교 과 목 414.519 강의록

수 치 선 박 유 체 역 학

- 보 텍 스 방 법 -

COMPUTATIONAL MARINE HYDRODYNAMICS

-VORTEX METHODS-

2019년 6월 27일

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A

NUMERICAL IMPLEMENTATION OF KUTTA CONDITION

A.1 Implementation of Kutta Condition in Two-Dimensions

The physical features behind the Kutta condition, although the interpretation is not complete, are complex as explained before. The object of the numerical implementation of the Kutta condition here is to determine the jump in the disturbance potential ($\triangle\phi_{\rm v}$) at the T. E. for which acceptable results may be obtained.

Following the feature of the 'Maskell' trailing-edge flow for the velocity at the T. E., we can define the tangential velocities (i.e., shed vorticity) on both the upper and lower surfaces at the T. E. by imposing a stagnation point at either the upper or the lower trailing-edge point. Then the difference of the velocity there is assigned to the shed vorticity.

Accordingly we can evaluate the jump in disturbance potential at the T. E. ($\triangle\phi_{\rm v}|_{TE}$) such that a surface potential distribution near the T. E. satisfies the behavior of the tangential velocities corresponding to the local flow characteristics near the T. E.

In practice, the Kutta condition is implemented first by attaching the wake to the T. E. (or by specifying a point where the vorticity leaves the body surface) and evaluating $\triangle\phi_{\rm v}|_{TE}$ from the values of the disturbance potential of the panels (ϕ_j).

Let us approximate the disturbance potential distributions (ϕ) on the upper and the lower surfaces near the T. E. as a parabolic form of the parameter s:

$$\phi_u\,(s) = a_u\,s^2 + b_u\,s + c_u\,, \tag{A.1}$$

$$\phi_\ell\,(s) = a_\ell\,s^2 + b_\ell\,s + c_\ell\,, \tag{A.2}$$

where the parameter s is arc-length along the body surface contour from the T. E. with positive taken as counterclockwise (see Figure **4.5**) and the subscripts u and ℓ refer to the upper and the lower surface, respectively. The coefficients b_u, b_ℓ, c_u and c_ℓ are to be determined by imposing the Kutta condition, but the coefficients a_u and a_ℓ (which have been kept in the following derivation) are ignored in the final expression (A.10) for $\triangle\phi_{\rm v}|_{TE}$ by assuming their contribution to be higher order.

Similar procedure has been presented by Ingham et al. (1981) [1] for the problems with two regions of different physical features, in which the two analytical

[1]Ingham, D. B., Heggs, P. J. and Manzoor, M. (1981), "The Numerical Solution of Plane Potential Problems by Improved Boundary Integral Equation Methods," *Journal of Computational Physics*, vol. 42, pp. 77–98.

solution forms of the Laplace equation for the two regions in the neighborhood of the discontinuity are introduced and then the appropriate physical matching conditions at the common interface are enforced to determine the coefficients associated with those forms. Another simple application of this procedure has been introduced by Batchelor (1967) to flow problem near a stagnation point. [2]

Then taking the gradient of (A.1) and (A.2) and then including the undisturbed velocity $\underline{q}_\infty (\equiv \underline{q}_o - \underline{q}_F)$ give the total tangential components (positive as counterclockwise) on the upper and the lower surface near the T. E. can be expressed as, respectively,

$$q_{tu}(s) = (\underline{q}_\infty + \nabla\phi_u)\cdot\underline{t}_u = \underline{q}_\infty\cdot\underline{t}_u + 2a_u\,s + b_u\,, \tag{A.3}$$

$$q_{t\ell}(s) = (\underline{q}_\infty + \nabla\phi_\ell)\cdot\underline{t}_\ell = \underline{q}_\infty\cdot\underline{t}_\ell + 2a_\ell\,s + b_\ell\,. \tag{A.4}$$

Then the potential jump at the T. E. from (A.1) and (A.2) can be written as

$$\triangle\phi_{\mathrm{v}}\big|_{TE} = \phi_u\,(0) - \phi_\ell\,(0) = c_u - c_\ell\,. \tag{A.5}$$

