

교 과 목 414.519 강의록

수 치 선 박 유 체 역 학

- 보 텍 스 방 법 -

COMPUTATIONAL MARINE HYDRODYNAMICS

-VORTEX METHODS-

2019년 6월 27일

Suh, Jung - Chun

서 정 천

Seoul National Univ., Dept. NAOE

서울대학교 공과대학 조선해양공학과

Contents

0. INTRODUCTION	1
0.1 Decomposition of Velocity Fields	1
0.2 Outline of Course Work	3
1. VECTOR ANALYSIS	5
1.1 Introduction	6
1.1.1 Definition of domain	7
1.1.2 Fundamental function analysis	9
1.2 Vector Calculus	11
1.2.1 Definition of vector quantity	11
1.2.2 Symbol of vectors	14
1.2.3 Basic unit tensors	14
1.2.3.1 Kronecker delta tensor	15
1.2.3.2 Permutation tensor	15
1.2.3.3 Multiplication of basic tensors	16
1.2.3.4 Example of permutation tensor	17
1.2.4 Multiplication of vectors	18
1.2.4.1 Scalar product	18
1.2.4.2 Vector product	18
1.2.4.3 Scalar triple product	18
1.2.4.4 Vector triple product	19

1.2.5	Vector derivatives	19
1.2.5.1	Gradient	19
1.2.5.2	Divergence	20
1.2.5.3	Curl	21
1.2.5.4	Laplacian	21
1.2.5.5	Differential operators	22
1.2.5.6	Directed derivative	22
1.2.6	Expansion formulas	23
1.3	Integral Theorems	24
1.3.1	Divergence theorem	24
1.3.2	Stokes theorem	26
1.3.3	Volume integrals of a vector	28
1.3.3.1	Volume integral of first moment	29
1.3.4	Surface integrals of a vector	30
1.3.4.1	Surface integrals of first moment	32
1.4	Curvilinear Coordinates on Lines and Surfaces	33
1.4.1	Intrinsic line frame	33
1.4.1.1	Example: Propeller pitch helix	36
1.4.1.2	Example: Streamline intrinsic frame	37
1.4.2	Curvilinear orthogonal coordinates	38
1.4.2.1	Line element	38
1.4.2.2	Gradient	40
1.4.2.3	Divergence	41
1.4.2.4	Curl	41
1.4.2.5	Laplacian	42
1.4.2.6	Convection term	43

1.5 Tensors of Second Order	44
1.5.1 Dyadic products	44
1.5.2 Gradient of a vector	45
1.6 Transport Theorem	46
1.7 Moving Coordinate Systems	49
1.7.1 Velocity due to rigid body rotation	49
1.7.2 Transformations of moving coordinates	50
1.8 Mathematical Identities	52
1.8.1 Green's scalar identity	52
1.8.2 Uniqueness of scalar identity	55
1.8.3 Type of boundary conditions	56
1.8.4 Vector identity	58
1.8.5 Integral expression of Helmholtz decomposition	62
1.8.6 Green functions	64
1.8.7 Uniqueness of vector identity	65
1.8.8 Classification of vector fields	66
1.9 Improper Integrals	67
1.9.1 Examples	67
1.9.2 Principal value integrals	68
2. BASIS OF FLUID FLOWS	71
2.1 Introduction	72
2.1.1 Basic definitions	72
2.1.2 Assumptions and axioms	73
2.1.3 Description of fluid motion	75
2.1.3.1 Lagrangian description	75
2.1.3.2 Eulerian description	75

2.1.4 Particle tracing lines	76
2.1.4.1 Example of particle tracing lines	78
2.2 Kinematics	81
2.2.1 Continuity	81
2.2.2 Vorticity, circulation, and velocity potential	83
2.2.2.1 Vorticity	83
2.2.2.2 Vortex line and vortex tube	84
2.2.2.3 Circulation and vorticity flux	84
2.2.2.4 Vortex strength	85
2.2.2.5 Velocity potential	86
2.2.3 Helmholtz decomposition of a velocity field	87
2.2.4 Velocity field of a vortex: Biot-Savart integral	89
2.3 Dynamics	93
2.3.1 Forces	93
2.3.1.1 Body forces	93
2.3.1.2 Surface forces	94
2.3.1.3 Stress and stress tensor	94
2.3.2 Example: Stress tensors for low Reynolds number flows	97
2.3.2.1 Velocity field	97
2.3.2.2 Stream function approach	99
2.3.2.3 Stress tensor and drag	100
2.3.3 Surface tension	101
2.3.4 Equations of motion: Navier-Stokes equations	103
2.3.5 Bernoulli equation	104
2.3.6 Kelvin's theorem	107
2.3.6.1 Viscous diffusion	109
2.3.6.2 Cases of inviscid flow	111

