Ship Stability

Ch. 1 Introduction to Ship Stability

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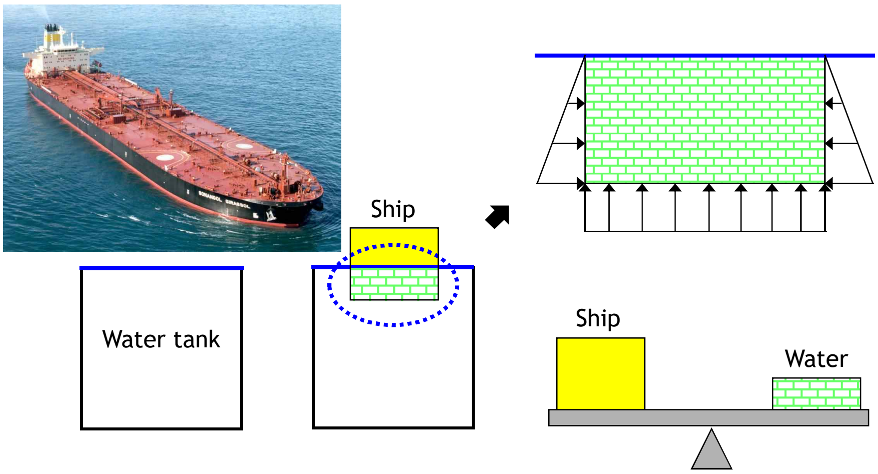
Ch. 1 Introduction to Ship Stability

1. Generals

How does a ship float? (1/3)

☑ The force that enables a ship to float ➡ " "

- It is .
- It has a magnitude equal to which is .



How does a ship float? (2/3)

☑ Archimedes' Principle

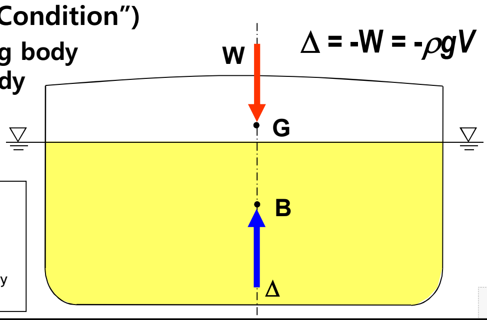
- The magnitude of the buoyant force acting on a floating body in the fluid is equal to the weight of the fluid which is displaced by the floating body.
- The direction of the buoyant force is opposite to the gravitational force.

Buoyant force of a floating body
= the weight of the fluid which is displaced by the floating body (" ")
➡ Archimedes' Principle

☑ Equilibrium State ("Floating Condition")

- Buoyant force of the floating body
  = of the floating body

∴ =



G: Center of gravity
B: Center of buoyancy
W: Weight, Δ: Displacement
ρ: Density of fluid
V: Submerged volume of the floating body
(Displacement volume, ∇)

How does a ship float? (3/3)

Displacement(Δ) = Buoyant Force = Weight(W)

$$\Delta = L \cdot B \cdot T \cdot C_B \cdot \rho$$
$$= W = LWT + DWT$$

☑ **Weight =** (Lightweight) + (Deadweight)

What is a "Hull form"?

☑ **Hull form**

that is streamlined in order to satisfy requirements of a ship owner such as a deadweight, ship speed, and so on

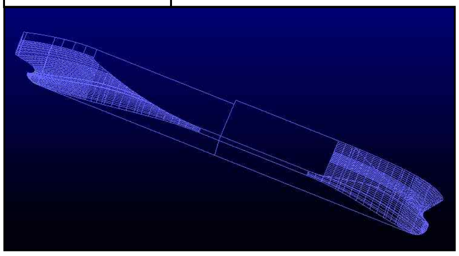
- Like a skin of human

☑ **Hull form design**

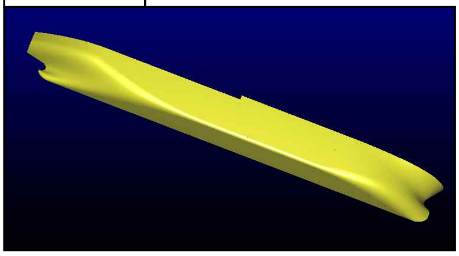
- Design task that designs the hull form

Hull form of the VLCC(Very Large Crude oil Carrier)

Wireframe model



Surface model



What is a "Compartment"?

☑ **Compartment**

- It is divided by a bulkhead which is a diaphragm or peritoneum of human.

☑ **Compartment design (General arrangement design)**

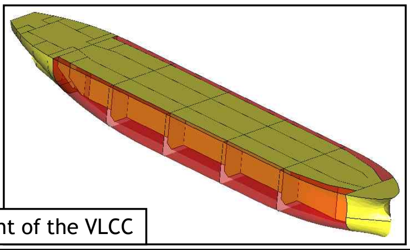
- Compartment modeling + Ship calculation

☑ **Compartment modeling**

- Design task that divides the interior parts of a hull form into a number of compartments

☑ **Ship calculation (Naval architecture calculation)**

- Design task that evaluates whether the ship satisfies the required cargo capacity by a ship owner and, at the same time, the international regulations related to stability, such as MARPOL and SOLAS, or not



Compartment of the VLCC

What is a "Hull structure"?

☑ **Hull structure**

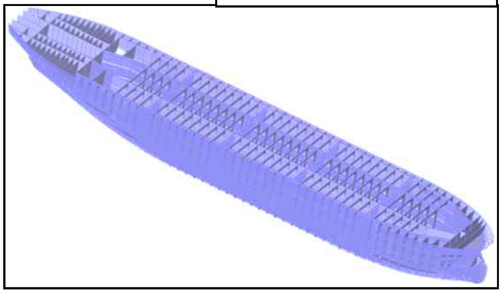
comprising of a number of hull structural parts such as plates, stiffeners, brackets, and so on

- Like a skeleton of human

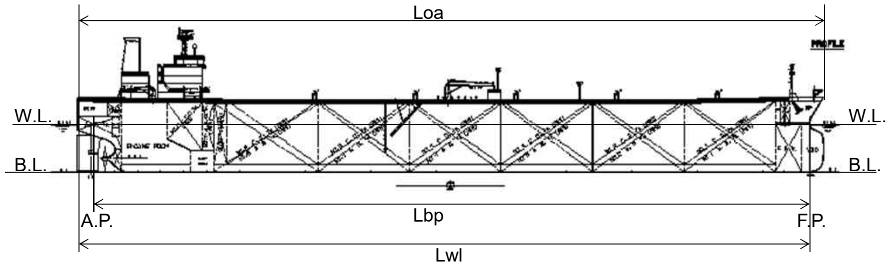
☑ **Hull structural design**

- Design task that determines the specifications of the hull structural parts such as the size, material, and so on

Hull structure of the VLCC



Principal Characteristics (1/2)



☑ **LOA (Length Over All) [m]: Maximum Length of Ship**

☑ **LBP (Length Between Perpendiculars (A.P. ~ F.P.)) [m]**

- A.P.: After perpendicular (normally, center line of the rudder stock)
- F.P.: Inter-section line between designed draft and fore side of the stem, which is perpendicular to the baseline

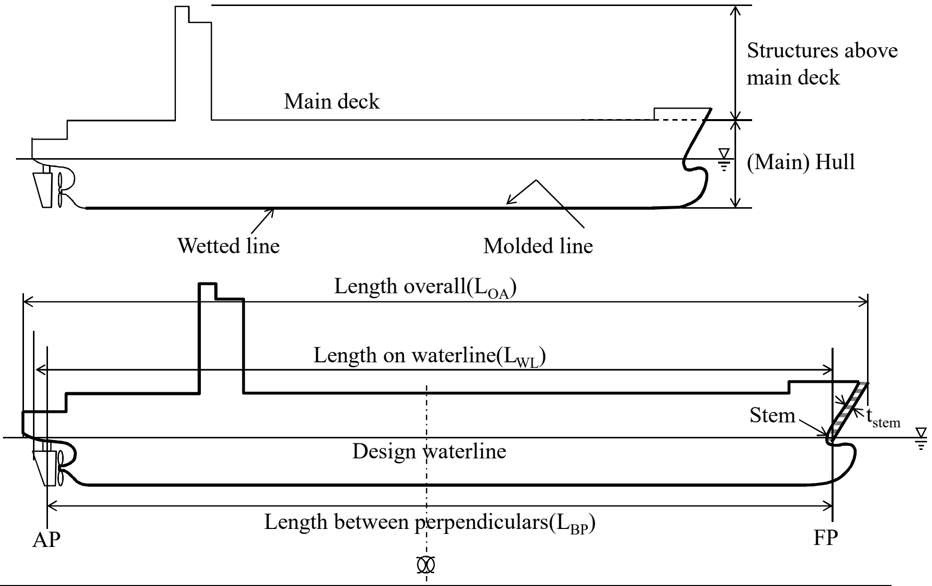
☑ **Lf (Freeboard Length) [m]: Basis of freeboard assignment, damage stability calculation**