This potential jump is expressed in terms of quantities in the panel-method approximation as:

$$\triangle\phi_{\mathrm{v}}\big|_{TE} = c_u - c_\ell = (\phi_1 - a_u\,s_1^2 - b_u\,s_1) - (\phi_N - a_\ell\,s_N^2 - b_\ell\,s_N), \tag{A.6}$$

where ϕ_1 and ϕ_N are the (unknown) disturbance potential, respectively, on the two adjacent panels to the T. E. (i.e., the 1st panel from the T. E. on the upper surface and the N-th panel on the lower surface) (see Figure **4.5**). But we specify the coefficients a_u , b_u , a_ℓ and b_ℓ by applying the Kutta condition at the T. E.

A.1.1 Steady flow cases

As a special case, first let us consider steady flow for which a stagnation point (for non-cusped foils) should be located at the T. E. Then it means $q_{tu}(0) = 0$

[2]See Batchelor, G. K. (1967), *An Introduction to Fluid Dynamics*, Cambridge University Press, Cambridge, p. 105.

and $q_{t\ell}(0) = 0$. Applying these constraints to (A.3) and (A.4) gives

$$b_u = -\underline{q}_\infty \cdot \underline{t}_u\Big|_{TE} \quad \text{and} \quad b_\ell = -\underline{q}_\infty \cdot \underline{t}_\ell\Big|_{TE} \,. \tag{A.7}$$

With these coefficients, (A.6) reduces to

$$\begin{aligned}
\triangle\phi_{\mathrm{v}}\big|_{TE} &= c_u - c_\ell \\
&= \phi_1 - a_u\, s_1^2 + \underline{q}_\infty \cdot \underline{t}_u\Big|_{TE} s_1 - \phi_N + a_\ell\, s_N^2 - \underline{q}_\infty \cdot \underline{t}_\ell\Big|_{TE} s_N \\
&= \phi_1 - \phi_N + a_\ell\, s_N^2 - a_u\, s_1^2 - \underline{q}_\infty \cdot (\underline{t}_N\, s_N) + \underline{q}_\infty \cdot (\underline{t}_1\, s_1) \\
&= \boxed{\phi_1 - \phi_N + \underline{q}_\infty \cdot \triangle\underline{r}}\,,
\end{aligned} \tag{A.8}$$

where the term $(a_\ell s_N^2 - a_u s_1^2)$ has been neglected, being of higher order compared with other terms and $\triangle\underline{r}(= \underline{r}_1 - \underline{r}_N)$ represents difference of position vectors of the control points of the two adjacent panels (Figure **4.5**).

Equation (A.8) is the same as the Kutta condition for steady two-dimensional lifting flow suggested first by Lee (1987). [3] He pointed out that the 'implicit' Kutta condition imposed just as $\triangle\phi_{\mathrm{v}}\big|_{TE} = \phi_1 - \phi_N$, [4] may lead to inaccurate results for extreme cases such as for a circular cylinder at 90^o angle of attack (for which the lift calculated by using this 'implicit' Kutta condition is incorrectly about half of the analytical one).

A.1.2 Unsteady flow cases

Next, we can follow a similar procedure for unsteady flow. First the tangential speed at the T. E. either on the upper or the lower surface, depending on the sign of $d\Gamma_B/dt$ as mentioned previously, should be specified in order to determine the unknown coefficients b_u and b_ℓ in (A.6). According to the behavior of the 'Maskell' trailing edge flow, the tangential speeds at the T. E. on both the upper and the lower surfaces are expressed in terms of a vortex strength at the T. E.

[3] Lee, J. T. (1987), *A Potential Based Panel Method for the Analysis of Marine Propellers in Steady Flow*, Department of Ocean Engineering, MIT, Report no. 87-13.

[4] For example, see Maskew, B. (1982), "Prediction of Subsonic Aerodynamic Characteristics: a Case for Low-Order Panel Methods," *Jounal of Aircraft*, vol. 19, no. 2, pp. 157–163.