2.4 Potential Flows. 111

2.4.1 Laplace equation 111

2.4.2 Kinematic boundary condition 113

 2.4.2.1 Alternative form. 113

2.4.3 Dynamic boundary condition: Free surface condition 114

2.4.4 Examples. 115

 2.4.4.1 Flow past a sphere 115

 2.4.4.2 Flow around a circular cylinder 116

3. SINGULARITY DISTRIBUTION METHODS 119

3.1 General Statements 120

3.1.1 Techniques for solving Laplace equation 120

3.1.2 Preview of singularity methods 121

3.1.3 Boundary integral forms 122

3.1.4 Disturbance flow about a body 124

3.2 Surface Distributions of Singularity 126

3.2.1 Interior flow field 126

3.2.2 Source distributions 128

3.2.3 Vortex distributions 128

3.2.4 Source and vortex distributions 129

3.2.5 Remarks for singularity distributions 129

3.2.6 Doublet distribution and solid angle 130

3.2.7 Equivalence of doublet and vortex distributions 132

3.3 Limiting Form of Expressions. 134

3.3.1 Introduction 134

3.3.2 Schematic implementation 136

3.3.3 Scalar functions 137

 3.3.3.1 Source distribution 138

 3.3.3.2 Doublet distribution. 139

3.3.4	Vector functions	139
3.4	Example : Circular Cylinder in Uniform Flow	140
3.4.1	Point doublet at center	140
3.4.2	Potential distribution	143
3.4.3	Stream function formulation	145
3.4.4	Source distribution	146
3.4.5	Vortex distribution	147
3.5	Direct Formulation for Surface Speed.	148
3.5.1	Boundary condition for interior flow	148
3.5.2	Example: Vortex distribution over a circle	150
3.6	Numerical Error	150
3.6.1	Error measures	150
4.	POTENTIAL BASED METHODS	153
4.1	Introduction.	154
4.2	Discretization of a Body Surface.	156
4.2.1	Evaluation of the integrals for a line element	157
4.3	Trailing Wake Sheet Behind a Lifting Body	159
4.3.1	Boundary condtions	159
4.3.2	Vortex distribution on wake sheet	159
4.3.3	Doublet distribution (potential jump) on wake sheet	161
4.3.4	Shedding vortex at trailing edge	161
4.4	Kutta Condition	162
4.4.1	Steady Kutta condition	162
4.4.2	Unsteady Kutta condition	163

4.5 Analytic Solution for Elliptic Section in Steady Uniformly Sheared Flows.	165
4.5.1 Conformal mapping	165
4.5.2 Mapping coefficients	167
4.5.3 Pressure, lift and moment	168
4.5.4 Summarized results.	170
4.6 Unsteady Lifting Flows for Two-Dimensional Hydrofoils . . .	172
4.6.1 Equations of motion in a moving frame	172
4.6.2 Representation of unsteady motion of a hydrofoil	173
4.6.3 Representation of velocity field in a moving frame	175
4.6.4 Formulation of boundary value problems for the disturbance potential.	175
4.6.5 Bernoulli-like equation in a moving frame	177
4.6.6 Integral equation for disturbance potential	179
4.6.7 Vortex model of shed wake sheet: Typical example.	181
4.6.8 Solution procedures	184
4.6.9 Numerical results: Steady flow cases	190
4.6.10 Numerical results: Unsteady flow cases	192
4.6.10.1 Start-up problems	192
4.6.10.2 Harmonic heave motion	196
4.6.10.3 Concluding remarks on combined motions	197
4.7 Formulation in Three-dimensions.	198
4.7.1 Extension to 3-D wing	198
4.7.2 Velocity components at a panel surface	200
4.7.3 Non-lifting flow about an ellipsoid.	201
4.7.4 Lifting flow about a circular wing	203

