Optimal Design of Energy Systems (M2794.003400)

Chapter 15. Dynamic Behavior of Thermal Systems

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Chapter 15. Dynamic Behavior of Thermal Systems

15.1 In What Situations is Dynamic Analysis Important?

Steady-state	Dynamic
More frequently than dynamic simulations	Address transient problems
Can be justified in the design	Can be corrected in in the field
Ex. Part-load efficiency, Potential operating problems	Ex. System shutdown, Damage the plant, Imprecise control

Dynamic Analysis : with respect to time, on/off, under control, disturbance

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15.2 Scope and Approach of This Chapter

Intention

- Concentration on thermal components
- Emphasis of behavior in the time domain
- The translation of physical situations into symbolic or mathematical representation

Object

- More comfortable in making dynamic analysis
- Representation of the performance in the time domain
- Experience in block diagram

15.3 One Dynamic Element in a Steady-State Simulation

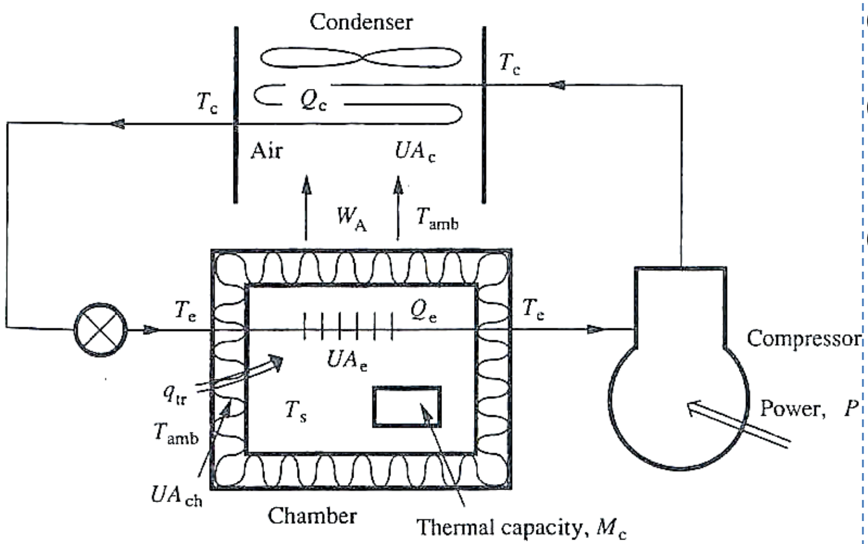

Fig. System with one dynamic element (refrigeration plant serving a cold room)

Compressor ref. capacity $\quad q_e = f_1(T_e, T_c)$

Compressor power $\quad P = f_2(T_e, T_c)$

Condenser $\quad q_c = \dot{m}c_{p,a}(T_c - T_{amb})(1 - e^{\frac{UA}{\dot{m}c_{p,a}}})$

$\rightarrow \varepsilon$

Evaporator $\quad q_e = (T_s - T_e)(UA_e)$

Energy balance $\quad q_c = P + q_e$

Heat transfer to chamber

$$q_e = q_{tr} = UA_{ch}(T_{amb} - T_s)$$

$\rightarrow$ steady-state

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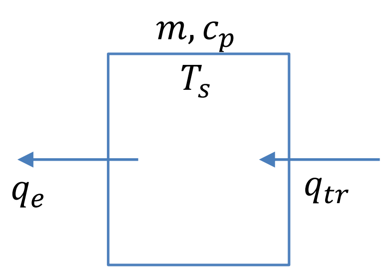
15.3 One Dynamic Element in a Steady-State Simulation

Pull-down

$$q_{tr} = UA_{ch}(T_{amb}, -T_s)$$

$$q_{tr} = q_e + mc_p\frac{dT_s}{dt}$$

Dynamic : during pull-down $q_{tr} \neq q_e$



15.4 Laplace transform

- Powerful tool in predicting dynamic behavior
- One way to solve ODE

$$L\{F(t)\} = \int_0^\infty F(t)e^{-st}dt = f(s)$$

$$L\{F'(t)\} = \int_0^\infty F'(t)e^{-st}dt$$

$$= e^{-st}F(t)]_0^\infty - \int_0^\infty F(t)(-s)\,e^{-st}dt$$

$$= -F(0) + sf(s)$$

$$L\{F''(t)\} = \int_0^\infty F''(t)e^{-st}dt$$

$$= e^{-st}F'(t)]_0^\infty - \int_0^\infty F'(t)(-s)\,e^{-st}dt$$

$$= -F'(0) + s[-F(0) + sf(s)]$$

$$= -F'(0) - sF(0) + s^2 f(s)$$

15.4 Laplace Transforms

Example 15.1 : What is the Laplace transform of the constant *c* ?

(Solution)

$$\mathcal{L}\{c\} = \int_0^{\infty} c\, e^{-st} dt = -\frac{c}{s} e^{-st}\Big]_0^{\infty} = \frac{c}{s}$$

Example 15.2 : What is the Laplace transform of *bt* ?

(Solution)

$$\mathcal{L}\{bt\} = \int_0^{\infty} bt\, e^{-st} dt = -b\frac{d}{ds}\int_0^{\infty} e^{-st}\, dt = -b\frac{d({}^1/_s)}{ds} = b/s^2$$

15.5 Inversion of Laplace Transforms $L^{-1}\{f(s)\} = F(t)$

Example 15.4 : Invert $\dfrac{s+10}{(s-2)^2(s+1)}$

(Solution) $$\frac{s+10}{(s-2)^2(s+1)} = \frac{A}{(s+1)} + \frac{B}{(s-2)^2} + \frac{B\prime}{s-2}$$

$$constants : 10 = 4A + B - 2B'$$

$$s : \quad 1 = -4A + B - B'$$

$$s^2 : \quad 0 = A + B'$$

$$A = 1,\ \ B = 4,\ \ B' = -1$$

$$\therefore\ L^{-1}\left\{\frac{s+10}{(s-2)^2(s+1)}\right\} = e^{-t} + 4te^{2t} - e^{2t}$$

15.5 Inversion of Laplace Transforms $L^{-1}\{f(s)\} = F(t)$

(Another Solution of Example 15.4)

- ✓ **For non-repeated roots**

$$\frac{N(s)}{D(s)} = \frac{A}{s-a} + \frac{B}{s-b} + \cdots \qquad A = \frac{N(s)(s-a)}{D(s)}\Big|_{s\to a} \qquad B = \frac{N(s)(s-b)}{D(s)}\Big|_{s\to b}$$

- ✓ **For repeated roots**

$$\frac{N(s)}{D(s)} = \frac{A}{s-a} + \frac{B}{(s-b)^2} + \frac{B'}{s-b} \qquad B = \frac{N(s)(s-b)^2}{D(s)}\Big|_{s\to b} \qquad B' = \frac{d}{ds}\left[\frac{N(s)(s-b)^2}{D(s)}\right]_{s\to b}$$

$$\to A = \frac{s+10}{(s-2)^2}\Big|_{s\to -1} = 1 \quad B = \frac{s+10}{s+1}\Big|_{s\to 2} = 4 \quad B' = \left(\frac{s+10}{s+1}\right)\Big|_{s\to 2} = -1$$

15.6 Solution of ordinary differential equations

Example 15.6 : Solve $\boldsymbol{Y''(t) + k^2 Y(t) = 0}$

(boundary conditions : $Y(0) = A$, $Y'(0) = B$)

(Solution)

Transform the differential equation

$$s^2 y(s) - sY(0) - Y'(0) + k^2 y(s) = 0$$

Boundary conditions

$$y(s) = \frac{As}{s^2 + k^2} + \frac{B}{s^2 + k^2}$$

Invert y(s)

$$Y(t) = Acos(kt) + (B/k)sin(kt)$$

15.7 Blocks, Block diagrams, and transfer functions

- Variable in S domain (not in time domain)

