Chapter 6. SYSTEM SIMULATION

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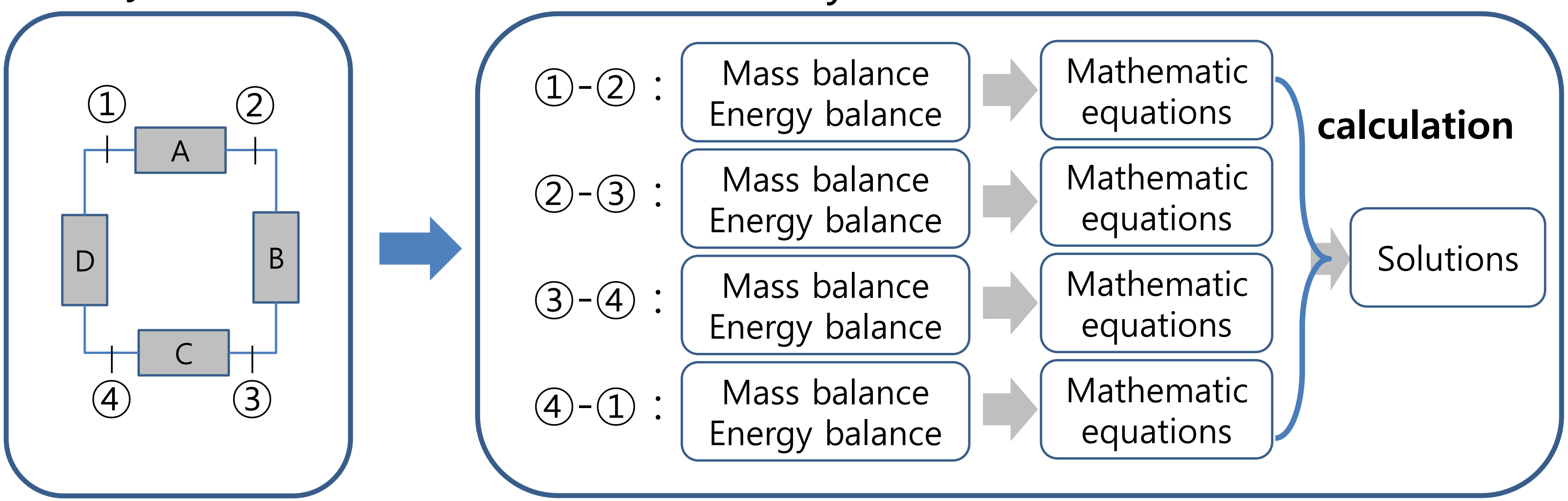
Chapter 6. SYSTEM SIMULATION

6.1 Description of system simulation

- **Calculation** of operating variables(pressure, temp., flow rate, etc…) in a thermal system operating in a steady state.
- **To presume performance** characteristics and equations for thermodynamic properties of the working substances.

System　　　　　　　　　　　　　System simulation



- characteristics of all components
- thermodynamic properties

- a set of simultaneous equations

Chapter 6. SYSTEM SIMULATION

6.2 Some uses of simulation

- Design stage – to achieve **improved design**
- Existing system – to explore prospective **modifications**

- Design condition
- Off design condition (part load / over load) ← normal operation

engineer

choose →

Operation variables

- Pressures
- Temperatures
- Flow rates

Components

- Compressors
- Pumps
- Heat exchangers
- Turbines

Overload

Part load

Cost of energy

Chapter 6. SYSTEM SIMULATION

6.3 Classes of simulation

System

- continuous – water(or air) flow
- discrete – human(or car) flow

- deterministic – input variables are precisely specified
- stochastic – input conditions are uncertain
  - Random
  - Probability distribution

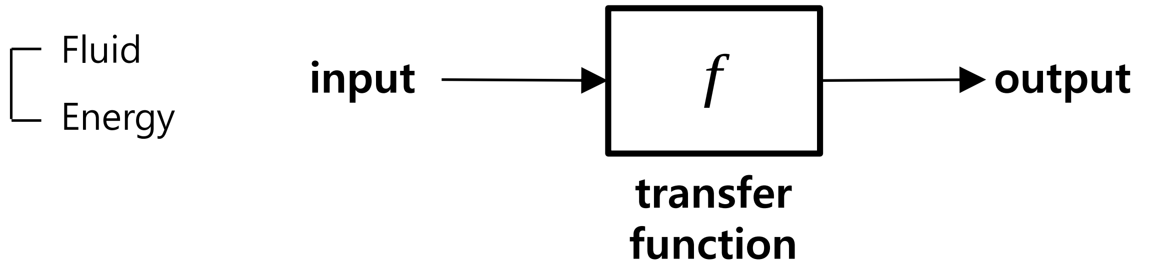
- steady state
- dynamic – changes with respect to time