(γ_{TE}) as:

$$\begin{cases} q_{t\ell} = \gamma_{TE}\ , \ q_{tu} = 0, & \text{if } d\Gamma_B/dt < 0 \\ q_{t\ell} = 0, \quad\quad q_{tu} = \gamma_{TE}\ , & \text{if } d\Gamma_B/dt > 0 \\ q_{t\ell} = 0, \quad\quad q_{tu} = 0, & \text{if } d\Gamma_B/dt = 0 \end{cases} \tag{A.9}$$

Substituting these relations into (A.3) and (A.4) in order find b_u and b_ℓ, then neglecting the term $(a_\ell\, s_N^2 - a_u\, s_1^2)$ as the steady flow cases and recalling $\underline{q}_\infty \equiv \underline{q}_o - \underline{q}_F$, we obtain the following expression for $\triangle\phi_{\rm v}|_{TE}$:

$$\boxed{\triangle\phi_{\rm v}|_{TE} = \begin{cases} \phi_1 - \phi_N + (\underline{q}_o - \underline{q}_F)_{TE}\cdot\triangle\underline{r} + \gamma_{TE}\, s_N, & \text{if } d\Gamma_B/dt < 0 \\ \phi_1 - \phi_N + (\underline{q}_o - \underline{q}_F)_{TE}\cdot\triangle\underline{r} - \gamma_{TE}\, s_1, & \text{if } d\Gamma_B/dt > 0 \\ \phi_1 - \phi_N + (\underline{q}_o - \underline{q}_F)_{TE}\cdot\triangle\underline{r}\ , & \text{if } d\Gamma_B/dt = 0 \end{cases} \quad (A.10)}$$

In a numerical code, from two circulation values at successive time steps an approximation for γ_{TE} may be used:

$$\gamma_{TE} = \frac{\Gamma_B^{(k-1)} - \Gamma_B^{(k)}}{\triangle {\rm v}_1} \tag{A.11}$$

Here $\triangle{\rm v}_1$ (that is given as an input parameter in a numerical code) is the length of the straight-line element of the wake sheet leaving the T. E. Consequently an iteration procedure is required to obtain unknown $\Gamma_B^{(k)}$ at the present instant of time, which is equal to the negative value of $\triangle\phi_{\rm v}|_{TE}$.

A.2 Implementation of Kutta Condition in 3-D Steady Flows

The Kutta condition has been applied originally in the steady two-dimensional flow case for uniqueness of solution mathematically and for regular flow in the vicinity of the trailing edge (T. E.) physically. It eventually implies that the rear stagnation point is at the T. E. for a non-cusped sharp-edged foil in order to satisfy both the pressure-equality condition and the condition of finite velocity at the T. E.. But if we applied this interpretation in steady three-dimensional flow, the two conditions of pressure equality and finite velocity can not be satisfied exactly at the T. E., since there is inherently a velocity difference across the sharp T. E.. and (iii) to satisfy the pressure equality condition at the T. E..

Let us approximate the disturbance potential distributions (ϕ) on the upper and the lower surfaces near the T. E. as a linear form of the local coordinates (geometrical parameters) ξ and η: [5]

$$\phi_u(\xi_u, \eta) = a_u\, \xi_u + b_u\, \eta + c_u\ , \tag{A.12}$$

$$\phi_\ell(\xi_\ell, \eta) = a_\ell\, \xi_\ell + b_\ell\, \eta + c_\ell\ , \tag{A.13}$$

where the parameter η is arclength along the T. E. positive taken as spanwise direction (see Fig. 2 in reference Mangler & Smoth (1970)) and the parameters ξ_u and ξ_ℓ are arclength along the upper surface and the lower surface, respectively, measured from the T. E. and normal to the T. E.. Here the subscripts u and ℓ refer to the upper and the lower surface, respectively. Then the potential jump at the T. E. from (A.12) and (3.8) can be written, including its spanwise variation term,

$$\triangle\phi = \phi_u\,(0,\eta) - \phi_\ell\,(0,\eta) = c_u - c_\ell + (b_u - b_\ell)\,\eta \tag{A.14}$$