Transfer function : $$TF = \frac{L\{O(t)\}}{L\{I(t)\}} = \frac{O(s)}{I(s)}$$

ratio of the output to the input

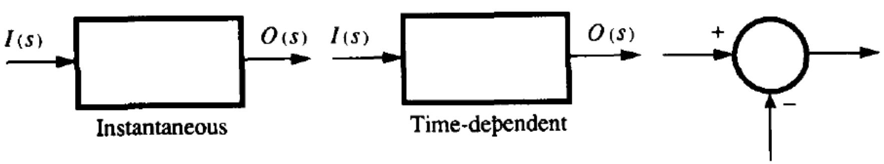


Fig. Symbols used in block diagrams

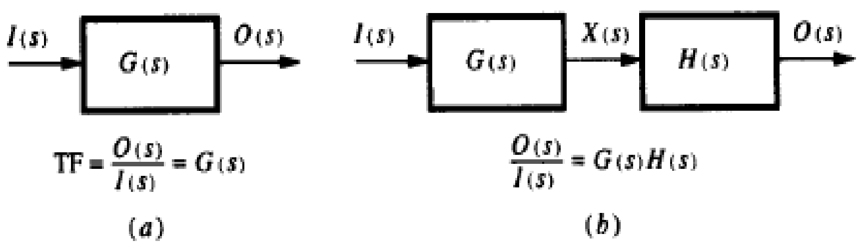


Fig. Transfer function and cascading of blocks

Proper T.F. = 분모 차수 ≥ 분자 차수

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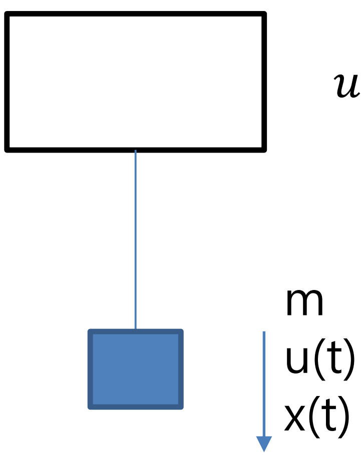
15.7 Blocks, Block diagrams, and transfer functions

- Variable in S domain (not in time domain)

Transfer function : $TF = \frac{L\{O(t)\}}{L\{I(t)\}} = \frac{O(s)}{I(s)}$

ratio of the output to the input



$$u(t) + mg - k * x(t) = m\ddot{x}(t)$$

$$0 - k\delta x + \delta u = m\delta\ddot{x}$$

Inverse Laplace

$$-k\Delta X(s) + \Delta U(s) = ms^2 \Delta X(s)$$

$$TF = \frac{\Delta X(s)}{\Delta U(s)} = \frac{1}{ms^2 + k}$$

15.8 Feedback Control Loop

Unity feedback $TF = \frac{G(s)}{1+G(s)}$

Non-unity feedback $TF = \frac{G(s)}{1+G(s)H(s)}$

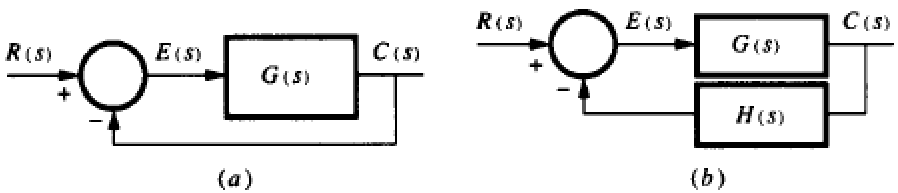
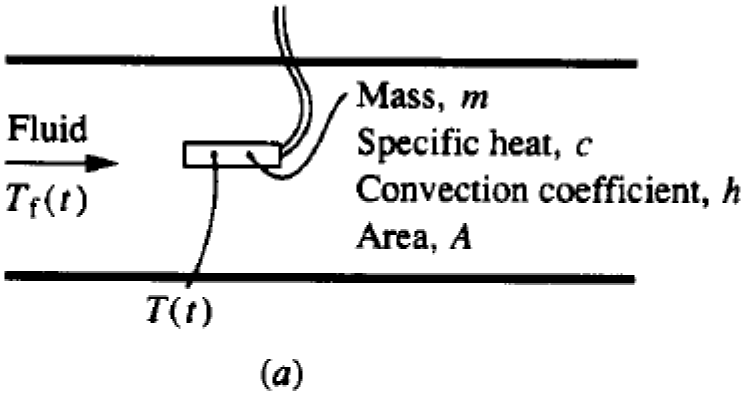


Fig. (a) Unity feedback loop

(b) Nonunity feedback loop

15.9 Time Constant Blocks



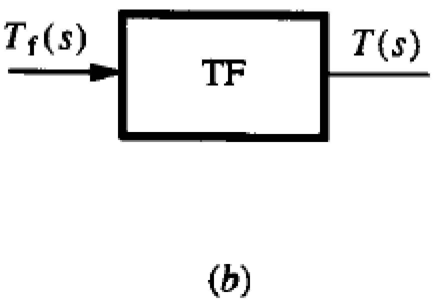


Fig. (a) Response of a temperature-sensing bulb to a change in fluid temperature
(b) Transfer function of this time-constant block

Standard technique for developing transfer function

1. Write differential equation $mc\frac{dT}{dt} = (T_f - T)hA$
2. Transform equation $\frac{mc}{hA}[sL(T) - T(0)] = L(T_f) - L(T)$
3. Solve for transfer function $(L\{O\}/L\{I\})$ $TF = \frac{T(s)}{T_f(s)} = \frac{1+T(0)\frac{B}{T_f(s)}}{1+Bs}$ $\left(B = \frac{mc}{hA}\right)$

For special case $T(0) = 0$: $TF = \frac{1}{1+Bs}$

15.9 Time Constant Blocks

$$mc\frac{d(T-T_0)}{dt}=[(T_f-T_0)-(T-T_0)]hA$$

$$TF=\frac{L\{T-T_0\}}{L\{T_f-T_0\}}=\frac{1}{Bs+1}$$

T_f : unit step increase $\quad T_f(s)=\frac{\Delta}{s}$

$\alpha\beta-\beta=0, \alpha=1, \beta=B$

$$L\{T-T_0\}=L\{T_f-T_0\}\frac{1}{(Bs+1)}=\frac{\Delta}{s(Bs+1)}=\Delta\left(\frac{\alpha}{s}-\frac{\beta}{Bs+1}\right)=\Delta(\frac{1}{s}-\frac{B}{Bs+1})$$

$$T-T_0=\Delta\left(1-e^{-\frac{t}{B}}\right)$$

$\left(B=\frac{mc}{hA}\right)$: time constant

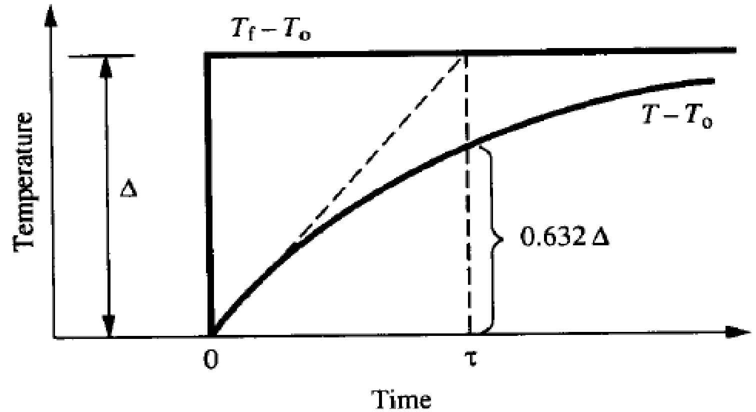
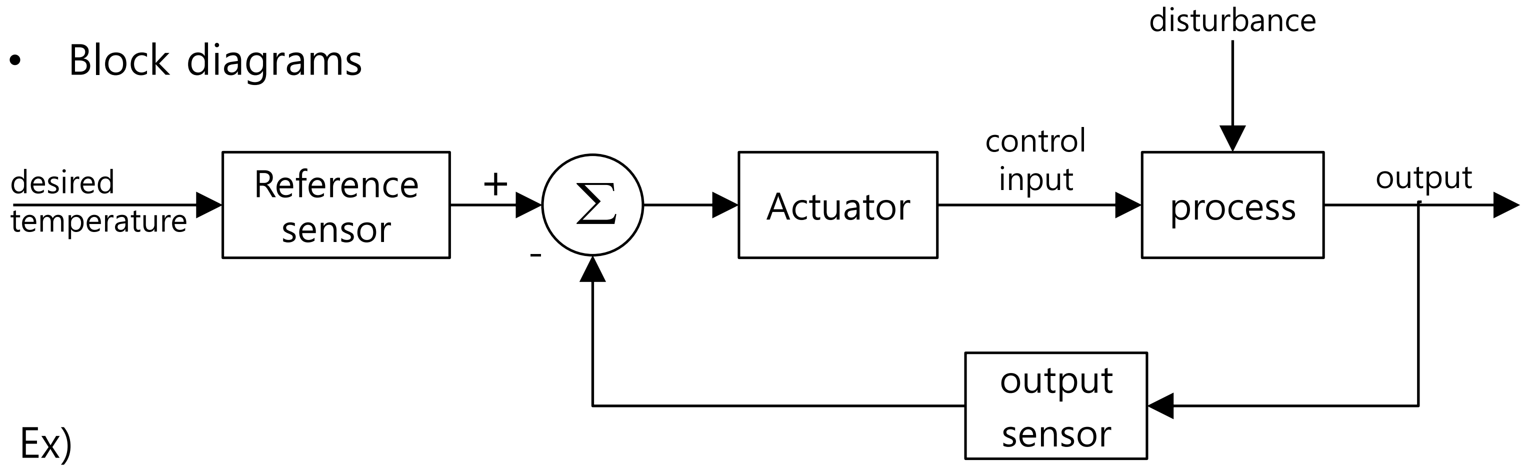