  (transient analysis)

  (dynamic simulation)

6.4 Information-flow diagrams

- A block signifies that an output can be calculated when the input is known



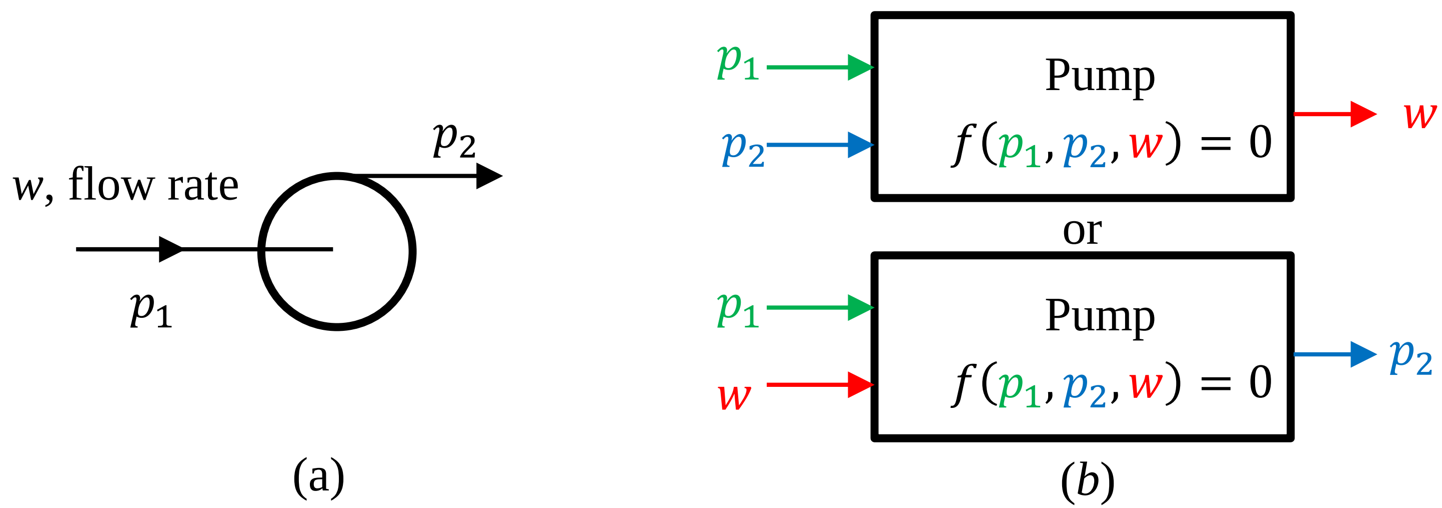


Fig. (a) Centrifugal pump in fluid-flow diagram (b) possible information-flow blocks representing pumps.

