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수 치 선 박 유 체 역 학

- 보 텍 스 방 법 -

COMPUTATIONAL MARINE HYDRODYNAMICS

-VORTEX METHODS-

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INTRODUCTION

0.1 Decomposition of Velocity Fields

Vortical flows are observed and conceived in nature. A vortex motion is the rotation of fluid elements. The rotational motion can be characterized by the vorticity $\underline{\omega} = \nabla \times \underline{q}$ where $\underline{q}$ is the fluid velocity. The vortical flow is said to be one of fluid region with relatively high vorticity.

Let us consider two partial diffential equations for $\underline{q}$:

$$\nabla \cdot \underline{q} = \theta \tag{1}$$

$$\nabla \times \underline{q} = \underline{\omega} \tag{2}$$

Our problem is to find the velocity field when the local rate of expansion (or compression) θ and the local vorticity (shearing prosess) $\underline{\omega}$ are specified throughout the fluid region with appropriate boundary conditions (and/or initial conditions).

Ignoring actual application of some boundary conditons, we consider 3 sets of differential equations:

$$\left.\begin{aligned} \nabla \cdot \underline{u}_\phi &= 0 \\ \nabla \times \underline{u}_\phi &= 0 \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} \nabla \cdot \underline{u}_\omega &= 0 \\ \nabla \times \underline{u}_\omega &= \underline{\omega} \end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned} \nabla \cdot \underline{u}_\theta &= \theta \\ \nabla \times \underline{u}_\theta &= 0 \end{aligned}\right\} \tag{5}$$

Then we may write the solution as a linear superposition of their individual solutions:

$$\boxed{\underline{q} = \underline{u}_\phi + \underline{u}_\omega + \underline{u}_\theta} \tag{6}$$

where $\underline{u}_\phi$ is a solenoidal and irrotational component of velocity field, $\underline{u}_\omega$ its rotational component, and $\underline{u}_\theta$ its non-zero divegence component. Note that the last one vanishes by the continuity equation (the principle of mass conservation) in the case of incompressible fluids. [1] The velocity field for an incompressible fluid has the form:

$$\boxed{\underline{q} = \underline{u}_\omega + \underline{u}_\phi} \tag{7}$$

It is well known in vector analysis that any vector function may be written as the sum of two vectors of the Helmholtz decomposition form.

$$\boxed{\underline{q} = \nabla \times \underline{A} + \nabla\phi,} \tag{8}$$

where $\underline{A}$ is '**vector potential (stream function)**' and ϕ is '**scalar potential (velocity potential)**'. [2]

[1] In the present course work, we consider only incompressible fluids unless stated otherwise.

[2] Even in the case of compressible fluids, we have the Helmholtz decomposition form; The non-zero divergence component $\underline{u}_\theta$ can be merged into the scalar potential component $\nabla\phi$ since $\underline{u}_\theta$ is irrotational such that $\underline{u}_\theta = \nabla\phi_\theta$ in which ϕ_θ becomes a solution of the Poisson-type equation.

0.2 Outline of Course Work

Our workscope is to construct the solution by numerical implementation based on the vorticity-velocity formulation. The fundamentals in the vorticity-velocity formulation are presented in Chapter 2 and Chapter 6.

Since the physical interpretation of the vorticity dynamics by Lighthill (1963) and Batchelor (1967), the vorticity-velocity formulation is one of candidates for solving Navier-Stokes (N.-S.) equations. The vorticity-velocity formulation is mathematically natural. The inertia force term in the N.-S. equations can be expressed as a Helmholtz decomposition form for which vorticity and pressure become a pair of potentials (Wu & Wu 1993).

The present course work would be focused on the vorticity-velocity formulation for the solution of unsteady incompressible Navier-Stokes equations, with two different numerical methods in a time domain analysis:

(1) Inviscid flow analysis
The panel method that was well established in the potential flow analysis is explained extensively in Chapter 3 through Chapter 5, and Appendix A and Appendix B. For preliminary studies, we will cover a background about mathematical and fluid basis in Chapter 1 and Chapter 2.

(2) Viscous flow analysis
The overall basic formulation and some results for simple bodies are presented in Chapter 6 through Chapter 8.

(a) Eulerian finite volume method
An integral approach is used, in conjunction with a finite volume scheme for solving the vorticity transport equation. The integral approach reflects the global coupling when imposed the boundary condition for vorticity at a solid surface. Mathematical identity for a vector or scalar field is used.

(b) Lagrangian vortex particle method
The main difference would be the discrete (particle) representation of the vorticity field. The main feature in the numerical scheme is

of a combination of the particle method and the boundary integral method (panel method). We also deal with the vortex-in-cell method as a hybrid method.