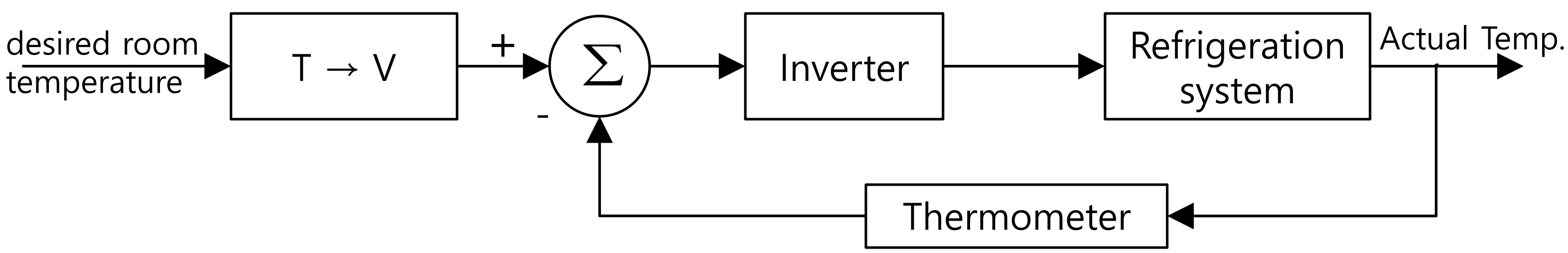
Fig. Step increase in fluid temperature T_f and response of the bulb temperature

cf) Time Constant Blocks - additional

- Control
- Feedback
- Block diagrams



Ex)



15.10 Cascade Time-constant Blocks

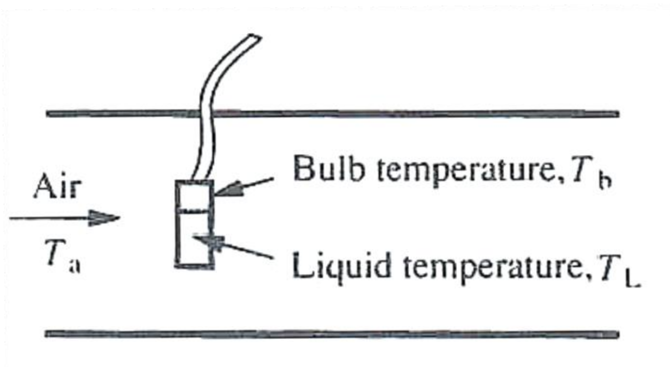


Fig. Response of liquid temperature T_L to a change in air temperature T_A

Heat balance equation :

$$(T_a - T_b)h_1A_1 = mc\frac{dT_b}{dt} + (T_b - T_L)h_2A_2$$

Neglect in order for the heat transfer from the air to the bulb to be represented by the time constant

$$(T_b - T_L)h_2A_2 = mc\frac{dT_L}{dt}$$

$$Let\ \tau_1 = \frac{mc}{h_1A_1}, \tau_2 = \frac{mc}{h_2A_2}$$

subscript 1 : air to bulb
subscript 2 : bulb to liquid

Suppose that T_a experiences a step increase of magnitude Δ from T_0

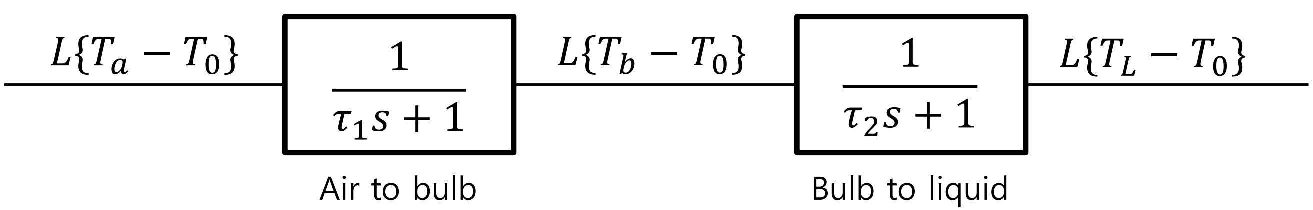


Fig. Cascaded time-constant blocks to represent the dynamic process

15.10 Cascade Time-constant Blocks

For unit step input

$$L\{T_L - T_0\} = \frac{\Delta}{s}\left(\frac{1}{\tau_1 s + 1}\right)\left(\frac{1}{\tau_2 s + 1}\right)$$

Inversion

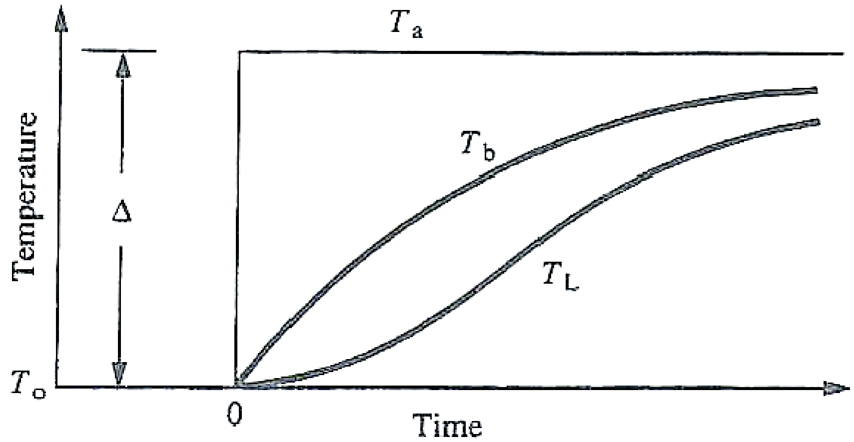
$$\frac{T_L - T_0}{\Delta} = 1 - \frac{\tau_1}{\tau_1 + \tau_2} e^{-\frac{t}{\tau_1}} - \frac{\tau_2}{\tau_2 + \tau_1} e^{-\frac{t}{\tau_2}}$$

① $t = 0,\ T_L - T_0 = 0$

② $\frac{d(T_L - T_0)}{dt} = 0 \ at\ t = 0$

③ $if\quad \tau_2 \ll \tau_1 \quad T_L - T_0 = \Delta(1 - e^{-t/\tau})$

④ $if\quad \tau_2 = \tau_1 \quad \frac{T_L - T_0}{\Delta} = 1 - e^{-t/\tau} - \frac{te^{-t/\tau}}{\tau}$



15.11 Stability Analysis

- Bode diagram expresses the frequency response of the system
- Bode diagram offers an excellent technique for explaining the mechanics of instability