Chapter 6. SYSTEM SIMULATION

6.4 Information-flow diagrams

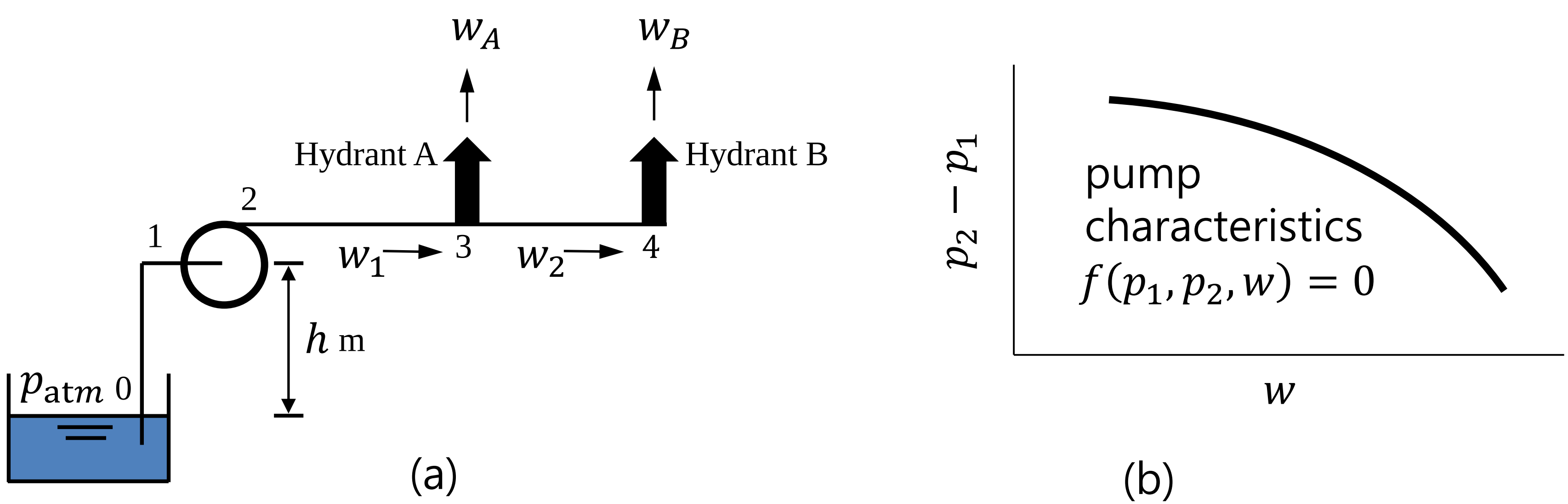


Fig. (a) Fire-water system (b) pump characteristics.

$$w_A = C_A\sqrt{p_3 - p_{\text{atm}}}$$

$$w_B = C_B\sqrt{p_4 - p_{\text{atm}}}$$

Section 0-1 : $p_{\text{atm}} - p_1 = C_1 w_1^2 + \rho g h$

Section 1-2 : $p_2 - p_3 = C_2 w_1^2$

Section 2-3 : $p_3 - p_4 = C_3 w_2^2$

$$f_1(w_A, p_3) = 0$$

$$f_2(w_B, p_4) = 0$$

$$f_3(w_1, p_1) = 0$$

$$f_4(w_1, p_2, p_3) = 0$$

$$f_5(w_2, p_3, p_4) = 0$$

Chapter 6. SYSTEM SIMULATION

6.4 Information-flow diagrams

Pump characteristics : $f_6(w_1, p_1, p_2) = 0$

Mass balance : $f_7(w_1, w_A, w_2) = 0$ or $w_1 = w_A + w_2$

$f_8(w_2, w_B) = 0$ or $w_2 = w_B$

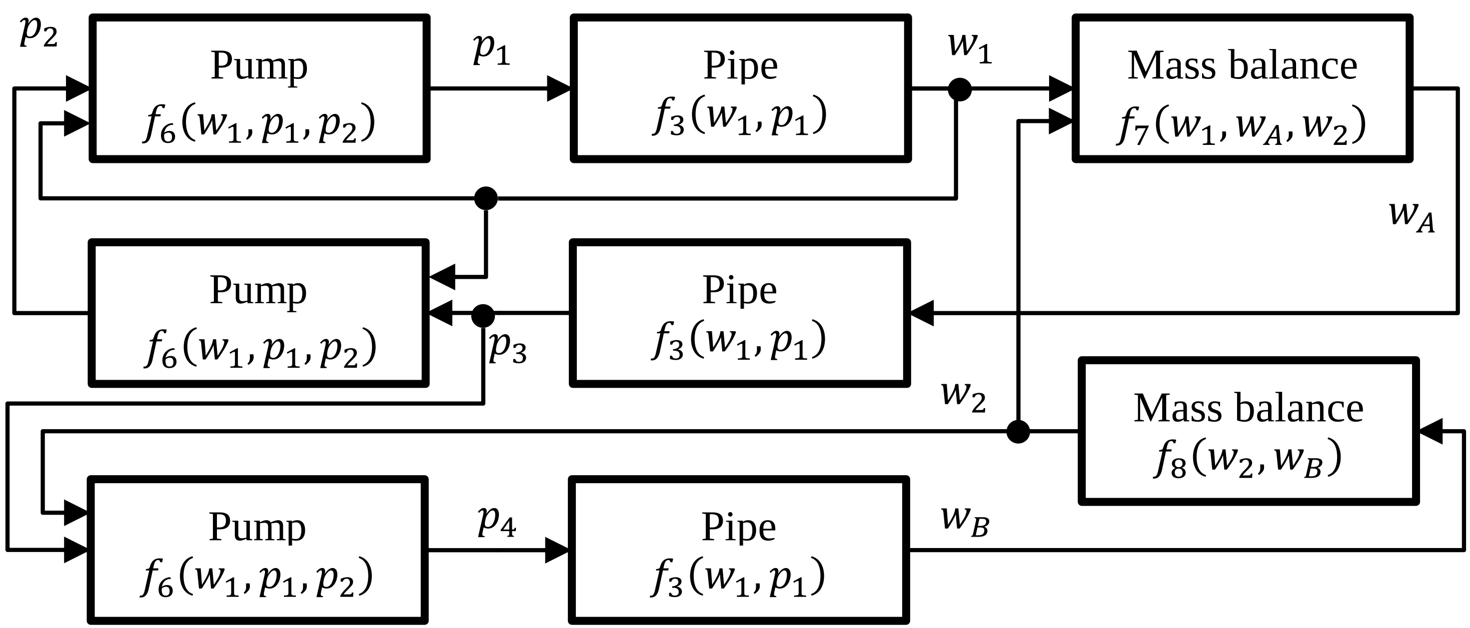
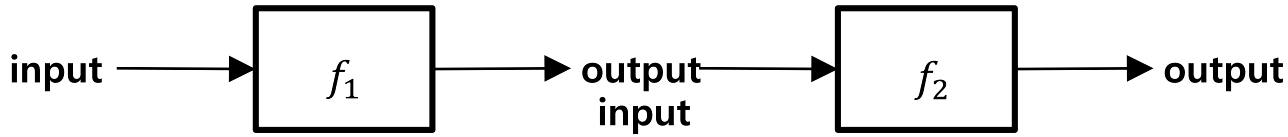