- 96% of Lwl at 0.85D or Lbp at 0.85D, whichever is greater

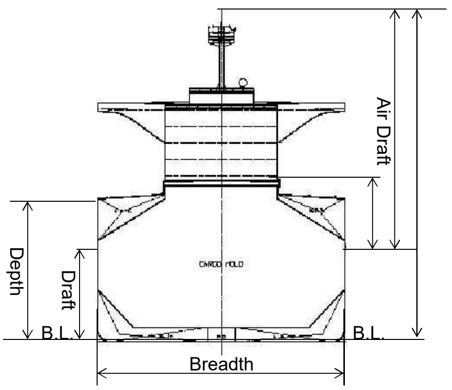
☑ **Rule Length (Scantling Length) [m]: Basis of structural design and equipment selection**

- Intermediate one among (0.96 Lwl at Ts, 0.97 Lwl at Ts, Lbp at Ts)

Definitions for the Length of a Ship



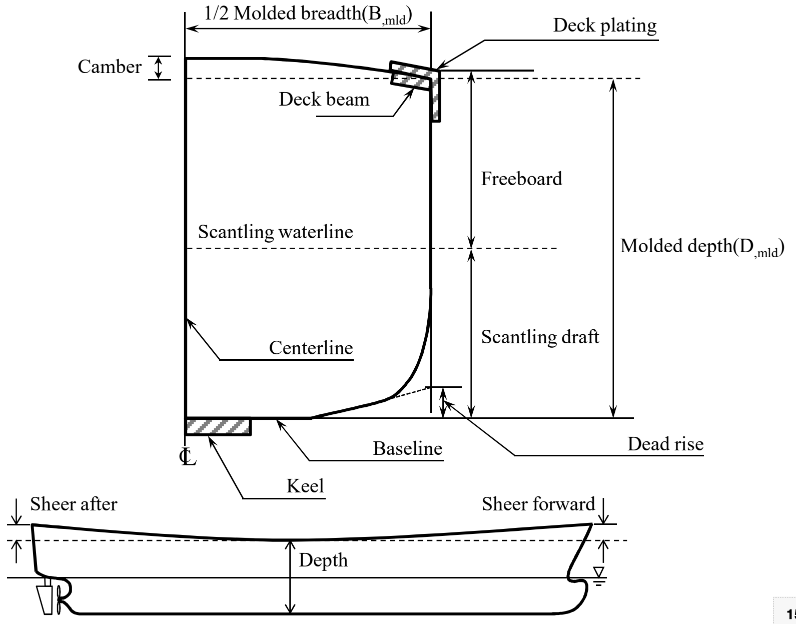
Principal Characteristics (2/2)



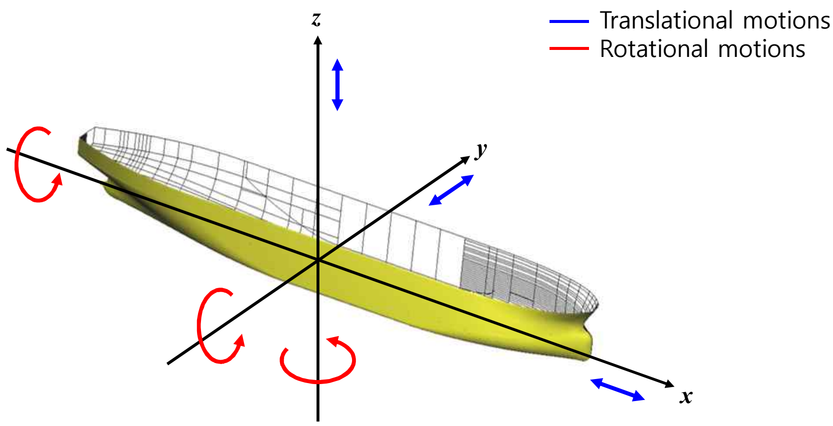
- B (Breadth) [m]: Maximum breadth of the ship, measured amidships
  - B_{molded}: excluding shell plate thickness
  - $B_{extreme}$: including shell plate thickness
- D (Depth) [m]: Distance from the baseline to the deck side line
  - D_{molded}: excluding keel plate thickness
  - $D_{extreme}$: including keel plate thickness
- Td (Designed Draft) [m]: Main operating draft
  - In general, basis of ship's deadweight and speed/power performance
- Ts (Scantling Draft) [m]: Basis of structural design

- Air Draft [m]: Distance (height above waterline only or including operating draft) restricted by the port facilities, navigating route, etc.
  - Air draft from baseline to the top of the mast
  - Air draft from waterline to the top of the mast
  - Air draft from waterline to the top of hatch cover
  - …

Definitions for the Breadth and Depth of a Ship



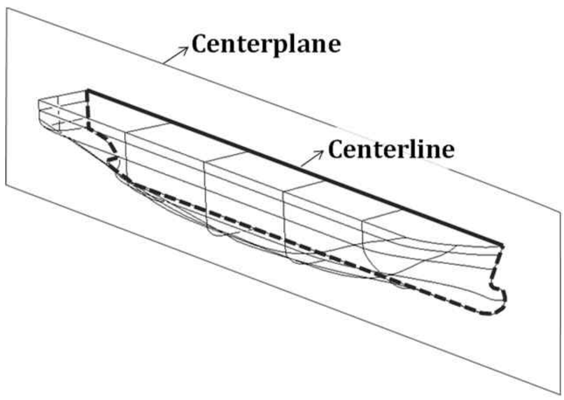
6 DOF Motions of a Ship



2. Static Equilibrium

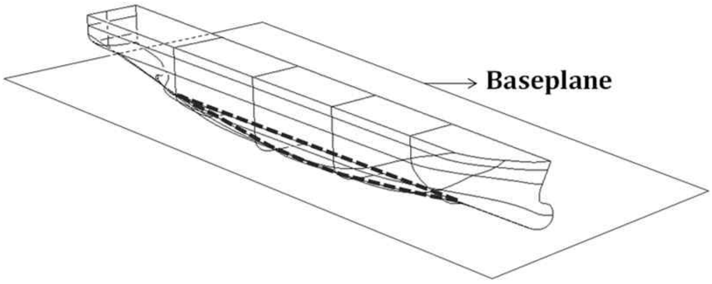
Center Plane

Before defining the coordinate system of a ship, we first introduce three planes, which are all standing perpendicular to each other.



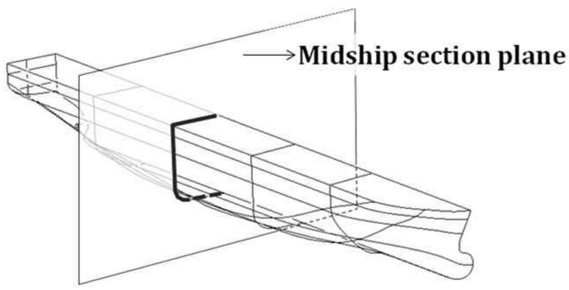
Generally, a ship is **symmetrical** about starboard and port.
The first plane is the vertical longitudinal plane of symmetry, or .

Base Plane



The second plane is the horizontal plane, containing the bottom of the ship, which is called .

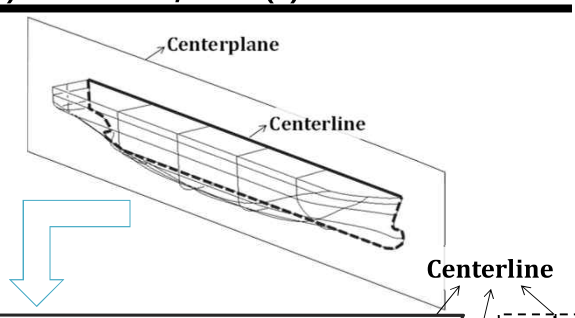
Midship Section Plane

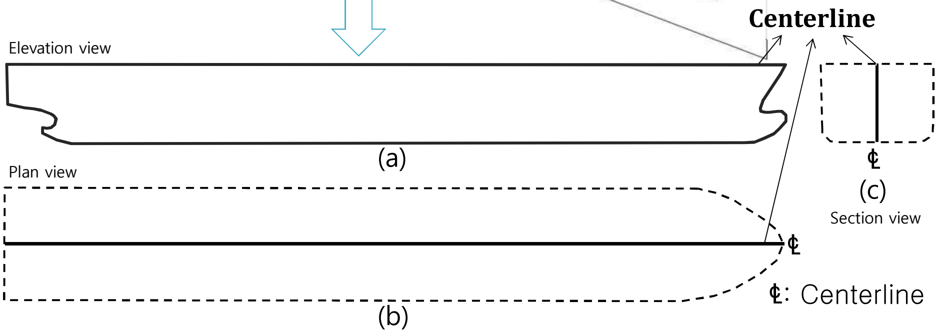


The third plane is the vertical transverse plane through the midship, which is called .