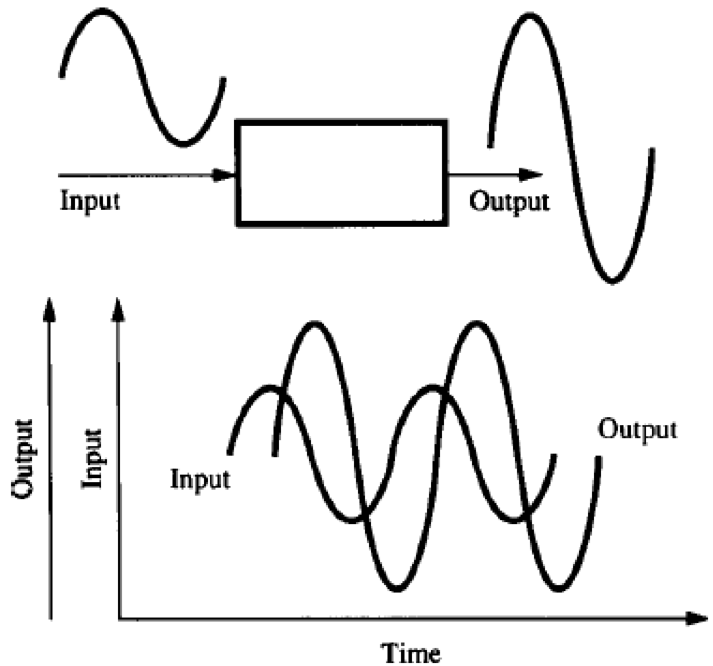


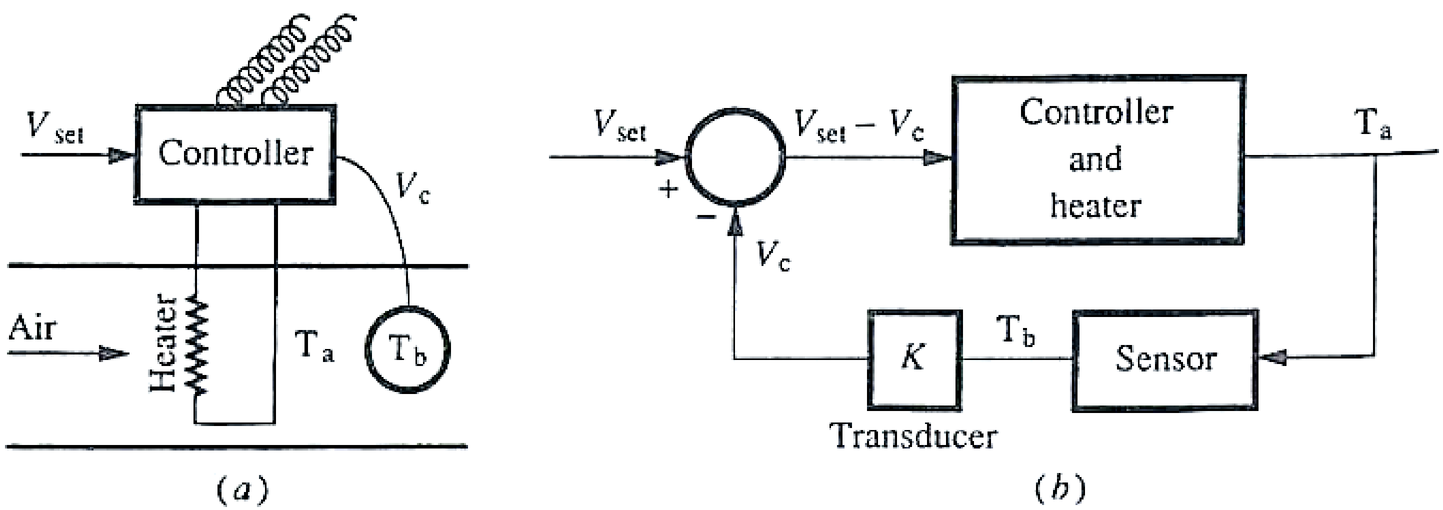
Fig. Frequency response input and output, Bode diagram

15.11 Stability Analysis

- The principle that leads to the Bode Criterion for stability is based on transmission of sine waves throughout the loop

- The **sum of the phase lags around the loop** and the **product of the amplification ratio** are computed

- Example : Air heater – controlled by a loop that senses the outlet air temperature T_a and converts this sensed temperature T_b to a control voltage V_c



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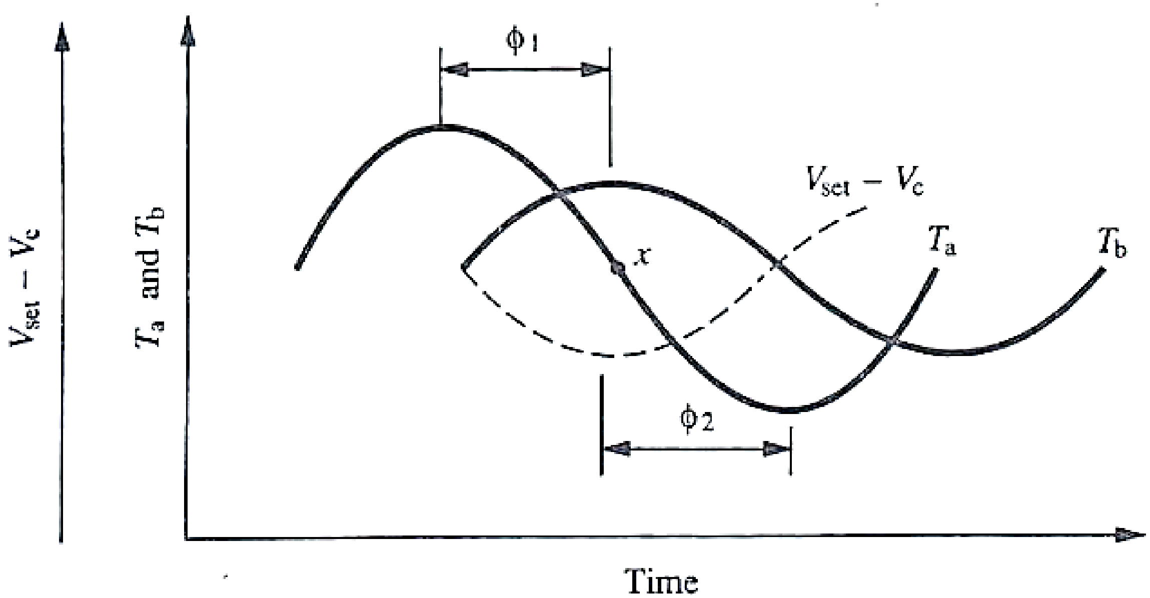
15.11 Stability Analysis

- Air temperature T_a experiences a disturbance that is the top half of a sine wave
- The sensed temperature T_b lags the variation in T_a by an angle ϕ_1
- The variation in T_b translates to a half sine wave of $V_{set} - V_c$, but reversed in sign

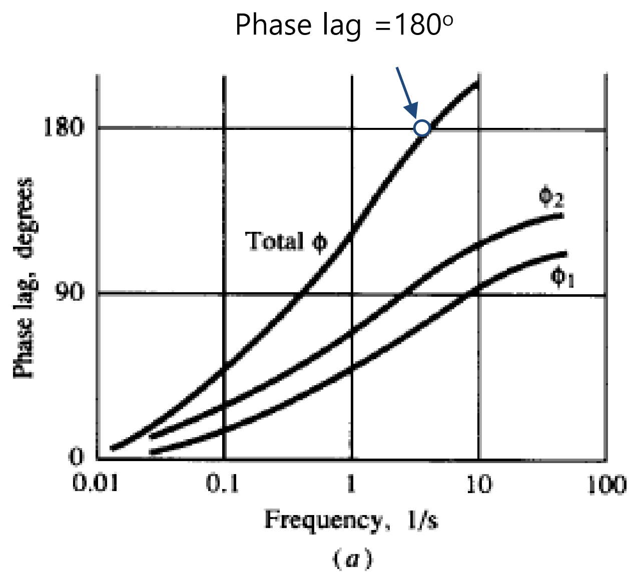


15.11 Stability Analysis

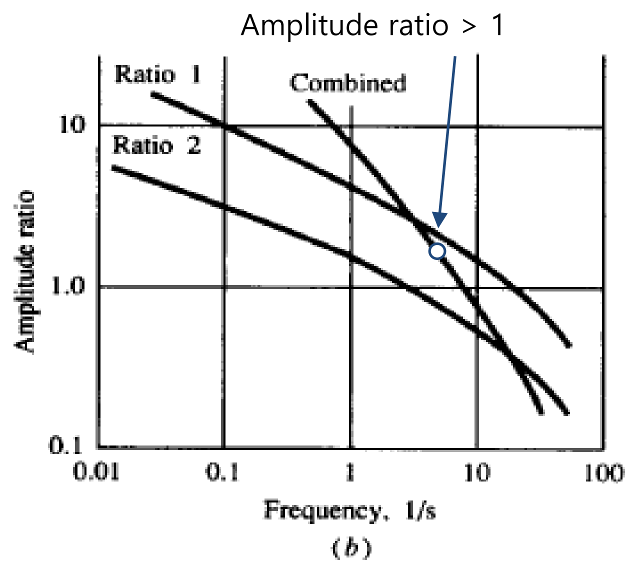
At the frequency f where **sum of phase lags** = 180° Fig.(a)

At the frequency f where **product of amplitude ratio** > 1 Fig.(b)

➡ the loop becomes unstable



(a)