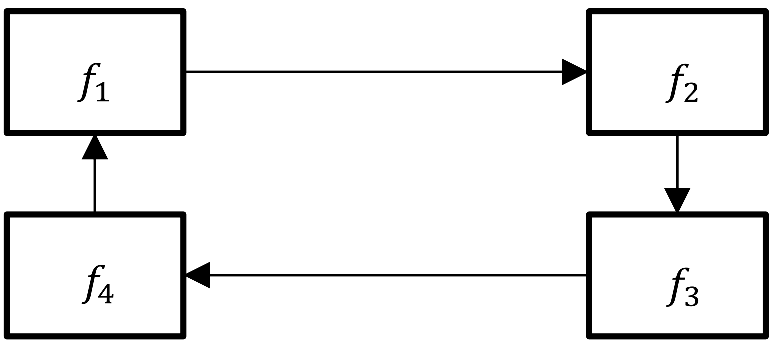
Fig. Information-flow diagram for fire-water system.

Chapter 6. SYSTEM SIMULATION

6.5 Sequential and simultaneous calculations

- Sequential calculation
- Simultaneous calculations





Chapter 6. SYSTEM SIMULATION

6.6 Two methods of simulation:

- Successive substitution
- Newton-Raphson

- To solve a set of simultaneous algebraic equations
- The **successive substitution** is a straight-forward technique and is closely associated with the information-flow diagram of the system.
- The **Newton-Raphson** method is based on a Taylor-series expansion.
- Each method has advantages and disadvantages

6.7 Successive substitution

- **Assume** a value of one or more variables and **begin** the calculation **until** the originally-assumed variables have been **recalculated**.
- The recalculated values are **substituted successively**, and **the calculation loop is repeated** until satisfactory convergence is achieved.

<Example 6.1>

Determine the values $\Delta p, w_1, w_2$ and w by using successive substitution.

<Given>

Pump 1: $\Delta p = 810 - 25w_1 - 3.75w_1^2$ [kPa]

Pump 2: $\Delta p = 900 - 65w_2 - 30w_2^2$ [kPa]

The friction in the pipe = $7.2w^2$

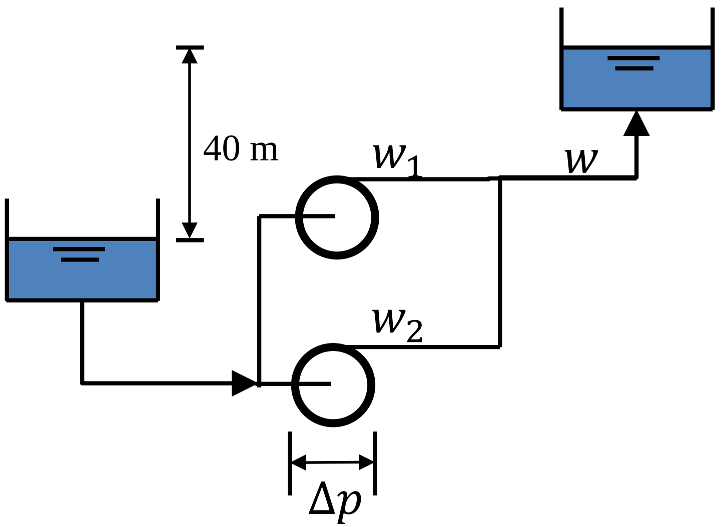


Fig. Water-pumping system

6.7 Successive substitution

<Solution>

✓ Pressure difference due to elevation and friction :

$$\Delta p = 7.2w^2 + \frac{(40\ \text{m})(1000\ \text{kg/m}^3)(9.807\ \text{m/s}^2)}{1000\ \text{Pa/kPa}}$$