Centerline in (a) Elevation view, (b) Plan view, and (c) Section view

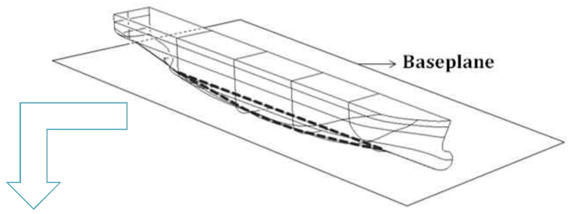
Centerline:
Intersection curve between center plane and hull form

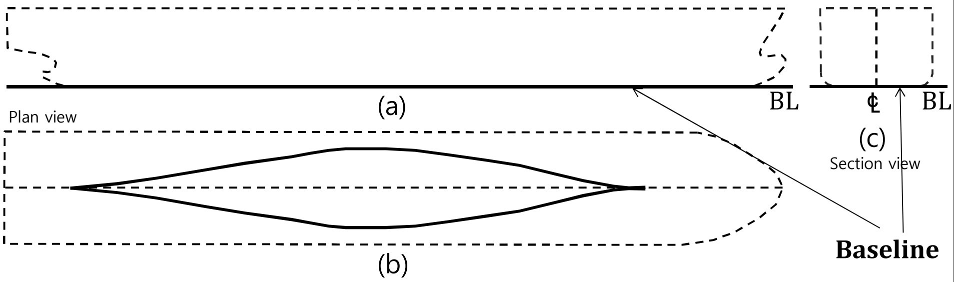




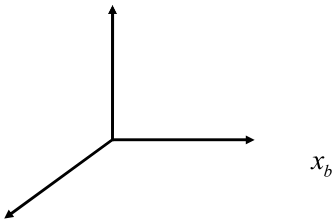
Baseline in (a) Elevation view, (b) Plan view, and (c) Section view

Baseline:
Intersection curve between base plane and hull form





System of Coordinates



n-frame: Inertial frame $x_n\, y_n\, z_n$ or $x\, y\, z$

Point E: Origin of the inertial frame(n-frame)

b-frame: Body fixed frame $x_b\, y_b\, z_b$ or $x'\, y'\, z'$

Point O: Origin of the body fixed frame(b-frame)

1) ***coordinate system***

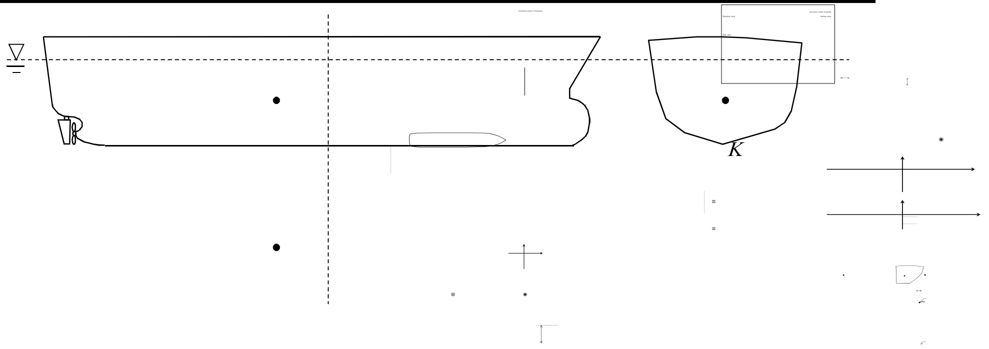
The right handed coordinate system with the axis called x_b(or x'), y_b(or y'), and z_b(or z') is **fixed to the object**. This coordinate system is called ***body fixed coordinate system*** or ***body fixed reference frame (b-frame)***.

2) ***coordinate system***

The right handed coordinate system with the axis called x_n(or x), y_n(or y) and z_n(or z) is **fixed to the space**. This coordinate system is called ***space fixed coordinate system*** or ***space fixed reference frame*** or ***inertial frame (n-frame)***.

In general, a change in the position and orientation of the object is described with respect to the inertial frame. Moreover Newton's 2nd law is only valid for the inertial frame.

Center of Buoyancy (B) and Center of Mass (G)



※ In the case that the shape of a ship is asymmetrical with respect to the centerline.

Center of buoyancy (B)

It is the point at which all the vertically upward forces of support ( ) can be considered to act.
It is equal to the center of volume of the submerged volume of the ship. Also, It is equal to the first moment of the submerged volume of the ship about particular axis divided by the total buoyant force (displacement).

Center of mass or Center of gravity (G)

It is the point at which all the vertically downward forces of weight of the ship ( ) can be considered to act.
It is equal to the first moment of the weight of the ship about particular axis divided by the total weight of the ship.

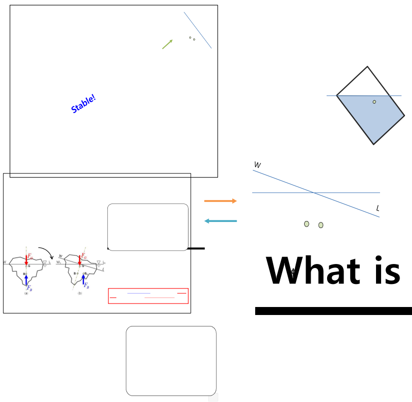
① **Newton's 2nd law**

$$ma = \sum F$$

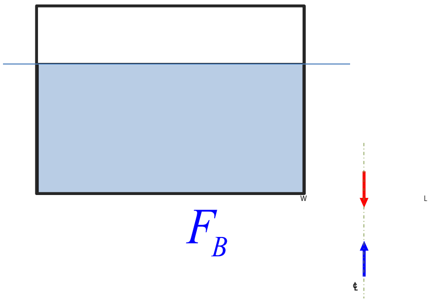
Static Equilibrium (2/3)

for the ship to be in static equilibrium

for the ship to be in static equilibrium



What is “Stability”?



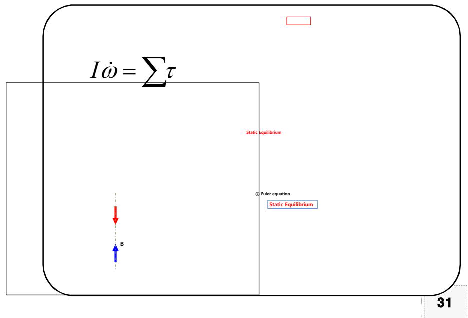
Stability = Stable + Ability

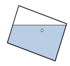
Stability of a Ship

- You have a torque on this object relative to any point that you choose. It does not matter where you pick a point.

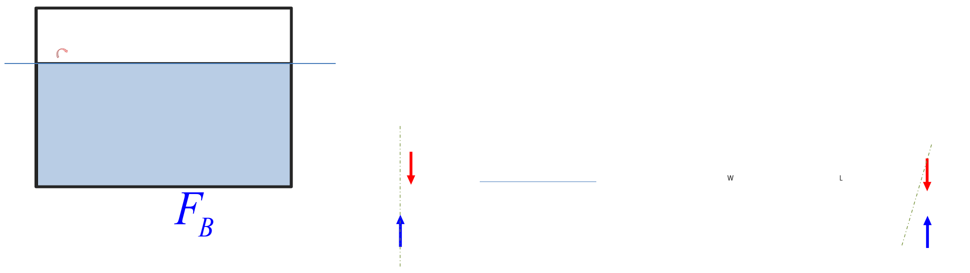
- The torque will only be zero when the buoyant force and the gravitational force are on one line. Then the torque becomes zero.

$$I\dot{\omega} = \sum \tau$$

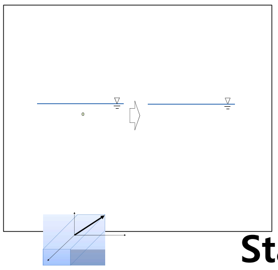




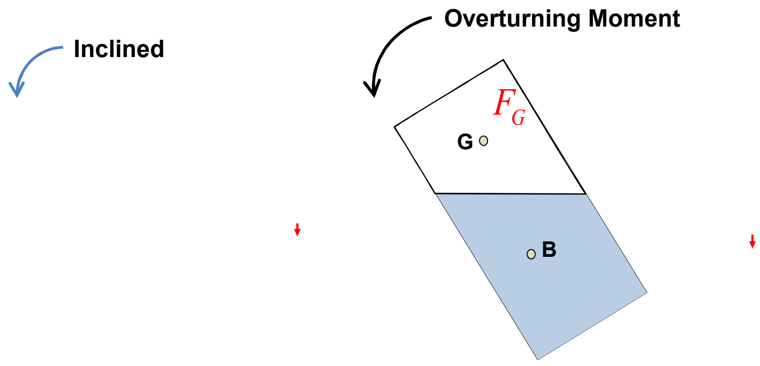
Interaction of Weight and Buoyancy of a Floating Body (2/2)



(a) (b)



Stability of a Floating Body (2/2)



(a) (b)

Floating body in unstable state

Equations for Static Equilibrium (1/3)

Suppose there is a floating ship. The states that the sum of total forces is zero.

$$\sum F = F_{G,z} + F_{B,z} = 0$$

, where
$F_{G.z}$ and $F_{B.z}$ are the z component of the gravitational force vector and the buoyant force vector, respectively, and all other components of the vectors are zero.