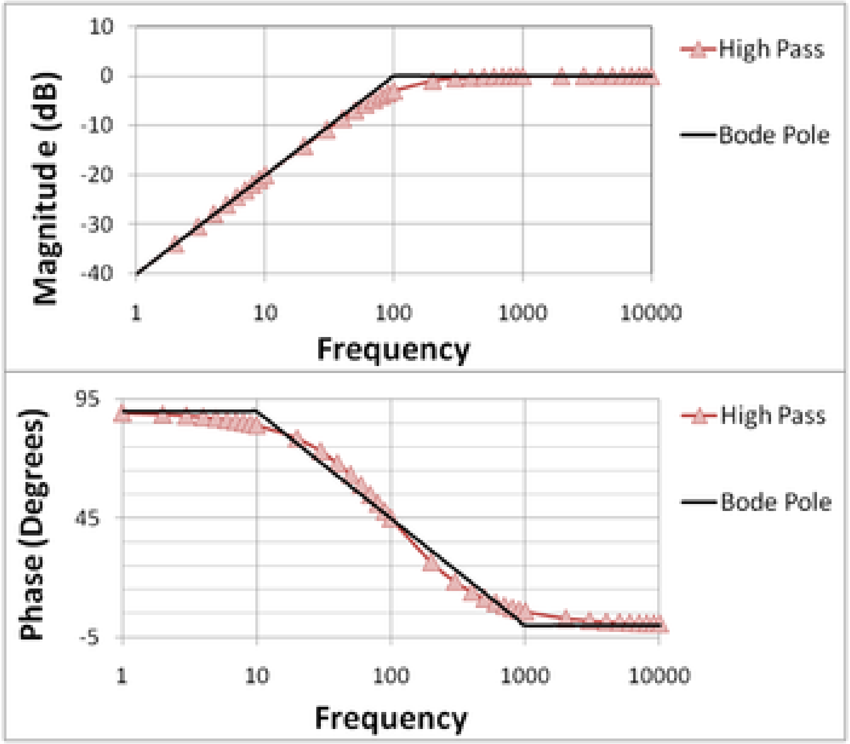


(b)

cf) Bode plot

At Linear time-invariant(LTI) system with **transfer function H(s)**, Bode plot consists of ***magnitude plot*** & ***phase plot***

➡ **Function of filter**

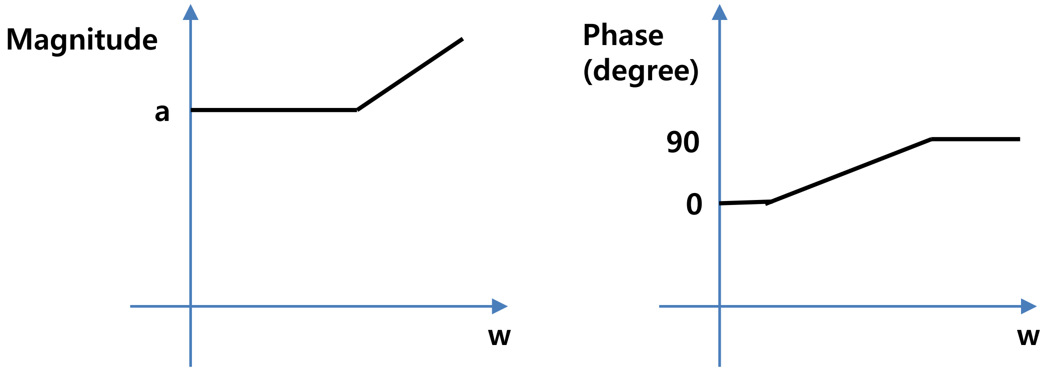


➡ **Represent the gain and phase of a system as a function of frequency**

ex) TF(s) = s+a

i) s = jw << a

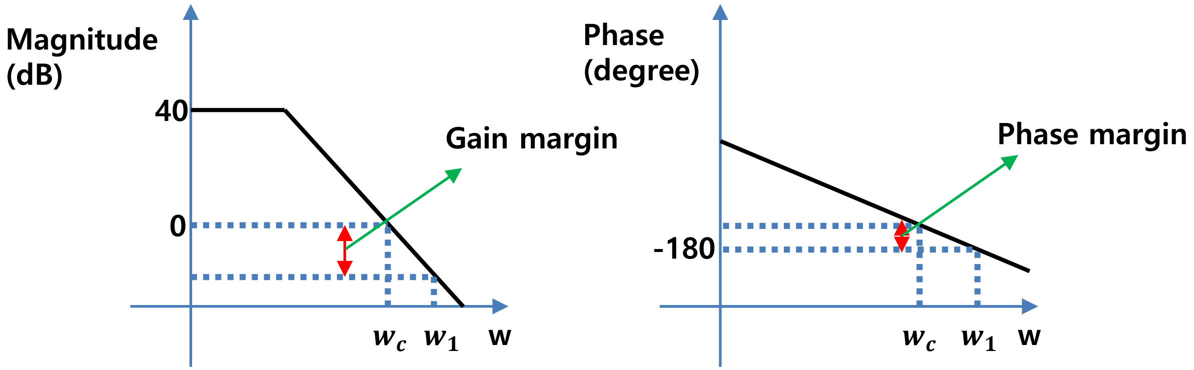
ii) s = jw >> a



ex) Why Bode plot?

$$dB = 20log_{10}|TF(s = jw)|$$

Crossover frequency w_c : $frequency\ w\ at\ 0dB$



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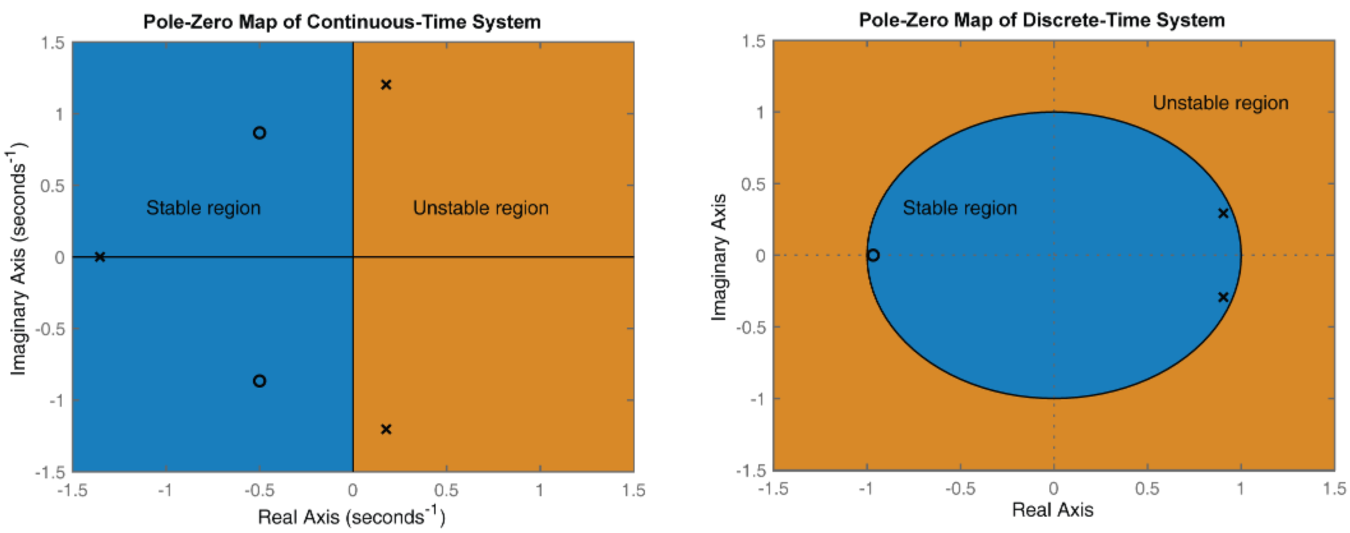
cf) Pole-zero plot

Pole-zero plot shows the location in the complex plane of the **poles and zeros** of the transfer function of a dynamic system, such as a controller, sensor, filter.