✓ Pump 1 : $\Delta p = 810 - 25w_1 - 3.75w_1^2$

✓ Pump 2 : $\Delta p = 900 - 65w_2 - 30w_2^2$

✓ Mass balance : $w = w_1 + w_2$

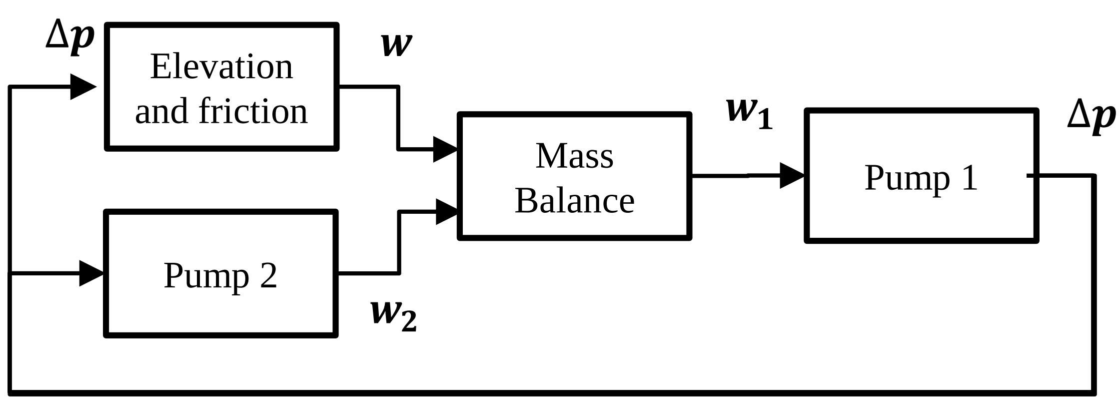


Fig. Information-flow diagram

Chapter 6. SYSTEM SIMULATION

6.7 Successive substitution

<Solution>

✓ Iteration with initial <u>assumption</u> : $w_1 = 4.2$

$$\Delta p = f_1(w_1)$$

$$w_2 = f_2(\Delta p)$$

$$w = f_3(\Delta p)$$

$$w_1 = f_2(w, w_2)$$

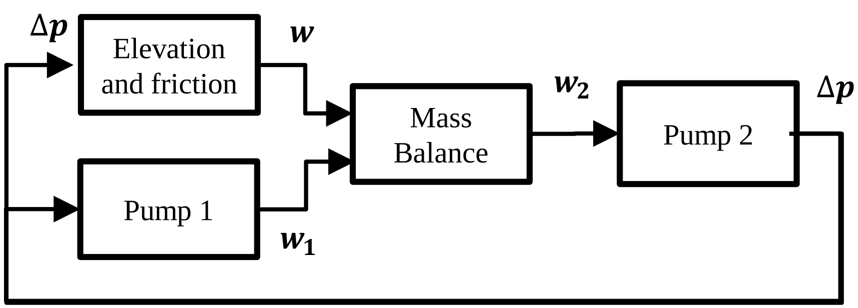
Iteration	Δp	w_2	w	w_1
1	638.85	2.060	5.852	3.792
2	661.26	1.939	6.112	4.174
3	640.34	2.052	5.870	3.818
4	659.90	1.946	6.097	4.151
⋮	⋮	⋮	⋮	⋮
47	649.98	2.000	5.983	3.983
48	650.96	1.995	5.994	3.999
49	650.04	2.000	5.983	3.984
50	650.90	1.995	5.993	3.998