Also the must be satisfied, this means, the resultant moment should be also zero.

where **M**$_G$ is the moment due to the gravitational force and **M**$_B$ is the moment due to the buoyant force.

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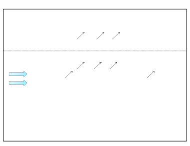
where **M**$_G$ is the moment due to the gravitational force and **M**$_B$ is the moment due to the buoyant force.

From the calculation of a moment we know that **M**$_G$ and **M**$_B$ can be written as follows:



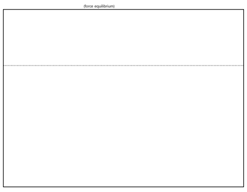
Equations for Static Equilibrium (3/3)

$$\sum \boldsymbol{\tau} = \mathbf{M}_G + \mathbf{M}_B = \mathbf{0}$$

where $\mathbf{M}_G$ is the moment due to the gravitational force and $\mathbf{M}_B$ is the moment due to the buoyant force.

Substituting

(force equilibrium)



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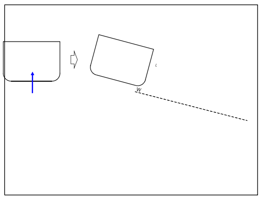
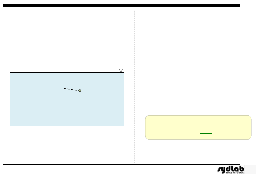
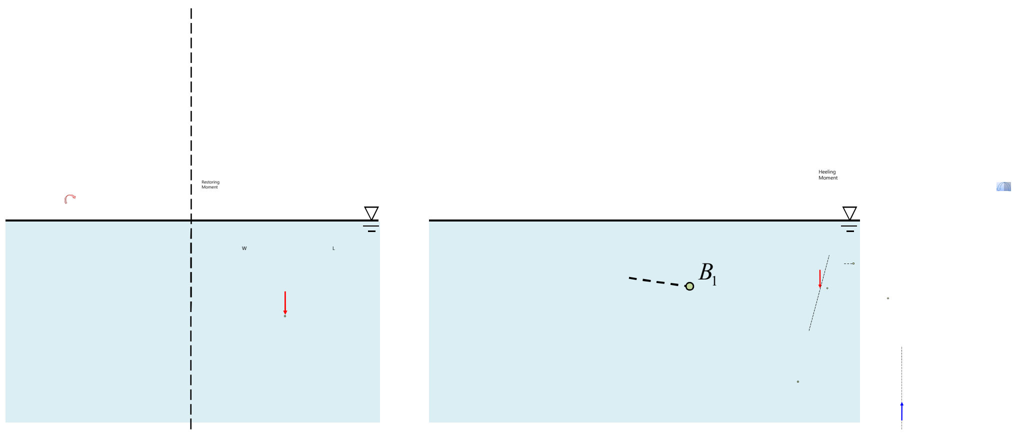


Restoring Moment Acting on an Inclined Ship

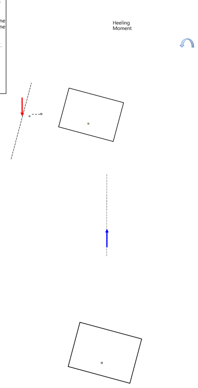


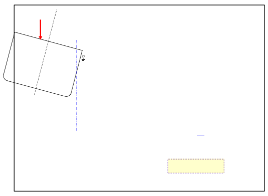


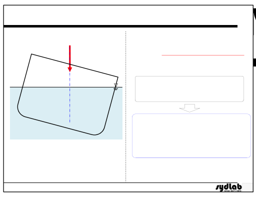
Restoring Moment

- The value of the restoring moment is found by multiplying the buoyant force of the ship (displacement), F_B, by the perpendicular distance from G to the line of action of F_B.
- It is customary to label as Z the point of intersection of the line of action of F_B and the parallel line to the waterline through G to it.
- This distance GZ is known as the '**restoring arm**' or '**righting arm**'.

Heeling Moment

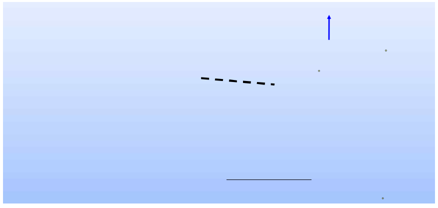






Metacenter (M)

$\tau_{restoring} = F_B \cdot GZ$



Z: The intersection point of the line of buoyant force through B_1 with the transverse line through G

Definition of M (Metacenter)

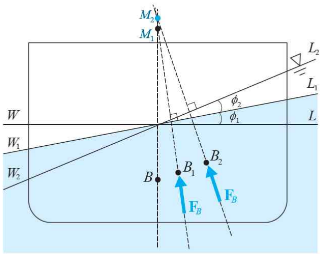
- The term **meta** was selected as a prefix for center because its Greek meaning implies **movement**. The **metacenter** therefore is a **moving center**.

- The intersection point of the vertical line through the center of buoyancy at position (B) with the vertical line through the center of buoyancy at position (B_1)

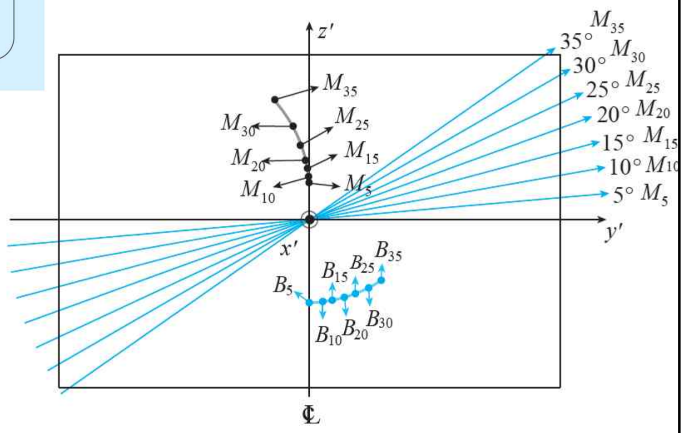
- GM ➡

For a small angle of inclination

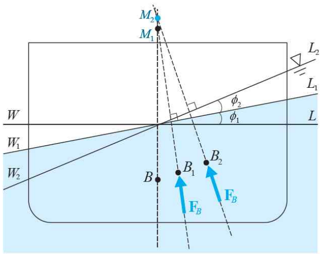
Restoring Moment at Large Angle of Inclination (2/3)

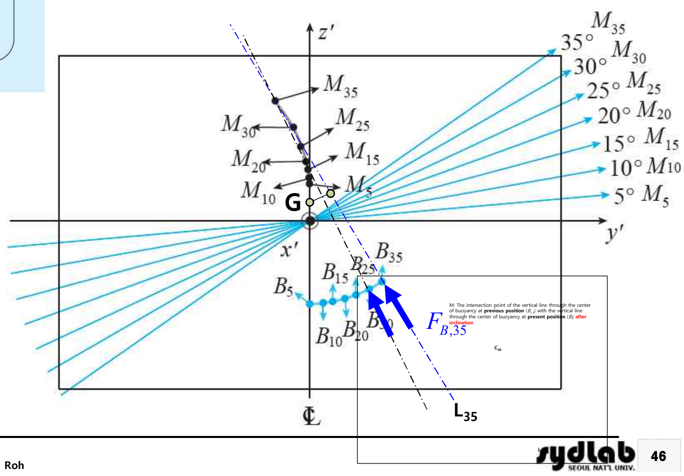


M: The intersection point of the vertical line through the center of buoyancy at **previous position** (B_{i-1}) with the vertical line through the center of buoyancy at **present position** (B_i) **after inclination**



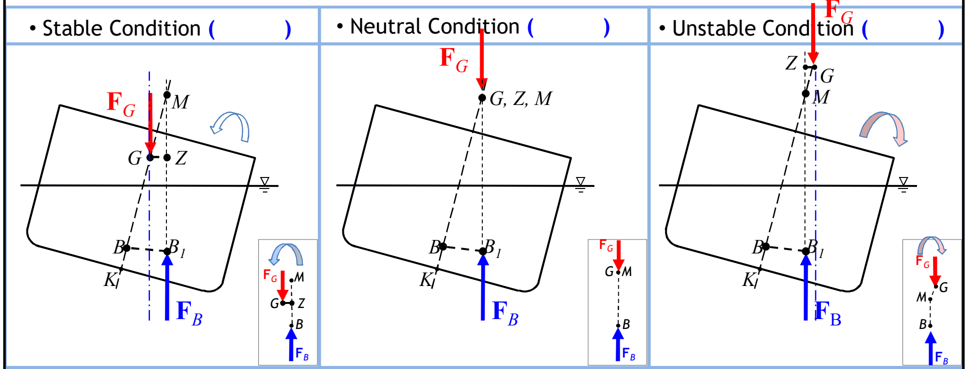
Restoring Moment at Large Angle of Inclination (3/3)