5. ANALYTICAL EVALUATION OF BOUNDARY INTEGRALS	205
5.1 Introduction	206
5.2 Transformation of the Surface Integrals to Contour Integrals	207
5.3 Constant Density Distributions over a Planar Polygon	209
5.3.1 Source distribution: Potential	209
5.3.2 Source distribution: Velocity	211
5.3.3 Doublet distribution: Potential	212
5.3.4 Doublet distribution: Velocity	213
5.3.5 Basic integrals	213
5.3.6 Test calculations for constant distributions	219
5.3.7 Extension to linear distributions	221
5.4 Bilinear Source and Doublet Distribution	224
5.4.1 Introduction	224
5.4.2 Transformation of the surface integrals for Stokes' theorem	224
5.4.3 Induced potential due to source distribution	228
5.4.4 Induced velocity due to source distribution	230
5.4.5 Induced potential and velocity due to doublet distribution	231
5.4.6 Closed-forms of the basic integrals	232
6. VORTICITY BASED METHODS	237
6.1 Introduction	238
6.1.1 Various vortical flows	239
6.1.2 Recent developments	241
6.1.2.1 CFD modeling	241
6.1.2.2 Physical interpretation	242
6.1.2.3 Vortex particle method	244
6.1.2.4 Vortex-In-Cell method	245

6.2 Vorticity-Velocity-Pressure Formulation 246

6.2.1 Navier-Stokes equations in Helmholtz decomposition 246

6.2.2 Vorticity transport equation 250

6.2.3 Pressure Poisson equation 251

6.2.4 Kinematic boundary condition 252

6.2.5 Dynamic boundary condition 252

6.2.6 Integral approach of formulation 253

 6.2.6.1 Two-dimensional formulation 255

6.2.7 Stream function approach: VIC method 256

6.2.8 Particle method in solving the vorticity transport equation 258

6.2.9 Hydrodynamic Forces 259

7. FINITE VOLUME METHODS 261

7.1 Introduction 262

7.2 Numerical Implementation 263

7.2.1 Vorticity transport equation 263

 7.2.1.1 Numerical schemes 264

 7.2.1.2 No-slip boundary condition with vorticity flux 265

7.2.2 Biot-Savart integral 267

 7.2.2.1 Evaluation of line integrals 267

 7.2.2.2 Computational enhancement 268

7.2.3 Pressure Poisson equation 270

 7.2.3.1 Formulation 270

 7.2.3.2 Application of panel methods 271

7.2.4 Computational procedure 272

7.3 Lid-driven Cavity Flows 275

7.3.1 Formulation 275

7.3.2 Comparison with analytic solution 277

7.4 Impulsively Started Circular Cylinder	281
7.4.1 General aspects	281
7.4.2 Computational grids	282
7.4.3 Numerical results	282
7.4.3.1 Analytic solution in early time stage	282
7.4.3.2 Time step	283
7.4.3.3 Computational domain	283
7.4.3.4 Reynolds number	287
7.4.3.5 Pressure, velocity and vorticity fields	287
7.5 Oscillating Circular Cylinder Problems	294
7.5.1 Key parameters	294
7.5.2 Flow characteristics	295
7.5.3 Formulation for moving frame fixed to cylinder	301
7.5.4 Numerical simulation	302
7.5.4.1 Case 1: $KC = 7, \beta = 143$ ($Re = 1000$)	303
7.5.4.2 Case 2: $KC = 10, \beta = 20$ ($Re = 200$)	306
7.5.4.3 Case 3: $KC = 16, \beta = 62.5$ ($Re = 1000$)	309
8. VORTEX PARTICLE METHODS	313
8.1 Introduction.	314
8.2 Numerical Implementation.	315
8.2.1 Particle representation of vorticity field	315
8.2.1.1 Two-dimensions	316
8.2.1.2 Three-dimensions	316
8.2.2 Velocity field	317
8.2.2.1 Regularized velocity field	318