Chapter 6. SYSTEM SIMULATION

6.7 Pitfalls in successive substitution

Diagram 2

✓ initial assumption : $w_2 = 2.0$

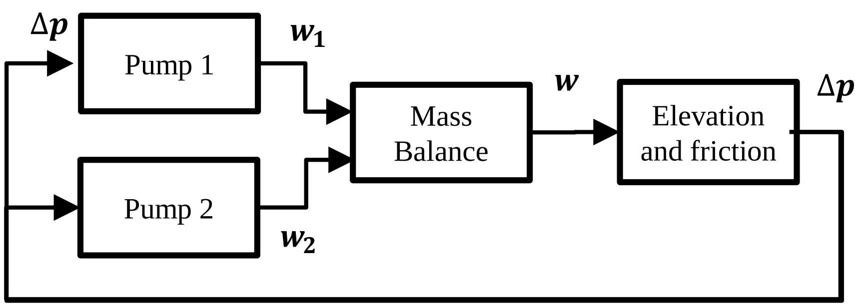


Iteration	w_1 [kg/s]	w_2 [kg/s]	w [kg/s]	Δp [kPa]
1	4.000	**2.000**	5.983	650.00
2	3.942	1.983	6.019	653.16
3	4.258	2.077	5.812	635.53
4	2.443	1.554	6.814	726.54
5	11.353	4.371	**divergence**	42.87

✓ Divergence occurs

Diagram 3

✓ initial assumption : $w = 6.0$



Iteration	w_1 [kg/s]	w_2 [kg/s]	w [kg/s]	Δp [kPa]
1	3.973	1.992	**6.000**	651.48
2	4.028	2.008	5.965	648.47
3	3.916	1.975	6.036	654.61
⋮	⋮	⋮	⋮	⋮
9	**divergence**		8.811	951.23

✓ Divergence occurs

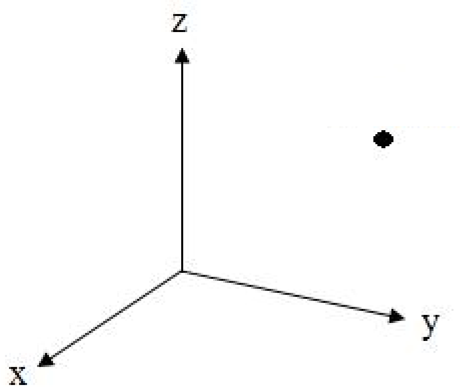
➡ Check the flow diagram in advance (Ch.14)

6.9 Taylor-series expansion

- The Newton-Raphson method is based on a Taylor-series expansion.

$$z = z(x, y)$$

Near the point $(a,\ b,\ z(a,b))$



$$z = c_0 + c_1(x-a) + c_2(y-b)$$
$$+c_3(x-a)^2 + c_4(x-a)(y-b) + c_5(x-a)^2 + \cdots$$

$$c_0 = z(a,b) \qquad c_3 = \frac{1}{2}\frac{\partial^2 z(a,b)}{\partial x^2}$$

$$c_1 = \frac{\partial z(a,b)}{\partial x} \qquad c_4 = \frac{\partial^2 z(a,b)}{\partial x \partial y}$$

$$c_2 = \frac{\partial z(a,b)}{\partial y} \qquad c_5 = \frac{1}{2}\frac{\partial^2 z(a,b)}{\partial y^2}$$

Chapter 6. SYSTEM SIMULATION

6.9 Taylor-series expansion

$$y = y(x)$$

$$y = d_0 + d_1(x-a) + d_2(x-a)^2 + \cdots$$

$$d_0 = y(a) \quad d_1 = \frac{dy(a)}{dx} \quad d_2 = \frac{1}{2}\frac{d^2 y(a)}{dx^2}$$

$$y = y(x_1, x_2, \cdots, x_n)$$

$$y = y(a_1, a_2, \cdots, a_n) + \sum_{j=1}^{n} \frac{\partial y(a_1, a_2, \cdots, a_n)}{\partial x_j}(x_j - a_j)$$

$$+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 y(a_1, a_2, \cdots, a_n)}{\partial x_i \partial x_j}(x_j - a_i)(x_j - a_j)$$

$$+ \cdots$$

Chapter 6. SYSTEM SIMULATION

6.9 Taylor-series expansion

<Example 6.2> Express $z = \ln(x^2 / y)$ as a Taylor-series expansion at (x=2,y=1)