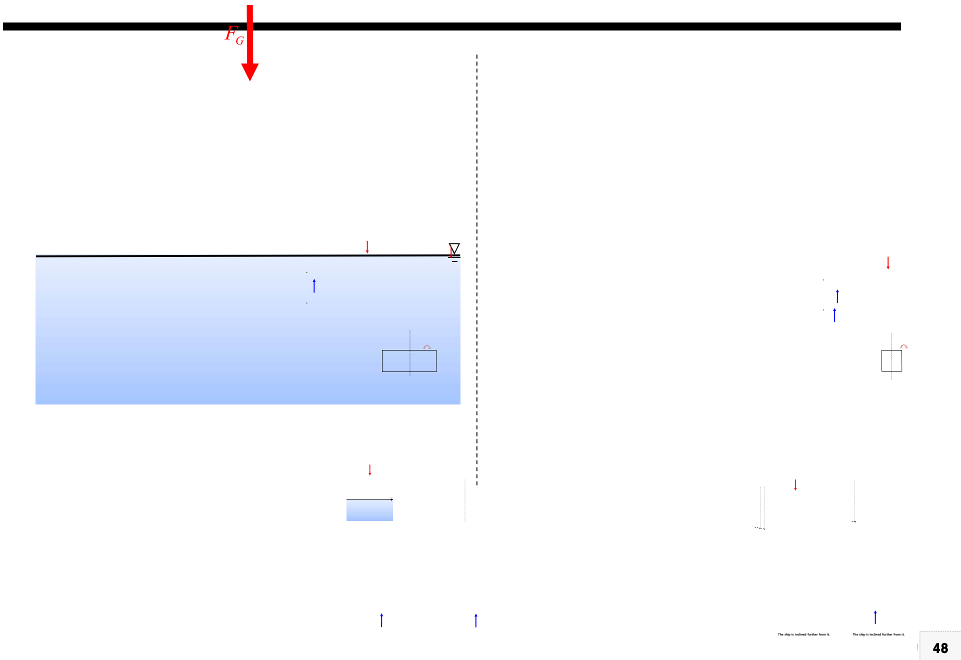


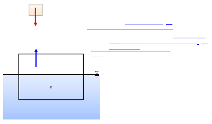
Stability of a Ship According to Relative Position between “G”, “B”, and “M” at Small Angle of Inclination

- **Righting (Restoring) Moment:** Moment to return the ship to the upright floating position
- **Stable / Neutral / Unstable Condition:** Relative height of G with respect to M is one measure of stability.



G: Center of mass   *K*: Keel
B: Center of buoyancy at upright position B_1: Changed center of buoyancy
F_G: Weight of ship F_B: Buoyant force acting on ship
Z: The intersection of the line of buoyant force through B_1 with the transverse line through G
M: The intersection of the line of buoyant force through B_1 with the centerline of the ship

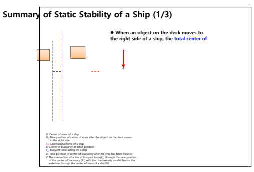


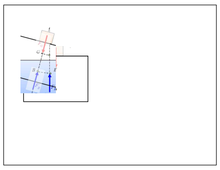


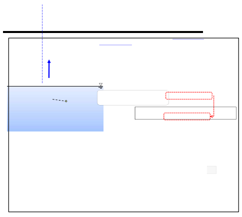
4. Ship Stability

G

● Because the buoyant force and the gravitational force are not on one line, the

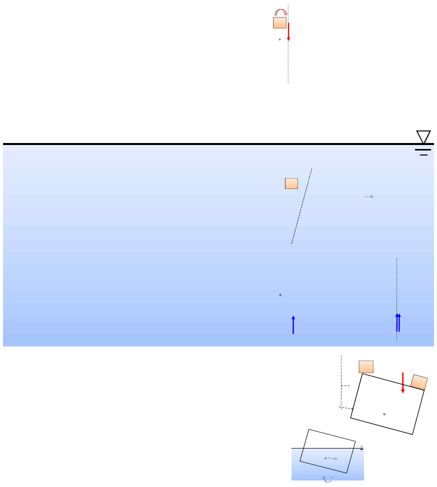






Summary of Static Stability of a Ship (2/3)

τ_e



● The total moment will only be zero when the buoyant force and the gravitational force are on one line. If the moment becomes zero, the ship is in **static equilibrium state**.

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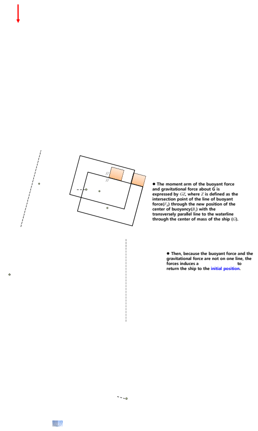
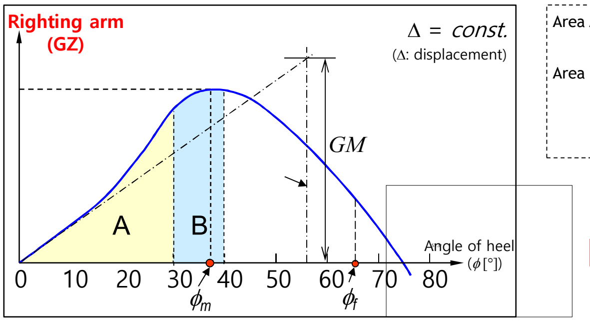


Evaluation of Stability
: Merchant Ship Stability Criteria – <u>IMO Regulations</u> for Intact Stability

(IMO Res.A-749(18) ch.3.1)

☑ **IMO recommendation on intact stability for passenger and cargo ships.**



Area A: Area under the righting arm curve between the heel angle of 0° and 30°

Area B: Area under the righting arm curve between the heel angle of 30° and min(40°, ϕ_f)

※ ϕ_f: Heel angle at which openings in the hull

ϕ_m: Heel angle of maximum righting arm

※ After receiving approval of calculation of IMO regulation from Owner and Classification Society, ship construction can proceed.

IMO Regulations for Intact Stability

(a) Area A ≥ 0.055 m-rad

(b) Area A + B ≥ 0.09 m-rad

(c) Area B ≥ 0.030 m-rad

}

The work and energy considerations (dynamic stability)

(d) GZ ≥ 0.20 m at an angle of heel equal to or greater than 30°

(e) GZ_{max} should occur at an angle of heel preferably exceeding 30° but not less than 25°.

(f) The initial metacentric height GM_o should not be less than 0.15 m.

}

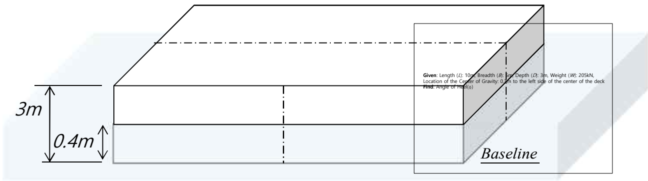
Static considerations

[Example] Equilibrium Position and Orientation of a Box-shaped Ship

Question 1) <u>The center of mass is moved to 0.3 [m] in the direction of the starboard side.</u>

A box-shaped ship of 10 meter length, 5 meter breadth and 3 meter height weighs 205 [kN].
The center of mass is moved 0.3 [m] to the left side of the center of the deck.
<u>When the ship is in static equilibrium state, determine the angle of heel (ϕ) of the ship.</u>

Assumption)
(1) Gravitational acceleration = 10 [m/s^2], Density of sea water = 1.025 [ton/m^3]
(2) When the ship will be in the static equilibrium finally, <u>the deck will not be immersed and the bottom will not emerge</u>.



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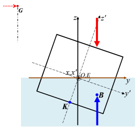
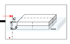
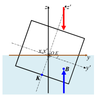
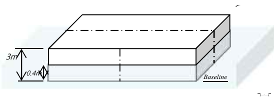
(1-2) Euler Equation: Moment Equilibrium
The resultant moment should be zero to be in static equilibrium.





Solution)
(1) Static Equilibrium (2/3)

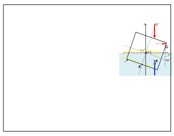


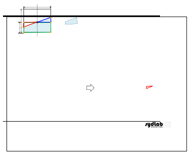


: Location of the center of mass of the ship

The first step is to satisfy the Newton-Euler equation which requires that the sum of total forces and moments acting on the ship is zero.

As described earlier, in order to satisfy a stable equilibrium, the buoyant force and gravitational force should act on the same vertical line, therefore, the moment arm of the buoyant force and gravitational force must be same.





Solution)

(2-1) Changed Center of Buoyancy, B_1, with Respect to the Body Fixed Frame

$y_G = y_B$

The centroi
body fixed

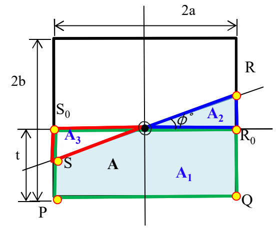
, where
A_A : the area of A
$M_{A,z'}$: 1st moment of area of A about z' axis
$M_{A,y'}$: 1st moment of area of A about y' axis.