Representing

- ✓ **stability**
- ✓ **causal / anti-causal system**
- ✓ **region of convergence**
- ✓ **minimum phase**

cf) Pole-zero plot

$$TF(s) = \frac{O(s)}{I(s)}$$

Pole : value of s that makes TF(s) -> ∞

zero : value of s that makes TF(s) -> 0

Stable : f(t) -> 0 as t -> ∞

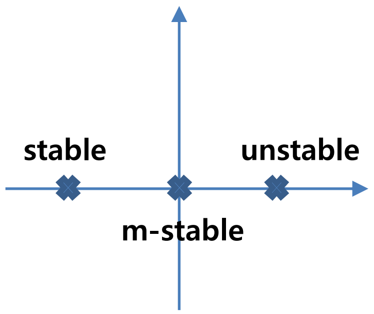
Unstable : f(t) -> ∞ as t -> ∞

Marginally stable : f(t) -> a or oscillation as t -> ∞

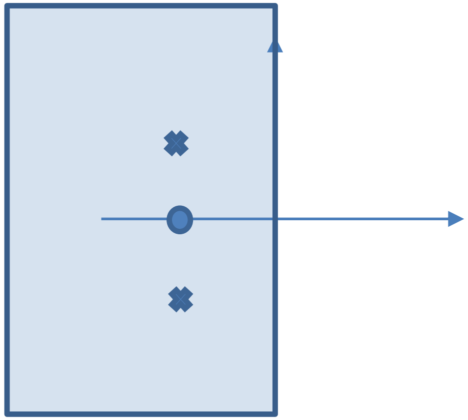
ex) TF(s) = $\boldsymbol{\frac{1}{s+a}}$ $\quad \boldsymbol{(pole : s = -a)}$

Inverse Laplace → $f(t) = e^{-at}$

a>0 : stable
a=0 : marginally stable
a<0 : unstable



ex) TF(s) = $\frac{s+1}{s^2+2s+2}$ $\quad (pole : s = -1 \pm j)$



Inverse Laplace

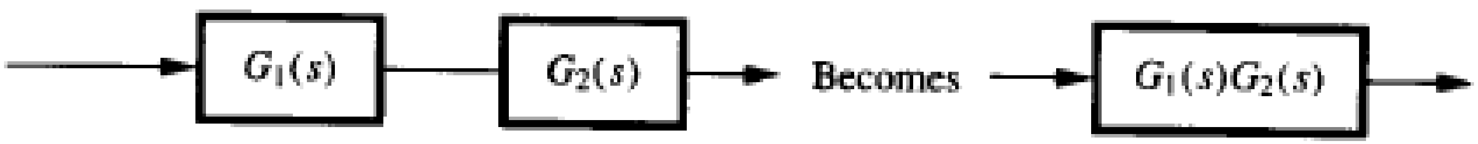
→ $f(t) = e^{-t}\cos(t)$ → **stable**

At proper TF, real negative pole only -> stable

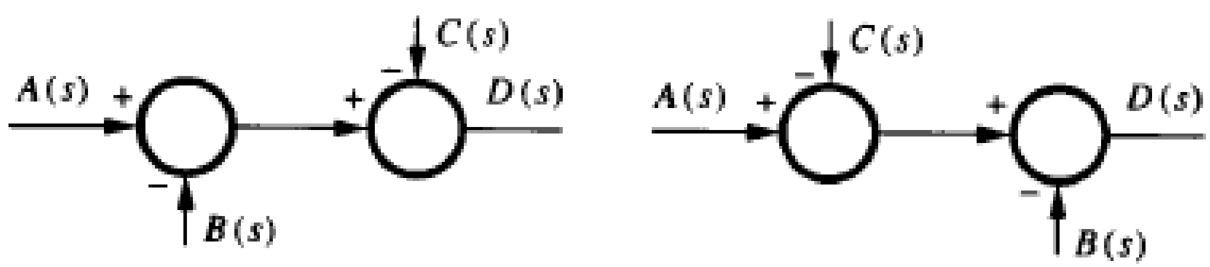
15.14 Restructuring the Block Diagram

- Reconstructing the block to simplify the loop

1. Combine two transfer function in series

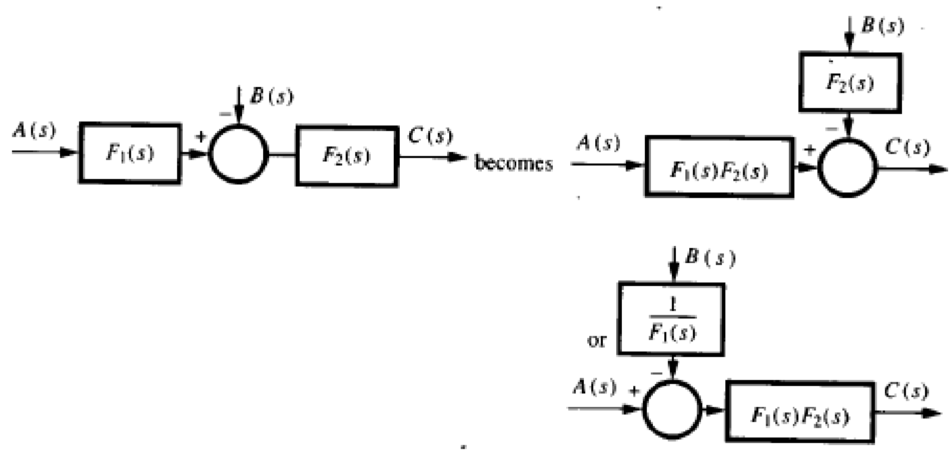


2. Exchange two adjacent summing points



15.14 Restructuring the Block Diagram

3. Move a summing point upstream or downstream of a block



4. Move a take off point

15.15 Translating the Physical situation into a Block Diagram

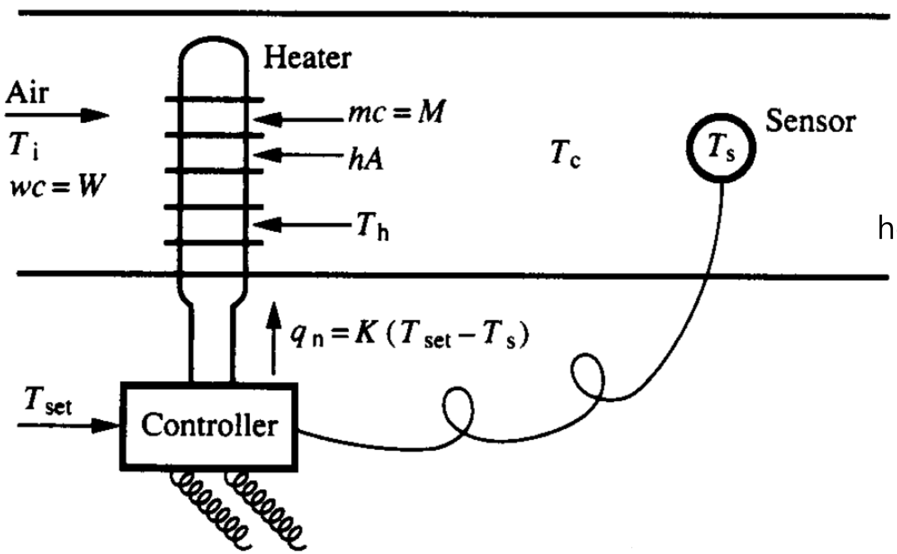


Fig. Air heating system and its control

$$q_h = K(T_{set} - T_s)$$

$$q_a = wc(T_h - T_i)\left(1 - e^{-\frac{hA}{wc}}\right) = W(T_h - T_i)\varepsilon$$

heater→air

↑ $wc = W$

$$q_h - q_a = mc\frac{dT_h}{dt}$$

$$q_h(s) - q_a(s) = M[sT_h(s) - T_h(0)]$$

15.15 Translating the Physical situation into a Block Diagram

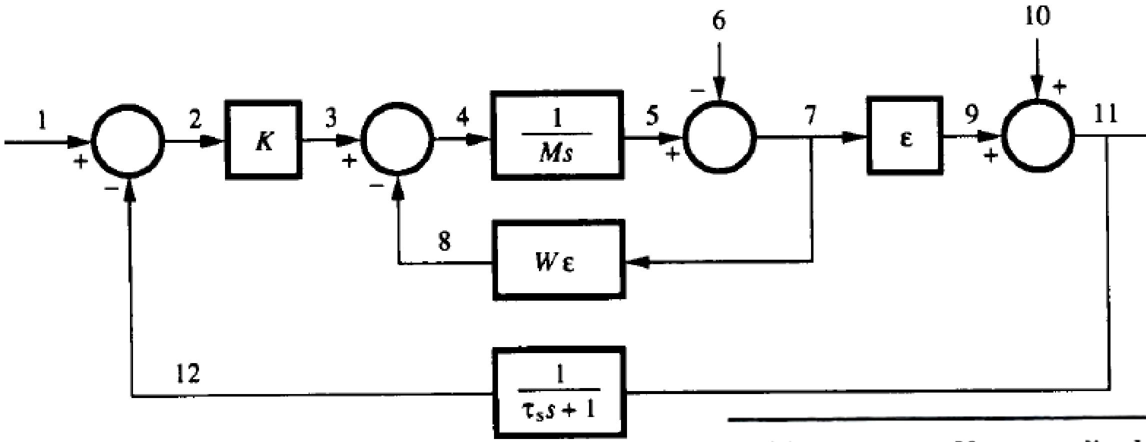


Fig. Air heating system and its control

Position	Nonnormalized	Normalized
1	T_{set}	$T_{set} - T_{set,0}$
2	$T_{set} - T_s$	$(T_{set} - T_s) - (T_{set,0} - T_{s,0})$
3	q_h	$q_h - q_{h,0}$
4	$q_h - q_a$	$(q_h - q_a) - (q_{h,0} - q_{a,0})$
5	T_h	$T_h - T_{h,0}$
6	T_i	$T_i - T_{i,0}$
7	$T_h - T_i$	$(T_h - T_i) - (T_{h,0} - T_{i,0})$
8	q_a	$q_a - q_{a,0}$
9	$T_c - T_i$	$(T_c - T_i) - (T_{c,0} - T_{i,0})$
10	T_i	$T_i - T_{i,0}$
11	T_c	$T_c - T_{c,0}$
12	T_s	$T_s - T_{s,0}$

Table. Designations of variables in block diagram of Fig.