8.2.3	Field viscous diffusion: PSE scheme	320
8.2.3.1	Image layer method in two-dimensions	321
8.2.3.2	Image layer method in three-dimensions	323
8.2.4	No-slip condition: Vorticity flux at wall	326
8.2.4.1	Wall viscous diffusion in two-dimensions	327
8.2.4.2	Wall viscous diffusion in three-dimensions	330
8.2.5	Pressure equation	331
8.2.6	Computational procedure	334
8.2.6.1	Redistribution	335
8.2.6.2	Force calculation	339
8.3	Some Comparative Results	340
8.3.1	Impulsively started cylinder	340
8.3.2	Impulsively started foil with varying angles of attack	350
8.3.2.1	Angle of attack : 90 deg.	356
8.4	Vortex-In-Cell Methods	359
8.4.1	Introduction	359
8.4.2	Rotational velocity component: FFT scheme based on regular grid	360
8.4.3	Potential velocity component: Panel method with linearly varying singularity distribution	363
8.4.4	Stretching term in 3-D	367
8.4.5	Stability issue	367
8.4.5.1	Stability criterion	368
8.4.6	Outline of the VIC scheme	369
8.4.7	Pressure calculation by panel method with a linearly varying singularity	372
8.5	Numerical Results by VIC Methods	374
8.5.1	Two dimensional flows	375
8.5.1.1	Impulsively started circular cylinder	375
8.5.1.2	Impulsively started NACA0012 hydrofoil	382

8.5.2 Three dimensional flows	387
8.5.2.1 Sphere	387
8.5.2.2 Rectangular wing	400
8.5.3 Features of vortex-in-cell method	401
8.6 Concluding Remarks	406
8.6.1 LES in vortex methods	407
8.6.2 Interaction between flow and bubble	408
8.6.2.1 Disturbance by volumetric motion	410
8.6.2.2 Disturbance by translational motion	411
A. NUMERICAL IMPLEMENTATION OF KUTTA CONDITION	415
A.1 Implementation of Kutta Condition in Two-Dimensions.	415
A.1.1 Steady flow cases.	417
A.1.2 Unsteady flow cases	418
A.2 Implementation of Kutta Condition in 3-D Steady Flows	419
B. INTEGRATION FOR SINGULARITY DISTRIBUTIONS	423
B.1 Introduction	424
B.1.1 Related work for closed-form expressions	425
B.1.2 Stokes' theorem	427
B.1.3 Basic vector operations	428
B.2 Induced Potential Due to Source Distribution.	429
B.2.1 Transformation of Eq. (B.18) into line integrals	431
B.3 Induced Velocity Due to Source Distribution	435
B.4 Induced Potential Due to Doublet Distribution.	437
B.5 Induced Velocity Due to Doublet Distribution	437

C. CODE <i>PRpan</i> FOR PANEL METHOD	441
C.1 Introduction	441
C.2 Program Lists of Subroutine <i>PRpan</i>	442
D. EVALUATION OF THE BIOT-SAVART INTEGRAL	459
D.1 Introduction	460
D.1.1 Integral representation	461
D.2 Biot-Savart Integral in 2-D	462
D.2.1 Transformation of integral	462
D.2.2 Analytic form of integrals	463
D.3 Biot-Savart Integral in 3-D	465
D.3.1 Transformation of integral	465
D.3.2 Specific line integrals	468
References	471
General References	473

List of Figures

1.1	Types of surfaces.	8
1.2	Two Cartesian coordinate systems rotated with respect to one another.	12
1.3	Intrinsic 3 orthonormal basis vectors along a curve in a local curvilinear coordinate system.	34
1.4	Cylindrical and spherical coordinate systems.	39
1.5	Rotation of a rigid body.	49
1.6	Moving coordinate system.	51
1.7	Two-dimensional drawing of a simply connected region for deriving the scalar identity.	53
1.8	Small sphere region containing a singular point.	55
1.9	Classification of vector fields.	66
2.1	Behavior of a solid and a fluid, under the action of constant shear force.	73
2.2	Example of various flow lines.	81
2.3	Integration region for Poisson's solution of vector fields.	89
2.4	Stress vector at surface.	94
2.5	Pressure diagram of a fluid.	96
2.6	Deformation of fluid element in 2-D flows.	96
2.7	Notation for a spherical bubble in uniform flow.	97
2.8	Force diagram for a spherical bubble with surface tension.	102
2.9	Intrinsic frame on the side surface of a vortex tube.	110