To obtain the centroid of A, the followings are required.
- **The area of A**
- **1st moment of area of A about z' axis**
- **1st moment of area of A about y' axis**

To calculate the centroid of A using the geometrical

Solution)

(2-2) Center of Buoyancy and Center of Gravity with Respect to the Body Fixed Frame (2/2)



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Centroid:

Moment of area about z' axis:

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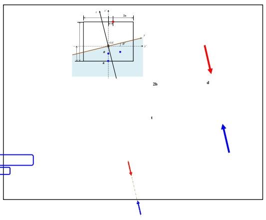
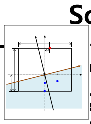
Moment of area about y' axis:



Solution)

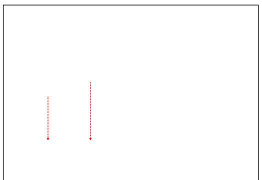


-3) Center of Buoyancy and Center of Gravity with Respect to the Body Fixed Frame (2/2)

2) Center of gravity, G, with respect to the body fixed frame

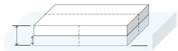
The center of gravity, G, with respect to the body fixed frame is given by geometrical relations as shown in the figure, which is

$$\left(y'_G, z'_G\right) = \left(d, 2b - t\right)$$



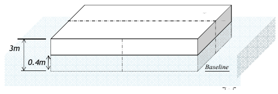
Solution)

3) Comparison between the Figure Describing the Ship Inclined and the Figure Describing the Water Plane Inclined (2/2)

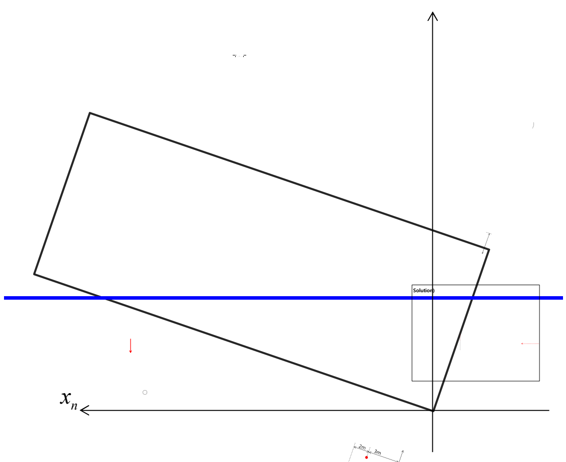


$$d \cdot \cos\phi + (2b - t) \cdot \sin\phi = \frac{\left\{-3t^2 + 2a^2 + a^2 \cdot (\tan\phi)^2\right\} \cdot \sin\phi}{6t}$$

Substituting a=2.5m, b=1.5m, t=0.4m, d=0.3m into this equation and rearranging

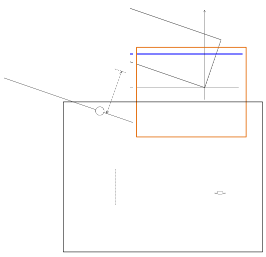


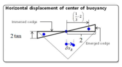
Solution)



The centers of buoyancy B and gravity G should be in the same vertical line.

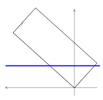
The centers of buoyancy B and gravity G should be in the same vertical line.

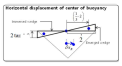


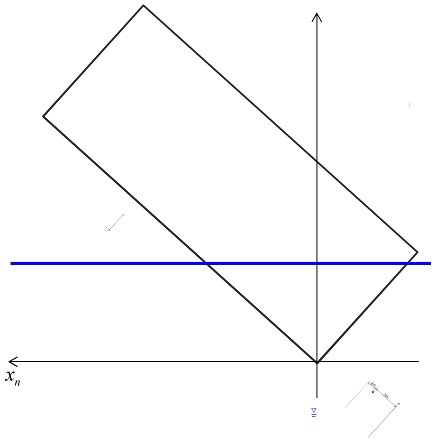


$$3\cos\alpha - 3\sin\alpha = \frac{a}{3}\cos\alpha - \frac{b}{3}\sin\alpha$$

Solution)







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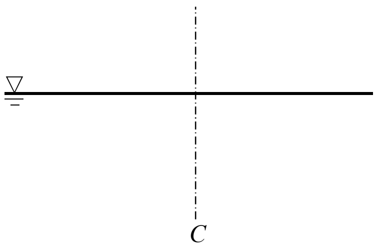
Question) Emergency circumstance happens in Ferry with displacement (mass) 102.5 ton. Heeling moment of 8 ton·m occurs due to passengers moving to the right of the ship. What will be an angle of heel?
Assume that wall sided ship with KB=0.6m, KG=2.4m, I_T=200m⁴.

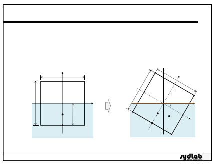
$$\left(0.2+\tan^2\phi\right)\sin\phi = 0.078$$

Because of nonlinear equation, solve it by numerical method.
Result of calculation is about ϕ=16.0°.

[Example] Change of Center Caused by Movement of Cargo

Question) As below cases partial weight w of the ship is shifted. What is the shift distance of center of mass of the ship?





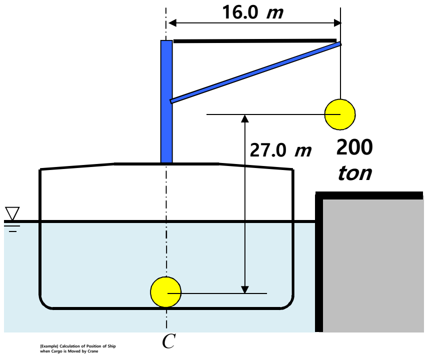
Question)

A cargo carrier of 18,000 ton displacement is afloat and has *GM* = 1.5*m.* And we want to transfer the cargo of 200 ton weight from bottom of the ship to land.

A lifting height of cargo is 27.0 m from the original position.

After lifting the cargo, turn the cargo to the right through a distance of 16.0 m from the centerline.

What will be the angle of heel of the ship?



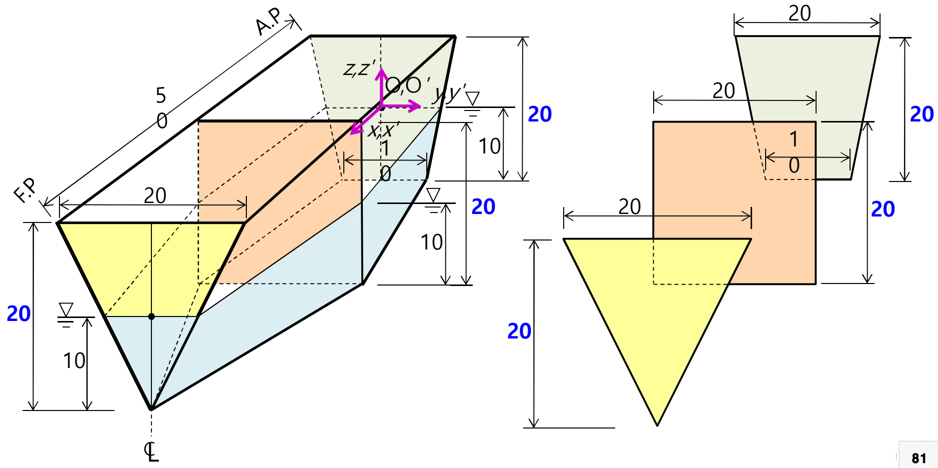
Hint) Use the Moment to Heel One Degree and the heeling moment caused by the movement of the cargo.

Problem to calculate the equilibrium angle of the ship when external force are applied.

[Example] Calculation of Center of Buoyancy of Ship with Various Station Shapes

A ship with three varied section shape is given. When this ship is inclined about x axis with an angle of -30°, calculate y and z coordinates of the center of buoyancy (with respect to the water plane fixed frame).

- Given: Length(L) 50m, Breadth(B) 20m, Depth(D) 20m, Draft(T) 10m, Angle of Heel(ϕ) -30˚
- Find: Center of buoyancy($y_{\nabla,\phi}$, $z_{\nabla,\phi}$) after heeling



Reference Slides

<1st moment of area>

$Area_a$

[Reference] Movement of Centroid Caused by Movement of Area (1/3)

Let us consider 1st moment of area about z axis through origin g.