<Solution>

$$z = \ln\frac{x^2}{y} = c_0 + c_1(x-2) + c_2(y-1) + c_3(x-2)^2 + c_4(x-2)(y-1) + c_5(y-1)^2 + \cdots$$

$$c_0 = \ln\frac{2^2}{1} = 1.39$$

$$c_1 = \frac{\partial z(2,1)}{\partial x} = \frac{2x/y}{x^2/y} = 1$$

$$c_2 = \frac{\partial z(2,1)}{\partial y} = -\frac{x^2/y^2}{x^2/y} = -1$$

$$c_3 = \frac{1}{2}\frac{\partial^2 z(2,1)}{\partial x^2} = \frac{1}{2}\left(-\frac{2}{x^2}\right) = -\frac{1}{4}$$

$$c_4 = \frac{\partial^2 z(2,1)}{\partial x \partial y} = 0$$

$$c_5 = \frac{1}{2}\frac{\partial^2 z(2,1)}{\partial y^2} = \frac{1}{2}\frac{1}{y^2} = \frac{1}{2}$$

$$\therefore \quad z = 1.39 + (x-2) - (y-1) - \left(\frac{1}{4}\right)(x-2)^2 + \left(\frac{1}{2}\right)(y-1)^2 + \cdots$$

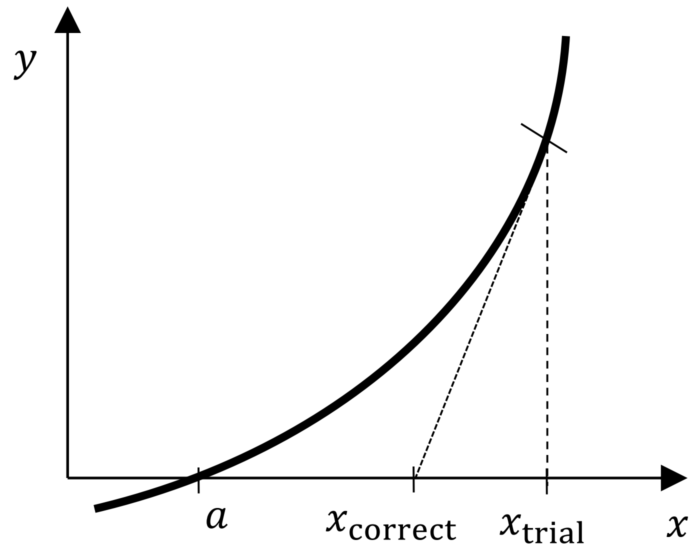
Chapter 6. SYSTEM SIMULATION

6.10 Newton-Raphson with one equation and one unknown

- In the Taylor-series expansion when x is close to a, the higher order terms become negligible.

$$y = y(a) + \frac{dy(a)}{dx}(x-a) + \left[\frac{1}{2}\frac{d^2y(a)}{dx^2}\right](x-a)^2 + \cdots$$

$$\rightarrow \quad y \approx y(a) + \frac{dy(a)}{dx}(x-a) = y(a) + y'(a)(x-a) \qquad (\because x \approx a)$$



$$y(x_t) = y(x_c) + \frac{y(x_t) - y(x_c)}{x_t - x_c}(x_t - x_c)$$

$$x_c = x_t - \frac{y(x_t)}{y'(x_t)}$$

$$x_{new} = x_{old} - (x_{trial} - x_{correct})$$

Chapter 6. SYSTEM SIMULATION

6.10 Newton-Raphson with one equation and one unknown

ex) $y(x) = x + 2 - e^{x}$ $\qquad\qquad y(x_c) = 0$

$x_t = 2 \longrightarrow y(x_t) = x_t + 2 - e^{x_t} = -3.39$

$$y \approx y(x_c) + y'(x_c)(x - x_c) \longrightarrow x_c = x_t - \frac{y(x_t)}{y'(x_t)} = 2 - \frac{-3.39}{1 - e^2} = 1.469 \longrightarrow x_{t,new}$$

Iteration	x_t	y(x)	y′(x)	x_c
1	2.000	-3.389	-6.389	1.470
2	1.470	-0.878	-3.347	1.207
3	1.207	-0.137	-2.345	1.149
4	1.149	-0.006	-2.154	1.146
5	**1.146**	0.000	-2.146	

6.11 Newton-Raphson with multiple equations and unknowns

$$f_1(x_1, x_2, x_3) = 0$$
$$f_2(x_1, x_2, x_3) = 0$$
$$f_3(x_1, x_2, x_3) = 0$$

trial value : $x_{1t},\ x_{2t},\ x_{3t}$

$$f_1(x_{1t}, x_{2t}, x_{3t}) = f_1(x_{1c}, x_{2c}, x_{3c}) + \frac{\partial f_1(x_{1t}, x_{2t}, x_{3t})}{\partial x_1}(x_{1t} - x_{1c})$$