3.1	Notation for unbounded flow fields.	126
3.2	Relationship between a constant doublet distribution and the solid angle of the distributed surface.	131
3.3	Equivalence of doublet and vortex distributions.	133
3.4	Schematic diagram of integration region for singular integrals.	135
3.5	Coordinate definition for region surrounding singular point.	136
3.6	Planar approximation of surface surrounding singular point.	137
3.7	Notation for flow about a circular cylinder in a uniform stream.	141
3.8	Streamlines around/inside a circle (about a doublet) in a uniform stream.	142
4.1	Notation for evaluation of induction integrals on a line element.	157
4.2	Flow past a foil without circulation, and with a properly selected circulation so that a stagnation point is at T.E.	163
4.3	Foil configuration for steady uniformly sheared onset flow.	166
4.4	The coordinate systems and a combined unsteady flow situation.	174
4.5	A doublet straight-line element attached to the trailing edge and a series of concentrated vortices.	182
4.6	Force diagram of 2-D foil.	188
4.7	Comparison of numerical and analytical disturbance potentials on the surface of an ellipse in steady uniform and shear flow.	190
4.8	Comparison of numerical and analytical total surface speeds on the surface of an ellipse in steady uniform and shear flow.	191
4.9	Comparison of numerical and analytical lift and moment coefficients versus angle of attack for a Moriya foil in steady shear flow.	193
4.10	Growth of lift for sudden start-up of NACA0006, NACA0012 and NACA 0018 foils in uniform onset flow.	194
4.11	Calculated location of vortex cores for start-up of an NACA0012 foil.	195
4.12	Calculated location of vortex cores for harmonic heave motion of an NACA0015 foil in uniform flow.	196
4.13	Magnitude of fluctuating lift with various reduced frequencies for heave motion of NACA0006, NACA0012, NACA0018 and NACA0024 foils in uniform flow.	197

7.8	Sensitivity of mesh size on vorticity, vorticity flux and pressure along the driven cavity wall for $Re = 100$.	279
7.9	Time evolution of the velocity along the center lines of the driven cavity for $Re = 100$.	280
7.10	Time evolution of kinetic energy of the driven cavity for $Re = 100$.	280
7.11	Streamline pattern, vorticity contour and pressure contour of the driven cavity for $Re = 100$.	280
7.12	Sensitivity of the time interval on the drag coefficients of the impulsively started circular cylinder at $Re = 60, 3000$ and 9500 .	284
7.13	Sensitivity of the outer radius on the drag coefficients of the impulsively started circular cylinder at $Re = 60, 3000$ and 9500 .	285
7.14	Sensitivity of the mesh size on the drag coefficients of the impulsively started circular cylinder at $Re = 60, 3000$ and 9500 .	286
7.15	Comparison of the computed surface vorticity with the analytical solution of the impulsively started circular cylinder at $Re = 3000$.	287
7.16	Time evolution of the primary separation position of the impulsively started circular cylinder at $Re = 9500$.	288
7.17	Surface pressure distribution of the impulsively started circular cylinder at several instants for $Re = 9500$.	288
7.18	Streamline patterns of the impulsively started circular cylinder for $Re = 9500$.	290
7.19	Vorticity contours of the impulsively started circular cylinder for $Re = 9500$.	291
7.20	Pressure contours of the impulsively started circular cylinder for $Re = 9500$.	292
7.21	Time-averaged vorticity fluxes ($\bar{\sigma}$) of the impulsively started circular cylinder in $t_1 - \Delta t < t < t_1$ and vorticity flux (σ) at $t = t_1$, where $t_1 = 2.5$ for $Re = 9500$.	293
7.22	Single pair regime of flow around a circular cylinder in oscillatory motion for $7 < KC < 15$.	297
7.23	Double-pair regime of flow around a circular cylinder in oscillatory motion for $15 < KC < 24$.	298

7.24	Three-pair regime of flow around a circular cylinder in oscillatory motion for $24 < KC < 32$	298
7.25	Classification of flows around a circular cylinder in oscillatory motion	299
7.26	Transverse vortex street pattern of flow around a circular cylinder in oscillatory motion at $T = 89$ for $KC = 7$, $\beta = 143$	303
7.27	Time history of drag and lift forces of flow around a circular cylinder in oscillatory motion for $KC = 7$, $\beta = 143$	304
7.28	Power spectra of drag and lift forces of flow around a circular cylinder in oscillatory motion for $KC = 7$, $\beta = 143$	305
7.29	Diagonally convected single-pair vortex pattern of flow around a circular cylinder in oscillatory motion at $T = 211.6$ for $KC = 10$, $\beta = 20$	306
7.30	Time history of drag and lift forces of flow around a circular cylinder in oscillatory motion at $T = 211.6$ for $KC = 10$, $\beta = 20$	307
7.31	Power spectra of drag and lift forces of flow around a circular cylinder in oscillatory motion for $KC = 10$, $\beta = 20$	308
7.32	Double-pair vortex convection pattern of flow around a circular cylinder in oscillatory motion at $T = 192.6$ for $KC = 16$, $\beta = 62.5$	309
7.33	Time history of drag and lift forces of flow around a circular cylinder in oscillatory motion for $KC = 16$, $\beta = 62.5$	310
7.34	Power spectra of drag and lift forces of flow around a circular cylinder in oscillatory motion for $KC = 16$, $\beta = 62.5$	311
8.1	Schematic diagram of the vortex particle method in two-dimensions.	315
8.2	Comparison of the image vortex layer of the present method with the image vortex system in Ploumhans & Wickelmans (2000)..	322
8.3	Example of the image vortex layer around an NACA 0012 hydrofoil.	322
8.4	Comparison of particle locations between the vortex particle method and the immersed boundary method in VIC..	325
8.5	Particles with respect to a panel for viscous wall diffusion..	328
8.6	Diffusion of vorticity from body boundary.	331
8.7	Numerical procedure of the vortex particle method.	334
8.8	Redistribution scheme for a general boundary in two-dimensions.	336