··· ①

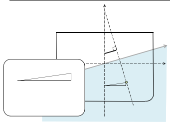
G_1 : Centroid of total area, $Area_A$: Total area
g : Centroid of the large circle, $Area_{A-a}$: Area of the large circle
g_1 : Centroid of the small circle, $Area_a$: Area of the small circle

[Reference] Movement of Centroid Caused by Movement of Area (2/3)

When the center of the small circle moves from g1 to g2, the total moment of area about z axis through origin g is

G_1 : Centroid of total area, $Area_A$: Total area
g : Centroid of the large circle, $Area_{A-a}$: Area of the large circle
g_1 : Centroid of the small circle, $Area_a$: Area of the small circle

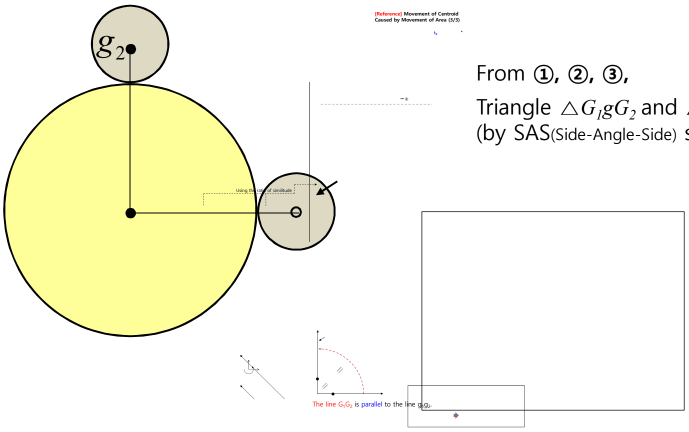
··· ②



… ③

From ①, ②, ③,

Triangle $\triangle G_1gG_2$ and $\triangle g_1gg_2$ are similar.
(by SAS(Side-Angle-Side) similarity theorem)



G_1 : Centroid of total area, $Area_A$: Total area
g : Centroid of the large circle, $Area_{A-a}$: Area of the large circle
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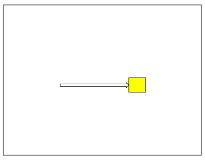
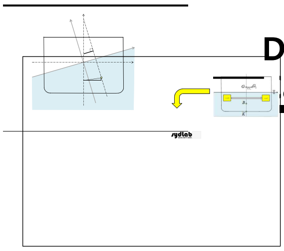
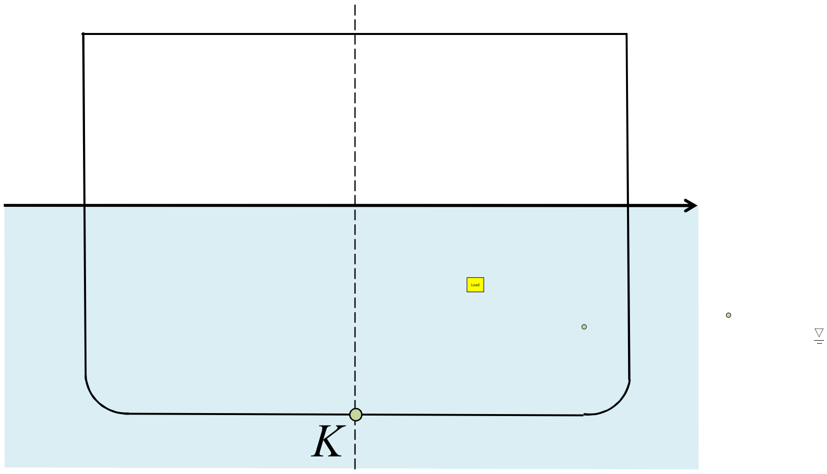
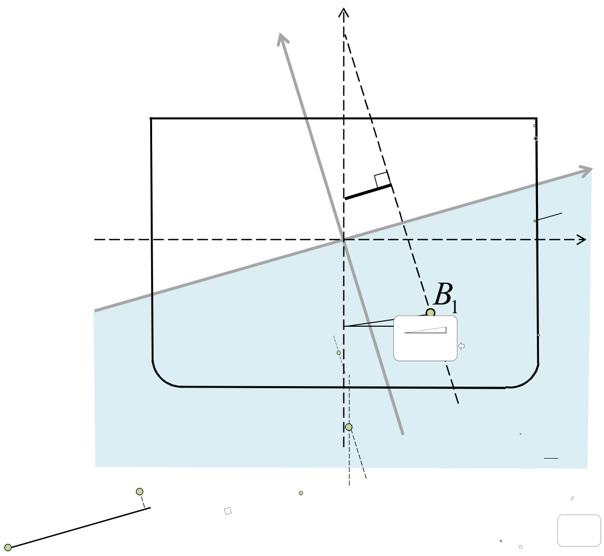




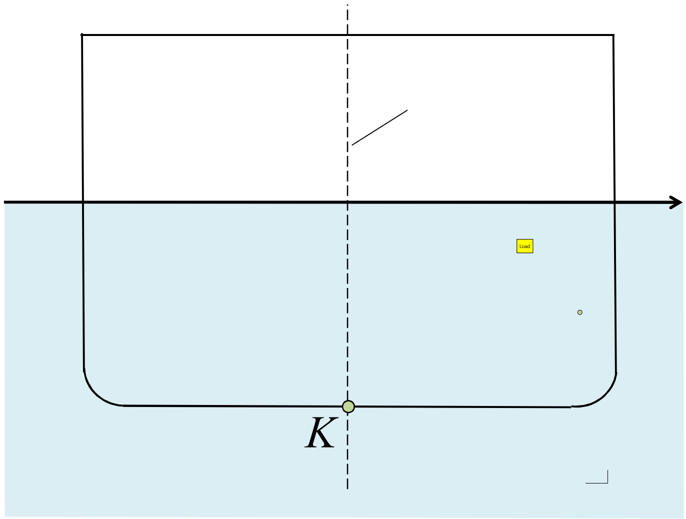
Determination of Heeling Angle for the Case of Moving a Cargo Only in Transverse Direction (1/4)



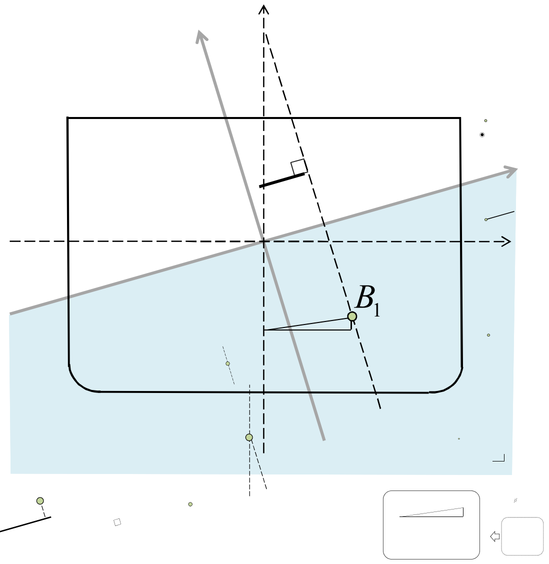
Determination of Heeling Angle for the Case of Moving a argo Only in Transverse Direction (3/4)



Determination of the Heeling Angle Due to the Movement of the Center of Gravity (1/4)

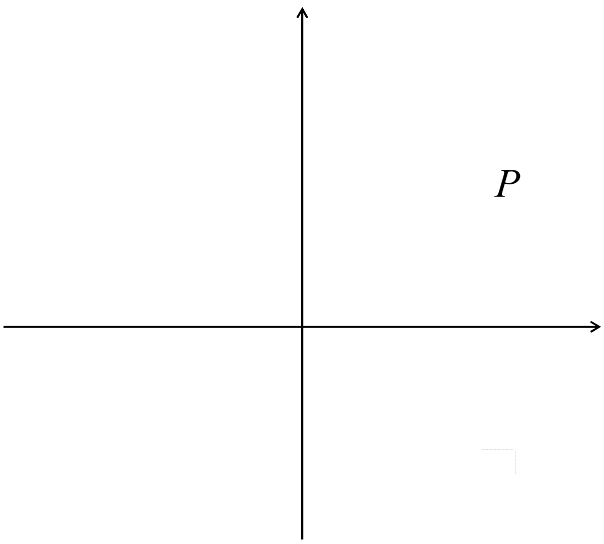


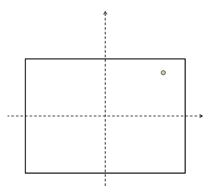
Determination of the Heeling Angle Due to the Movement of the Center of Gravity (3/4)



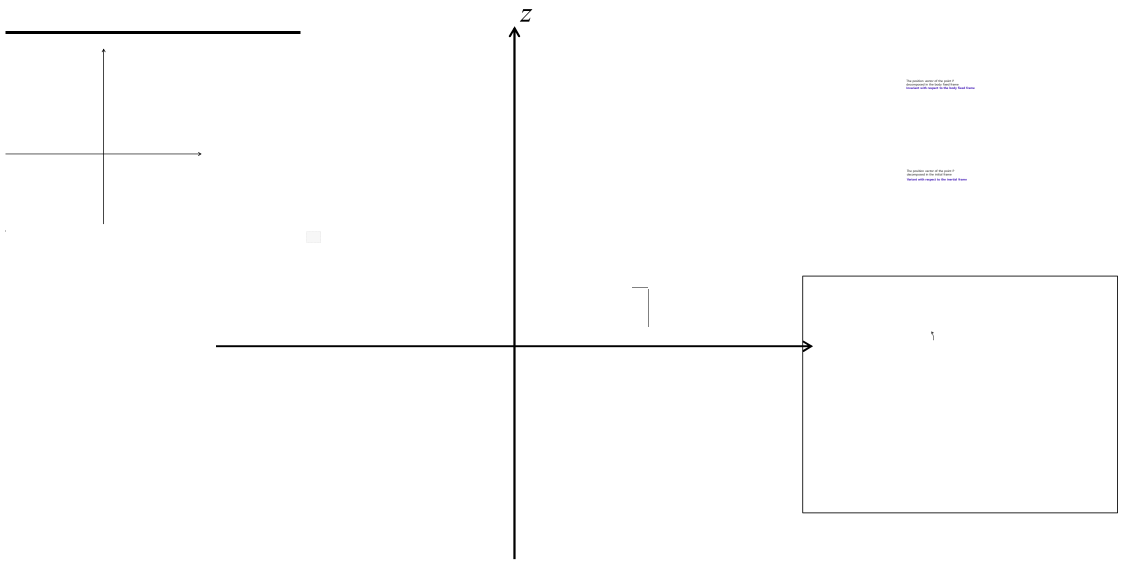
[Appendix] Rotational Transformation of a Position Vector to a Body in Fluid