15.15 Translating the Physical situation into a Block Diagram

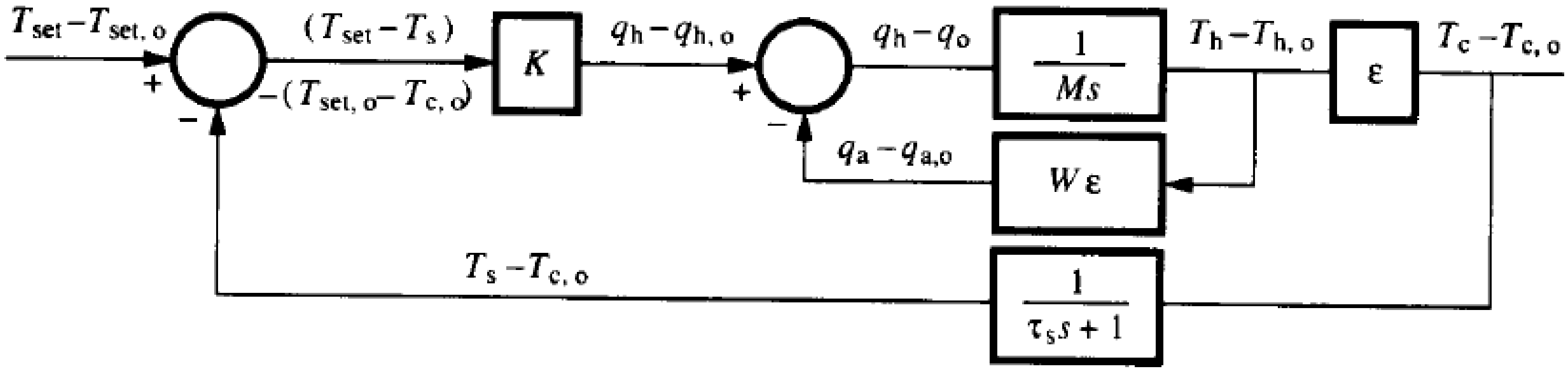


Fig. Diagram after elimination of two summing points

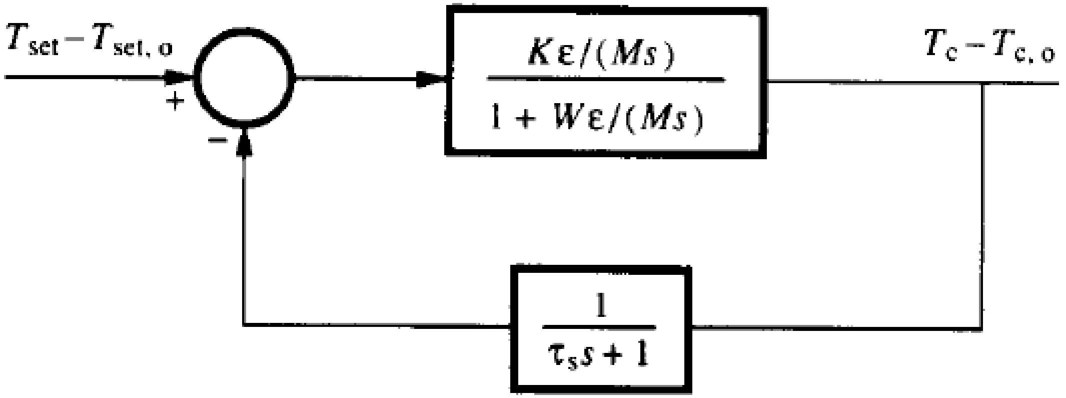


Fig. Simplified nonunity feedback loop for air heater controller

15.16 Proportional Control

$$q_h = K_p \underbrace{(T_{set} - T_s)}_{\text{error}}$$

K↑ unstable

K↓ offset

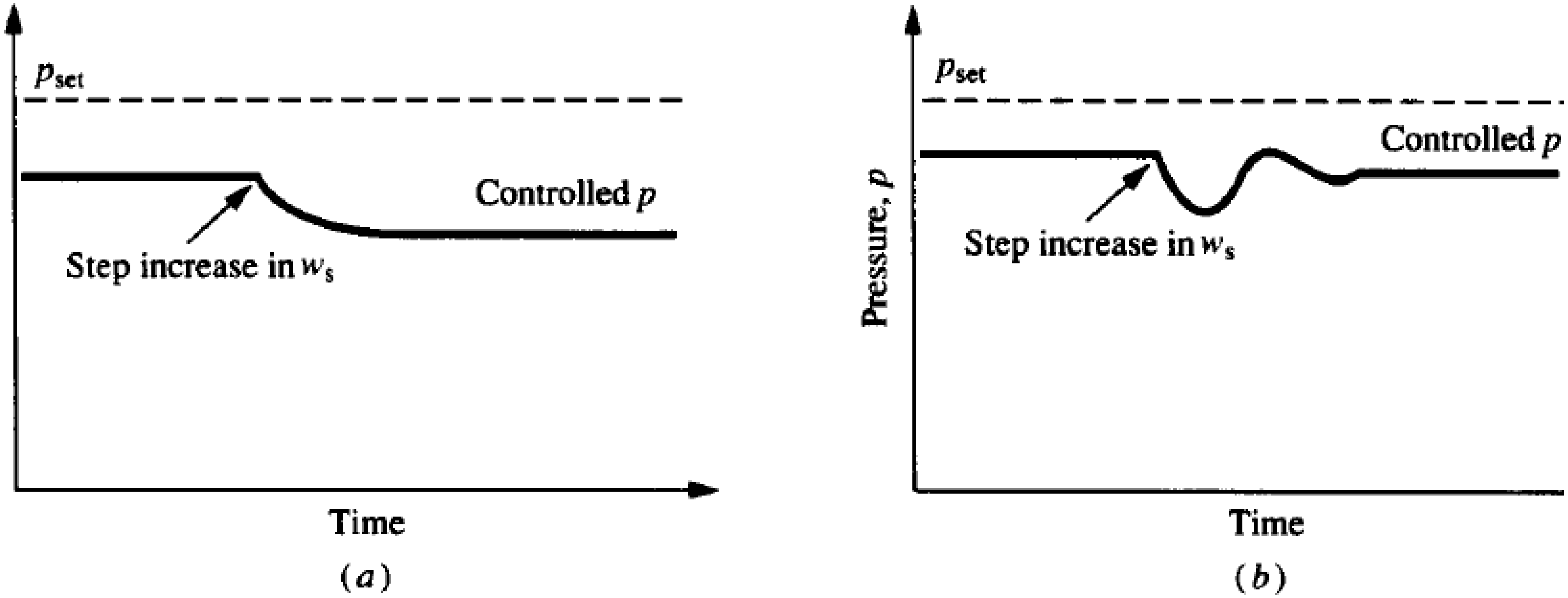


Fig. Pressure controller (a) with low gain (b) with high gain

15.17 Proportional – Integral (PI) Control

- to eliminate the offset

$$K_I \int (error)dt$$

$$TF = \frac{K_I}{s} \leftarrow \frac{K_I \Delta / s^2}{\Delta / s}$$

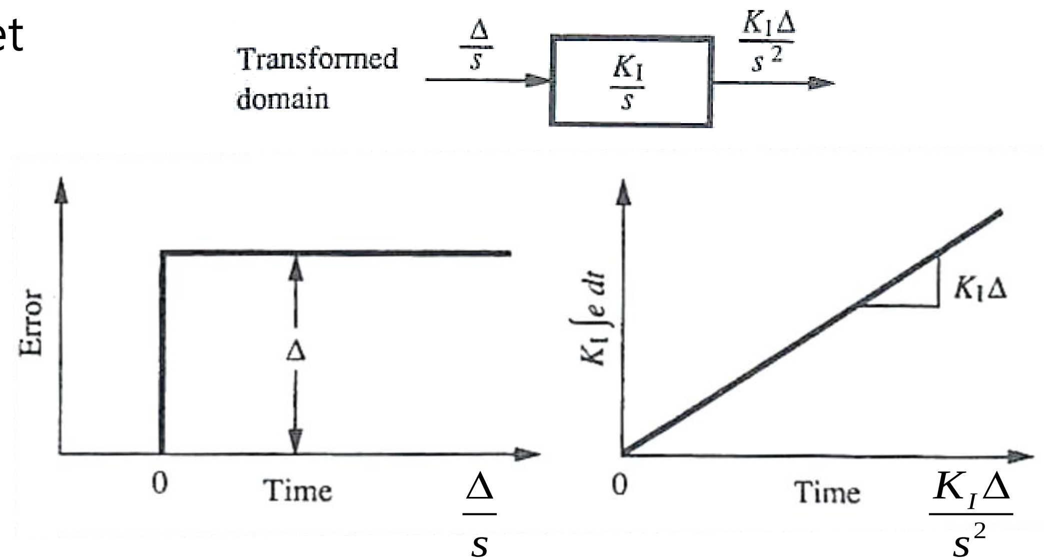


Fig. Transfer function of the I-mode

- PI control

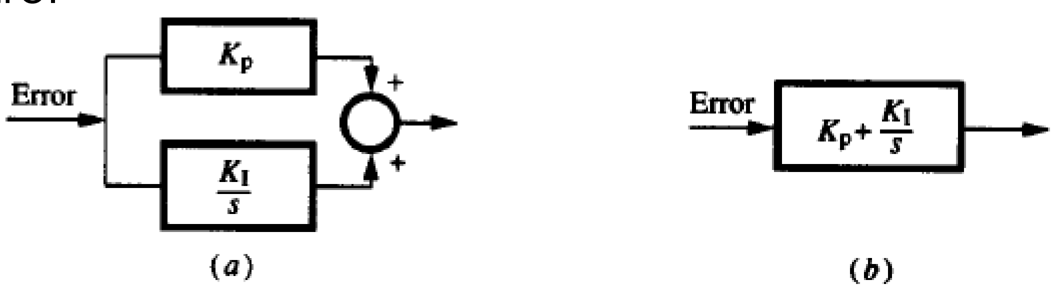