8.9	Two-dimensional redistribution scheme for a particle near a boundary.	338
8.10	Comparison of the accumulated spurious slip velocity distribution on the cylinder surface.	341
8.11	Comparison of I_x for the impulsively started cylinder problem ($0 < T < 0.25$).	342
8.12	Comparison of I_x for the impulsively started cylinder problem ($0 < T < 4$).	343
8.13	Comparison of C_D for the impulsively started cylinder problem ($0 < T < 0.25$).	344
8.14	Comparison of C_D for the impulsively started cylinder problem ($0 < T < 4$).	344
8.15	Comparison of the surface vorticity for the impulsively started cylinder problem for $Re = 550$ at $T = 0.5$ and $T = 4.0$	345
8.16	Comparison of the streamline patterns for the impulsively started cylinder problem for $Re = 550$ at $T = 1$, $T = 2$, $T = 3$ and $T = 4$	347
8.17	Comparison of the vorticity contours for the impulsively started cylinder problem for $Re = 550$ at $T = 1$, $T = 2$, $T = 3$ and $T = 4$	348
8.18	Comparison of the pressure contours for the impulsively started cylinder problem for $Re = 550$ at $T = 1.0$, $T = 2.0$, $T = 3.0$ and $T = 4.0$	349
8.19	Comparison of C_p for the impulsively started cylinder problem for $Re = 550$ at $T = 1$ and $T = 4.0$	350
8.20	Streamline patterns, vorticity contours and pressure contours for the impulsively started NACA0021 at $Re = 550$, $\alpha = 5^\circ$ and $T = 4.0$	353
8.21	Streamline patterns, vorticity contours and pressure contours for the impulsively started NACA0021 foil at $Re = 550$, $\alpha = 10^\circ$ and $T = 4.0$	354
8.22	Comparison of drag and lift for the impulsively started NACA0021 foil at $Re = 550$ and $\alpha = 5$	355
8.23	Comparison of drag and lift for the impulsively started NACA0021 foil at $Re = 550$ and $\alpha = 10$	355
8.24	Iso-contours of vorticity around NACA 0012 hydrofoil at $\alpha = 90^\circ$ and $Re = 1200$	356
8.25	Streamlines around NACA 0012 hydrofoil at $\alpha = 90^\circ$ and $Re = 1200$	357
8.26	Comparison of the streamlines around NACA 0012 hydrofoil with the experimental result at $\alpha = 90^\circ$ and $Re = 1200$	358

