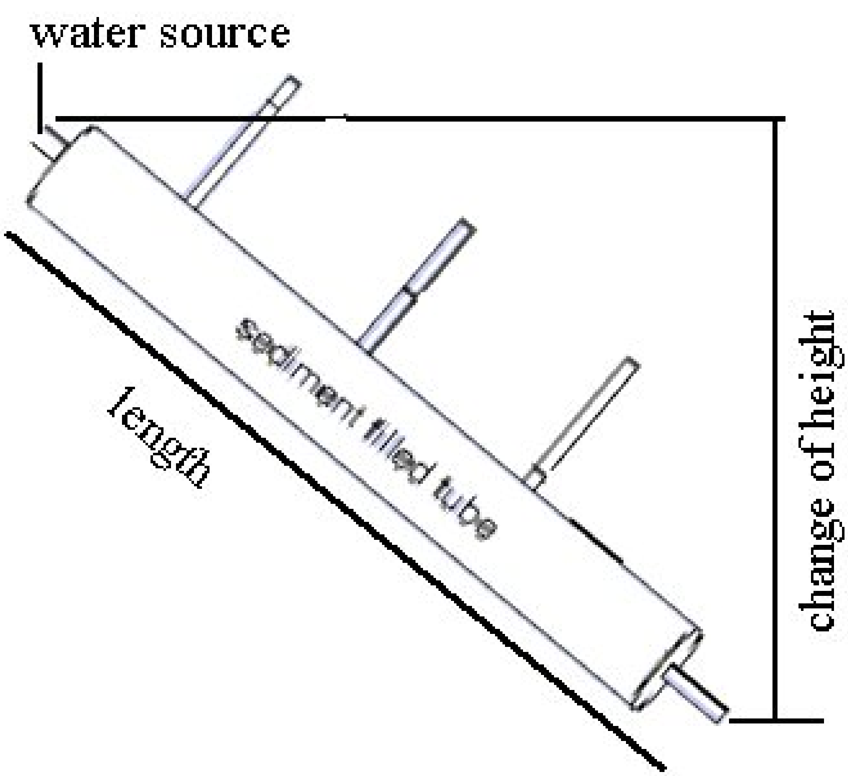
Darcy's Law - Darcy experiment

Darcy, 17th century Paris:
Darcy measured the discharge of a tube by measuring the volume of water that flows out the outlet during a fixed period of time (e.g., 1 minute) (discharge in terms of the milliliters per minute (a liter is 10^{-3} m^3 and a ml is 10^{-6} m^3)). The discharge is usually expressed in units of m^3/sec.

His experiments were meant to investigate the fundamental controls on ground water flow. It uses a "Darcy tube", a tube that is packed with sediment through which water can flow.





Water flows from an elevated reservoir into the inlet of the tube, through the sediment and then out the outlet of the Darcy tube. Can vary the drop in hydraulic head that occurs across the tube by changing the elevation of inlet reservoir.
Darcy related the discharge (Q) to the hydraulic gradient (head drop/distance), cross-sectional area (A) of the tube and to a property of the sediment that he referred to as the hydraulic conductivity (K).

Quantitative expression of the above mentioned relationship:

Discharge proportional to properties of material and pressure head (gradient),

$Q \propto (A * P)/l$

A = cross sectional area
P = pressure
l = path length

and k = proportionality constant, **hydraulic conductivity**

$$Q = k \frac{A \bullet P}{l}$$

P can be expressed as difference in elevation (i.e., elevation or pressure head) between the start of flow length h_1 and end h_2, or in the field, as the change in elevation of saturated zone.

$$Q = k \bullet \frac{A \bullet (h_1 - h_2)}{l}$$

and $(h_1-h_2)/l$ = the hydraulic gradient

Units:

$$\frac{L^3}{T} = k \bullet L^2 \frac{(L - L)}{L} = k \bullet L^2$$

therefore, the unit of k should be L/T

Permeability: ability of water to flow through soil/sediment/rock.

controlled by:

- porosity
- grain size
- interconnection of the pores
  sand/gravel: high permeability- high porosity, good interconnection of pores. [k = 0.1 - 1000's m/day]
    - clay: very low permeability- high porosity, moderate interconnection of the pores, very small pore size [k = <0.01 m/day]
    - sedimentary rocks: high to very low permeability- high to very low porosity, good to poor int erconnection of the pores
        - sandstone [k = 0.3-3 m/day]
        - shale [k = <0.01 m/day]
    - crystalline rocks: moderate to very low permeability- usually low porosity, often good interconnection of fractures [k = 0.0003-3 m/day]

Hydraulic conductivity: an indication of an aquifer's ability to transmit water

$k = k_{intrinsic}\ (\rho g/\mu)$

k; hydraulic conductivity (m/day, cm/sec)
$k_{intrinsic}$; intrinsic permeability (m^2, darcy; 1 darcy = 0.987 μm^2)
ρ; fluid density
g; gravitational constant
μ; dynamic viscosity

k depends on the medium and fluid properties (ground water hydrology).

(e.g.) sand 10^{-2} cm/sec
silt 10^{-4} cm/sec
clay 10^{-7} cm/sec

$k_{intrinsic}$ is a function of the medium only, independent of fluid property (petroleum industry).