Fig. Block diagram symbols of the PI control

15.18 Proportional-Integral-Derivative(PID) Control

$$K_p + \frac{K_I}{s} + K_D s$$

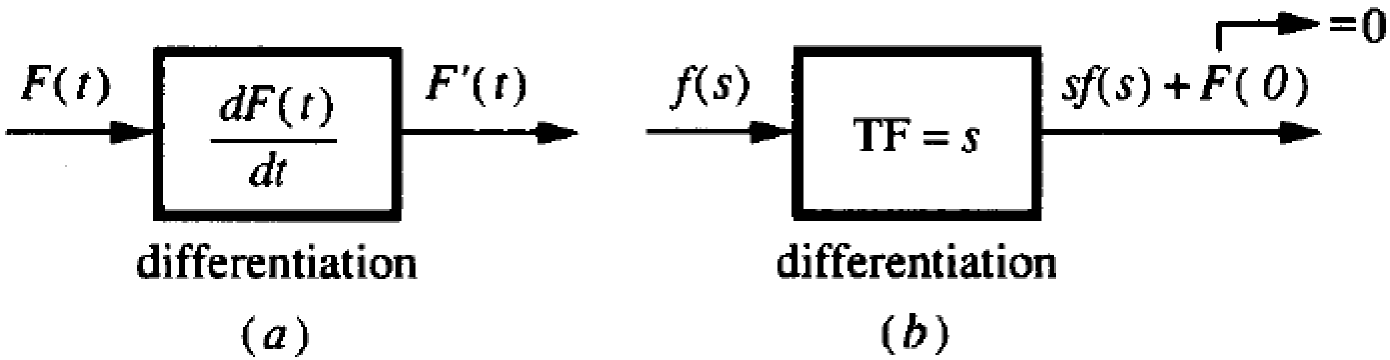
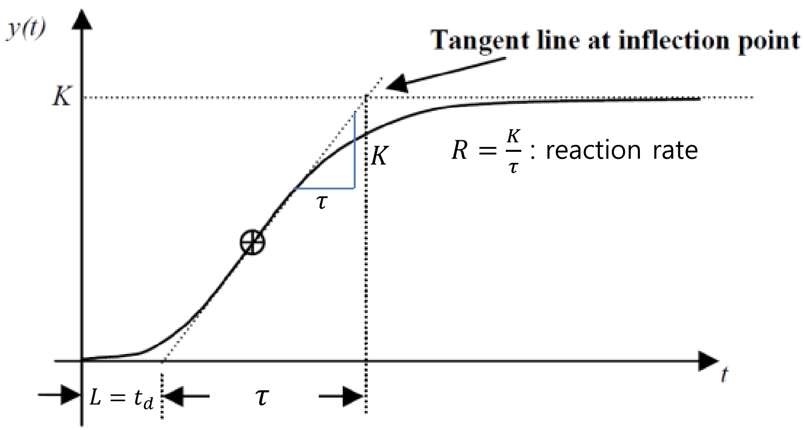


Fig. The differentiation process in (a) the time domain, (b) the transformed domain

cf) PID control – Ziegler-Nichols Tuning of PID controller (1942)



Time delay of t_d seconds

TF may be approximated by $TF = \dfrac{Ke^{-t_d s}}{\tau s + 1}$

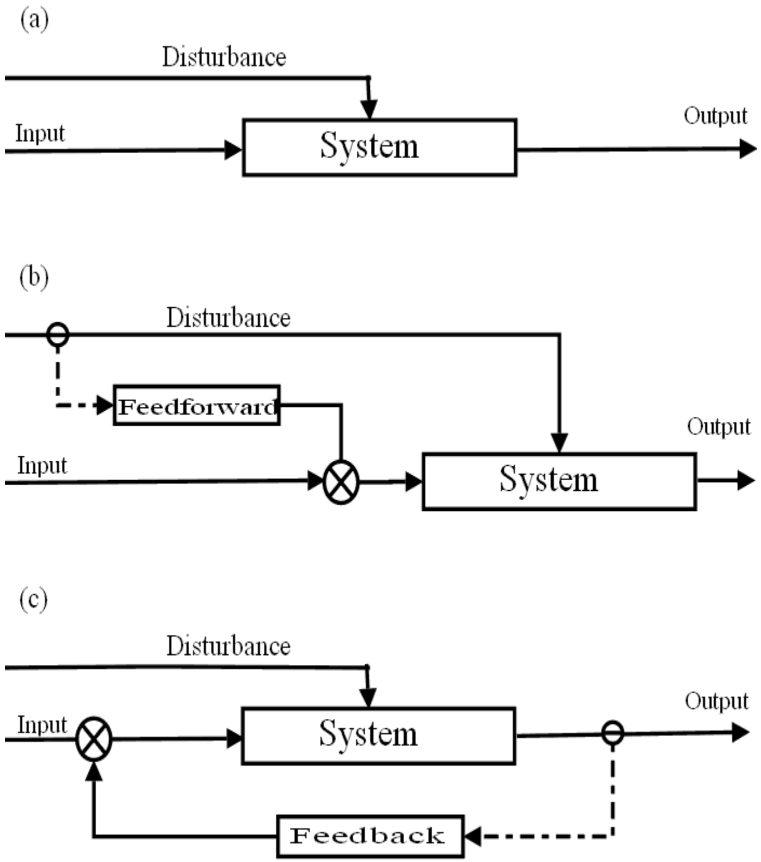
$e^{-t/\tau}$

$$D(s) = K\left(1 + \frac{1}{T_1 s} + T_D s\right)$$

$$= K + \frac{K}{T_I s} + KT_D s = K_p + \frac{K_I}{s} + K_D s$$

cf) Feedforward control

└ open loop control



(a) No control

(b) Feed forward control

(c) Feedback control