8.27	Comparison of CPU times for velocity evaluations in 3-D.	359
8.28	Regular immersed grids for FFT.	360
8.29	Two types for enforcement of the no-penetration flow condition in the regular grid system.	364
8.30	Schematic arrangement of a field point k due to a singularity distribution element composed of several triangular panels.	366
8.31	Behavior of the maximum residual slip velocity during the iteration.	368
8.32	Diffusion of vorticity on a regular Cartesian grid in VIC methods.	372
8.33	Schematic arrangement for boundary condition of the pressure head H	373
8.34	Comparison of I_x for an impulsively started circular cylinder at $Re = 550$	377
8.35	Drag coefficient of an impulsively started circular cylinder at early stage of times for $Re = 550$	378
8.36	Drag coefficient of an impulsively started circular cylinder for $Re = 550$	378
8.37	Velocity distribution along wake centerline for an impulsively started circular cylinder for $Re = 550$	379
8.38	Instantaneous streamlines around impulsively started circular cylinder at $Re = 550$	380
8.39	Comparison of streamlines for an impulsively started circular cylinder for $Re = 550$	381
8.40	Comparison of I_x for an impulsively started NACA0012 hydrofoil at zero angle of attack for $Re = 1200$	383
8.41	Vorticity contours for an impulsively started NACA0012 hydrofoil for $\alpha = 30^\circ$ and $Re = 1200$	384
8.42	Comparison of streamlines at $T = 1.0$ with the experimental snapshot for an impulsively started NACA 0012 hydrofoil at $\alpha = 30^\circ$ for $Re = 1200$	385
8.43	Comparison of streamlines at $T = 2.0$ with the experimental snapshot for an impulsively started NACA 0012 hydrofoil at $\alpha = 30^\circ$ for $Re = 1200$	386
8.44	Surface panel discretization of a sphere.	387
8.45	Comparison of drag coefficient of a sphere with experiments.	388
8.46	Comparison of drag coefficient of a sphere with the numerical one by Johnson & Patel (1999).	389

8.47	Streamlines about an impulsively started sphere for $Re = 100$.	390
8.48	Vorticity contours for an impulsively started sphere for $Re = 100$.	391
8.49	Pressure coefficient contours for an impulsively started sphere for $Re = 100$.	392
8.50	Comparison of streamlines about a sphere for $Re = 100$ with the numerical ones by Johnson & Patel (1999).	393
8.51	Comparison of wake pattern for a sphere with the numerical one by Johnson & Patel (1999).	394
8.52	Comparison of pressure contours for a sphere for $Re = 100$ with the numerical one by Johnson & Patel (1999).	395
8.53	Comparison of vorticity contours for a sphere for $Re = 100$ with the numerical one by Johnson & Patel (1999).	396
8.54	Comparison of streamlines for a sphere for $Re = 100$ with the experimental ones by Taneda (1956).	397
8.55	Comparison of streamlines between two Reynolds numbers.	398
8.56	Comparison of pressure coefficient contours between two Reynolds numbers.	399
8.57	Comparison of vorticity contours between two Reynolds numbers.	400
8.58	Surface panel discretization of a rectangular wing.	402
8.59	Comparison of streamtraces and pressure coefficient for a rectangular wing for $Re = 100$ with the results obtained by FLUENT.	403
8.60	Streamwise vorticity contours at downstream locations $x = 0.67, 0.8$, and 1.0 at $t = 1.92$.	404
8.61	Location of the tip vortex center along downstream.	405
8.62	Turbulent flow past a cylinder by VIC method.	408
8.63	Schematic diagram of interaction between the motion of a single bubble and the ambient viscous flow.	410
8.64	Local coordinates for the hydrodynamic impulse of the bubble	412
8.65	Schematic of the vorticity generation.	413
8.66	Bubble behavior for two different cavitation numbers.	414

D.1 Definition of a quadrilateral element.	463
D.2 Definition sketch of the local coordinate system (x', y')	464

List of Tables

5.1 Comparison of the Basic Integrals by Analytic and Numerical Calculation at Point $P5(0.0, 0.0, +0.00001)$	220
5.2 Comparison of Potentials and Velocities by Analytic and Numerical Calculation at Point $P2(0.5, 6.0, +0.00001)$	220
6.1 Comparison of CPU times between vortex-in-cell method and finite difference method for 2-D driven cavity flow for various Reynolds numbers.	246
7.1 Regimes of flow around a circular cylinder in oscillatory motion at $Re = 10^3$	295
7.2 Principal features of the flows classified in eight regimes of flow around a circular cylinder in oscillatory motion with KC and β	300
7.3 Fundamental lift frequencies of the observed flow around a circular cylinder in oscillatory motion.	301
8.1 Parameters used in the numerical simulation of the flow around an impulsively started circular cylinder.	341
8.2 Parameters used in the numerical simulation of the flow around an impulsively started NACA 0021 hydrofoil.	351
8.3 Parameters used in the numerical simulation of the flow around an impulsively started circular cylinder.	376
8.4 Parameters used in the numerical simulation of the flow around an impulsively started NACA 0012 hydrofoil.	382



INTRODUCTION

0.1 Decomposition of Velocity Fields

Vortical flows are observed and conceived in nature. A vortex motion is the rotation of fluid elements. The rotational motion can be characterized by the vorticity $\underline{\omega} = \nabla \times \underline{q}$ where \underline{q} is the fluid velocity. The vortical flow is said to be one of fluid region with relatively high vorticity.