***transmissivity (m^2/day); k * (saturated thickness of aquifer)**

Specific velocity: average velocity of flow across a cross-section of an aquifer

$$v = Q/A$$

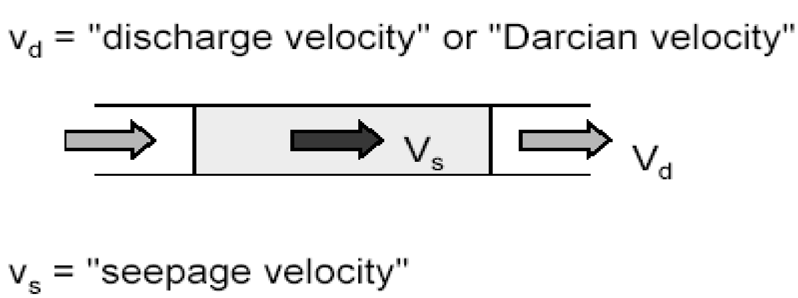
Now we can express Darcy's Law as velocity and in differential form:

$$\frac{Q}{A} = v = k\left(\frac{dh}{dl}\right)$$

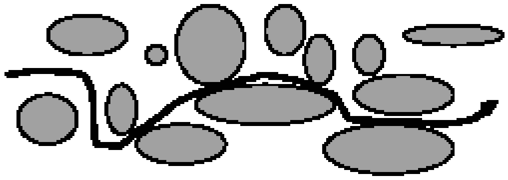
v = Darcy velocity, Darcy flux. This v is for macroscopic scale - it represents the combination of all the impossible to measure individual velocities through the porous media. It is a **" macroscopically averaged description of microscopic processes**".

(* specific velocity = specific discharge = Darcy velocity = Darcy flux)

Seepage velocity: actual average velocity of water particles



Actual velocity is higher because the water flows a longer path through the soil particles.



"Tortuosity" is a measure of the distance traveled vs. linear distance.

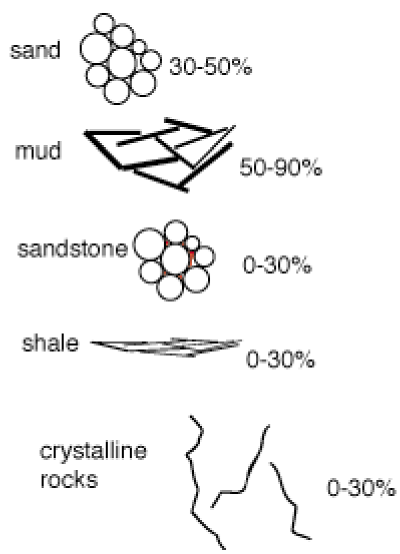
$$v_p = \frac{v}{n}; \;\; pore\ velocity = \frac{Darcy\ velocity}{porosity}$$

and

$$v_p = \frac{k}{n}\frac{dh}{dl}$$

Porosity ranges from 0 to <100% therefore, seepage velocity is always higher than Darcy velocity (by a factor of 3 or 4)

Porosity: % open space/void in a rock or sediment

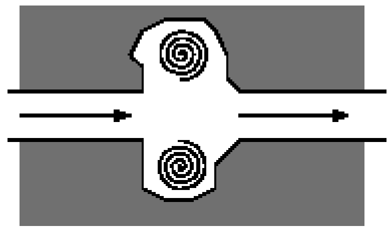


- sand/gravel: voids between grains. 30-50% porosity
- clay: voids between sheets of clay. Charges on the clay particles produce a house of cards like structure. 50-90% porosity (often very high)
- sedimentary rocks:
    - sandstone is like sand except that the pores are partially filled with mineral cement. 0-30% porosity
    - shale is like mud except that compaction has closed many of the pores. 0-30% porosity (often very low)

- crystalline rocks: fracture porosity. 0-30% porosity (often very low)

Effective porosity (n_e):

Not all pores in the soil (aquifer) conduct flow. Some are clogged (i.e, dead-end pores)



therefore,

$$\mathbf{n_e = \lambda n}$$

λ is effective porosity factor (determined experimentally)
$\lambda \approx 1$ for sands and gravels
$\lambda = 0.01$ to 0.5 for clays

Seepage velocity ($v_p = \frac{k}{n}\frac{dh}{dl}$) is more appropriate if n is substituted with n_e;

$$v_p = \frac{k}{n_e}\frac{dh}{dl}$$

Darcy's law applies to laminar flow in porous media:

"Reynolds number (R_e)" as the boundary between laminar and turbulent flow

$$R_e = \frac{vD\rho}{\mu}$$

where
v = discharge velocity [ft/s]
D = mean particle size [ft]
ρ = mass density of water [1.94 lb-sec^2/ft^4]
μ = dynamic viscosity of water [2.35×10^{-5} lb-sec/ft^2]

In natural conditions (aquifers), it is known that the boundary occurs at R_e <1 to 10

In most cases, ground water travels at slow enough velocity for Darcy's law to be valid (i.e., true laminar flow)

Groundwater contamination migrates with the water. The rate and direction is controlled by Darcy's law.