This potential jump is expressed in terms of unknown quantities in the panel-method approximation as:

$$\triangle\phi = (\phi_1 - \phi_N) - (a_u\xi_{u1} - a_\ell\xi_{\ell N}) - (b_u\eta_1 - b_\ell\eta_N) + (b_u - b_\ell)\eta \tag{A.15}$$

where ϕ_1 and ϕ_N are the (unknown) disturbance potential, respectively, at the control points of the two adjacent panels to the T. E. (i.e., the 1st panel from the T. E. on the upper surface and the N-th panel on the lower surface). $\xi_{u1}, \xi_{\ell N}, \eta_1, \eta_N$ are the local coordinates of the control points. Then taking the gradient of (A.12) and (3.8) and then including the undisturbed velocity $\underline{q}_o$ give

[5] Similar procedure has been presented by Ingham et al. (1981) for the problems with two regions of different physical features, in which the two analytical solution forms of the Laplace equation for the two regions in the neighborhood of the discontinuity are introduced and then the appropriate physical matching conditions at the common interface are enforced to determine the coefficients associated with those forms. Also this procedure has been applied to irrotational solenoidal flow near a stagnation point (Batchelor (1967)).

the total tangential speeds on the upper and the lower surface near the T. E.:

$$q_u^2 = (\underline{q}_o \cdot \underline{e}_{\xi u} + a_u)^2 + (\underline{q}_o \cdot \underline{e}_\eta + b_u)^2 \tag{A.16}$$

$$q_\ell^2 = (\underline{q}_o \cdot \underline{e}_{\xi \ell} + a_\ell)^2 + (\underline{q}_o \cdot \underline{e}_\eta + b_\ell)^2 \tag{A.17}$$

where $\underline{e}_{\xi u}$, $\underline{e}_{\xi \ell}$ and $\underline{e}_\eta$ are the unit vectors of the local coordinate system at the trailing edge point.

According to the Mangler and Smith's analysis, vanishing the tangential speed at the T. E. either on the upper or the lower surface allows us to determine the unknown coefficients a_u and a_ℓ in (A.15):

$$\begin{cases} a_u = -\underline{q}_o \cdot \underline{e}_{\xi u} + \sqrt{-D}\,,\ a_\ell = -\underline{q}_o \cdot \underline{e}_{\xi \ell}, & \text{if } D < 0 \\ a_u = -\underline{q}_o \cdot \underline{e}_{\xi u}\,,\ a_\ell = -\underline{q}_o \cdot \underline{e}_{\xi \ell} + \sqrt{D}, & \text{if } D > 0 \\ a_u = -\underline{q}_o \cdot \underline{e}_{\xi u}\,,\ a_\ell = -\underline{q}_o \cdot \underline{e}_{\xi \ell}, & \text{if } D = 0 \end{cases} \tag{A.18}$$

where $D = 2(\underline{q}_o \cdot \underline{e}_\eta)(b_u - b_\ell) + (b_u^2 - b_\ell^2)$. Here b_u and b_ℓ are still unknown representing variation of the perturbation potential in η-direction on the upper surface and the lower surface at the T. E. panels. Consequently this model requires an iteration procedure to determine these coefficients by fitting the potential values at the T. E. panels in that direction.

As a special case of two-dimensional steady flow, (for which a stagnation point should be located at the T. E.) it holds $q_u = 0$ and $q_\ell = 0$. Applying these constraints to (A.16) and (A.17) gives

$$a_u = -\underline{q}_o \cdot \underline{e}_{\xi u} \quad \text{and} \quad a_\ell = -\underline{q}_o \cdot \underline{e}_{\xi \ell} \tag{A.19}$$

With these coefficients, (A.15) reduces to

$$\triangle\phi = \phi_1 - \phi_N + \underline{q}_o \cdot \triangle\underline{r} \tag{A.20}$$

where $\triangle\underline{r} (= \underline{r}_1 - \underline{r}_N)$ denotes difference of position vectors of the control points of the two adjacent panels. Lee, J. T. (1987) suggested this equation as the Kutta condition for steady two-dimensional lifting flows, by which he has shown significant improvement on accuracy of numerical solutions compared to those obtained by the so-called Morino's Kutta condition. Accordingly Eq. (A.15) contains the Kutta condition Eq. (A.8) for two-dimensional flows.