$$+ \frac{\partial f_1(x_{1t}, x_{2t}, x_{3t})}{\partial x_2}(x_{2t} - x_{2c})$$

$$f_2 = \cdots$$

$$+ \frac{\partial f_1(x_{1t}, x_{2t}, x_{3t})}{\partial x_3}(x_{3t} - x_{3c})$$

$$f_3 = \cdots$$

$$+ \cdots$$

Chapter 6. SYSTEM SIMULATION

6.11 Newton-Raphson with multiple equations and unknowns

$$\begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} & \dfrac{\partial f_1}{\partial x_3} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} & \dfrac{\partial f_2}{\partial x_3} \\ \dfrac{\partial f_3}{\partial x_1} & \dfrac{\partial f_3}{\partial x_2} & \dfrac{\partial f_3}{\partial x_3} \end{bmatrix} \begin{bmatrix} x_{1t} - x_{1c} \\ x_{2t} - x_{2c} \\ x_{3t} - x_{3c} \end{bmatrix} = \begin{bmatrix} f_1(x_{1t}, x_{2t}, x_{3t}) \\ f_2(x_{1t}, x_{2t}, x_{3t}) \\ f_3(x_{1t}, x_{2t}, x_{3t}) \end{bmatrix}$$

$$\rightarrow \quad x_{i,new} = x_{i,old} - (x_{i,t} - x_{i,c})$$

Chapter 6. SYSTEM SIMULATION

6.11 Newton-Raphson with multiple equations and unknowns

<Example 6.3> Solve Example 6.1 by Newton-Raphson method

<Solution>

- ✓ requirement : $f_1 = \Delta p - 7.2w^2 - 392.28 = 0$

$$f_2 = \Delta p - 810 + 25w_1 + 3.75{w_1}^2 = 0$$

$$f_3 = \Delta p - 900 + 65w_2 + 30{w_2}^2 = 0$$

$$f_4 = w - w_1 + w_2 = 0$$

- ✓ trial value : $\Delta p = 750,\ \ w_1 = 3,\ \ w_2 = 1.5,\ \ w = 5$

$$\rightarrow\ \ f_1 = 177.7,\ \ f_2 = 48.75,\ \ f_3 = 15.0,\ \ f_4 = 0.50$$

	$\partial/\partial\Delta p$	$\partial/\partial w_1$	$\partial/\partial w_2$	$\partial/\partial w$
$\partial f_1/\partial$	1	0	0	$-14.4w$
$\partial f_2/\partial$	1	$25+7.5w_1$	0	0
$\partial f_3/\partial$	1	0	$65+60w_2$	0
$\partial f_4/\partial$	0	-1	-1	1

Chapter 6. SYSTEM SIMULATION

6.11 Newton-Raphson with multiple equations and unknowns

<Solution>

- ✓ $$\begin{bmatrix} 1.0 & 0.0 & 0.0 & -72.0 \\ 1.0 & 47.5 & 0.0 & 0.0 \\ 1.0 & 0.0 & 155.0 & 0.0 \\ 0.0 & -1.0 & -1.0 & 1.0 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix} = \begin{bmatrix} 177.7 \\ 48.75 \\ 15.0 \\ 0.50 \end{bmatrix} \qquad \Delta x_i = x_{i,t} - x_{i,c}$$
- ✓ $\Delta x_1 = 98.84,\ \Delta x_2 = -1.055,\ \Delta x_3 = -0.541\ \Delta x_4 = -1.096$
- ✓ corrected variable : $\Delta p = 750 - 98.84 = 651.16\ w_1 = 4.055,\ w_2 = 2.041,\ w = 6.096$

Iteration	w_1 [kg/s]	w_2 [kg/s]	w [kg/s]	Δp [kPa]	f_1	f_2	f_3	f_4
1	3.000	1.500	5.000	750.00	177.720	48.750	15.000	0.500
2	4.055	2.041	6.096	651.16	-8.641	4.171	8.778	0.000
3	3.992	1.998	5.989	650.48	-0.081	0.015	0.056	0.000
4	3.991	1.997	5.988	650.49	0.000	0.000	0.000	0.000

$$\therefore\ \Delta p = 650.49,\ w_1 = 3.991,\ w_2 = 1.997,\ w = 5.988$$

Chapter 6. SYSTEM SIMULATION

6.13 Overview of system simulation

- The system simulation operate to achieve **improved design** or to explore prospective **modifications**
- Choosing the combinations of dependent equation is important. But in large system is may not be simple to choose it.
- Successive substitution is a straight-forward technique and is usually easy to program. Its disadvantages are that sometimes the sequence may either converge very slowly or diverge.
- The Newton-Raphson technique is based on a Taylor-series expansion. It is powerful, but it is able to diverge on specific equations