Let us consider two partial differential equations for \underline{q} :

$$\nabla \cdot \underline{q} = \theta \quad (1)$$

$$\nabla \times \underline{q} = \underline{\omega} \quad (2)$$

Our problem is to find the velocity field when the local rate of expansion (or compression) θ and the local vorticity (shearing process) $\underline{\omega}$ are specified throughout the fluid region with appropriate boundary conditions (and/or initial conditions).

Ignoring actual application of some boundary conditons, we consider 3 sets of differential equations:

$$\left. \begin{aligned} \nabla \cdot \underline{u}_\phi &= 0 \\ \nabla \times \underline{u}_\phi &= 0 \end{aligned} \right\} \quad (3)$$

$$\left. \begin{aligned} \nabla \cdot \underline{u}_\omega &= 0 \\ \nabla \times \underline{u}_\omega &= \underline{\omega} \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} \nabla \cdot \underline{u}_\theta &= \theta \\ \nabla \times \underline{u}_\theta &= 0 \end{aligned} \right\} \quad (5)$$

Then we may write the solution as a linear superposition of their individual solutions:

$$\boxed{\underline{q} = \underline{u}_\phi + \underline{u}_\omega + \underline{u}_\theta} \quad (6)$$

where \underline{u}_ϕ is a solenoidal and irrotational component of velocity field, \underline{u}_ω its rotational component, and \underline{u}_θ its non-zero divergence component. Note that the last one vanishes by the continuity equation (the principle of mass conservation) in the case of incompressible fluids.¹ The velocity field for an incompressible fluid has the form:

$$\boxed{\underline{q} = \underline{u}_\omega + \underline{u}_\phi} \quad (7)$$

It is well known in vector analysis that any vector function may be written as the sum of two vectors of the Helmholtz decomposition form.

$$\boxed{\underline{q} = \nabla \times \underline{A} + \nabla \phi,} \quad (8)$$

where \underline{A} is ‘**vector potential (stream function)**’ and ϕ is ‘**scalar potential (velocity potential)**’.²

¹In the present course work, we consider only incompressible fluids unless stated otherwise.

²Even in the case of compressible fluids, we have the Helmholtz decomposition form; The non-zero divergence component \underline{u}_θ can be merged into the scalar potential component $\nabla \phi$ since \underline{u}_θ is irrotational such that $\underline{u}_\theta = \nabla \phi_\theta$ in which ϕ_θ becomes a solution of the Poisson-type equation.

0.2 Outline of Course Work

Our workscope is to construct the solution by numerical implementation based on the vorticity-velocity formulation. The fundamentals in the vorticity-velocity formulation are presented in Chapter 2 and Chapter 6.

Since the physical interpretation of the vorticity dynamics by Lighthill (1963) and Batchelor (1967), the vorticity-velocity formulation is one of candidates for solving Navier-Stokes (N.-S.) equations. The vorticity-velocity formulation is mathematically natural. The inertia force term in the N.-S. equations can be expressed as a Helmholtz decomposition form for which vorticity and pressure become a pair of potentials (Wu & Wu 1993).

The present course work would be focused on the vorticity-velocity formulation for the solution of unsteady incompressible Navier-Stokes equations, with two different numerical methods in a time domain analysis:

(1) Inviscid flow analysis

The panel method that was well established in the potential flow analysis is explained extensively in Chapter 3 through Chapter 5, and Appendix A and Appendix B. For preliminary studies, we will cover a background about mathematical and fluid basis in Chapter 1 and Chapter 2.

(2) Viscous flow analysis

The overall basic formulation and some results for simple bodies are presented in Chapter 6 through Chapter 8.

(a) Eulerian finite volume method

An integral approach is used, in conjunction with a finite volume scheme for solving the vorticity transport equation. The integral approach reflects the global coupling when imposed the boundary condition for vorticity at a solid surface. Mathematical identity for a vector or scalar field is used.

(b) Lagrangian vortex particle method

The main difference would be the discrete (particle) representation of the vorticity field. The main feature in the numerical scheme is

of a combination of the particle method and the boundary integral method (panel method). We also deal with the vortex-in-cell method as a hybrid method.