Representation of a Point "P" on the Object with Respect to the Body Fixed Frame (Decomposed in the Body Fixed Frame)

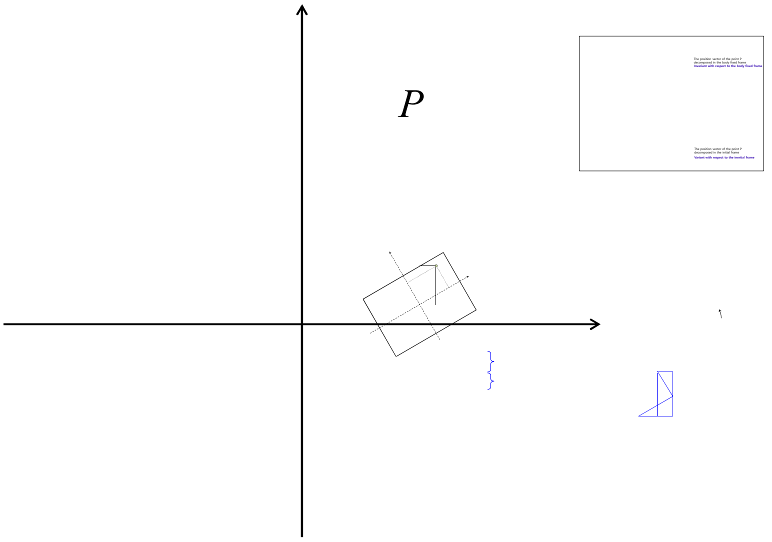




Rotation of the Object with an Angle of ϕ and then Representation of the Point "P" on the Object with Respect to the Inertial Frame



Coordinate Transformation of a Position Vector



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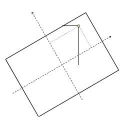
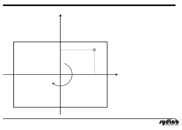
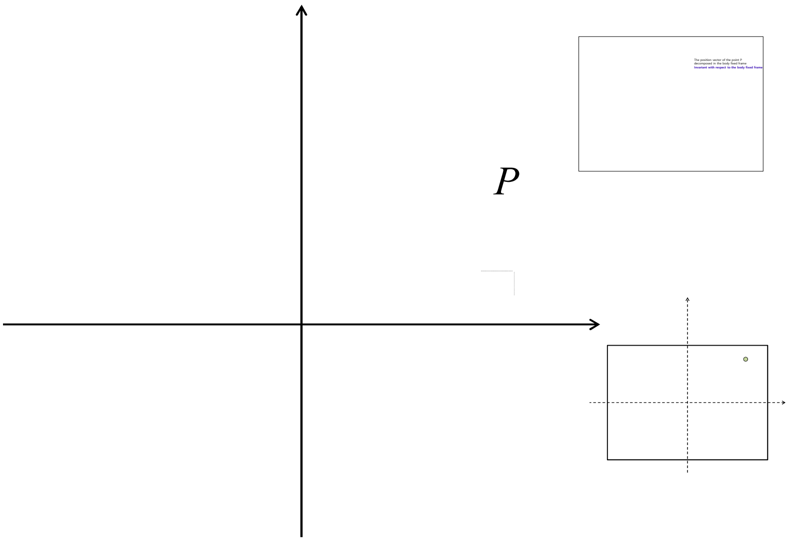
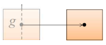
The position vector of the point P decomposed in the body fixed frame
Invariant with respect to the body fixed frame
The position vector of the point P decomposed in the initial frame
Variant with respect to the inertial frame

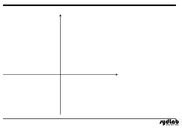




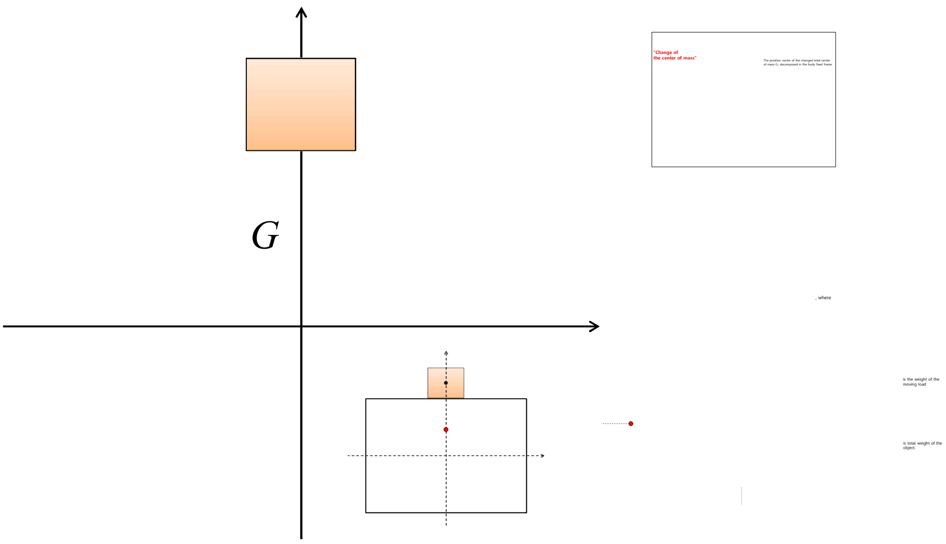
Representation of a Point "P" on the Object with Respect to the Body Fixed Frame (Decomposed in the Body Fixed Frame)

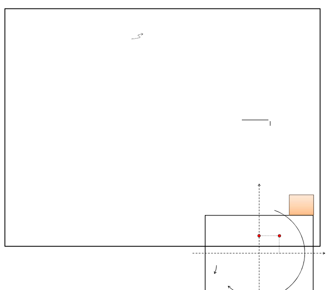


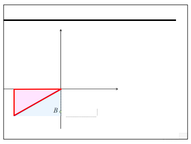




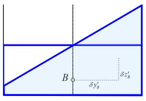
Change of the Total Center of Mass Caused by Moving a Load of Weight "w" with Distance "d" from "g" to "g_1"

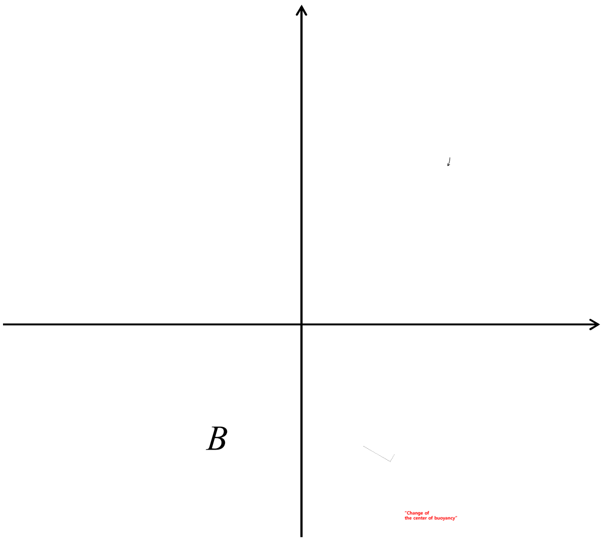


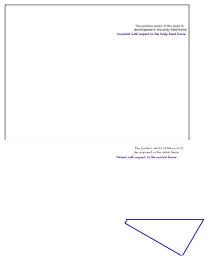


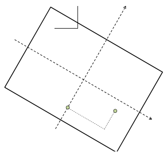


Change of the Center of Buoyancy Caused by Changing the Shape of Immersed Volume









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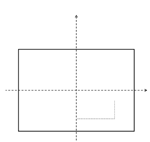
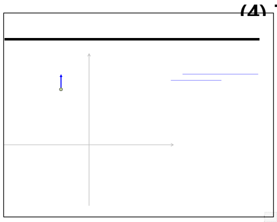
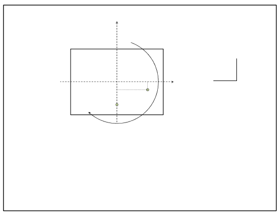


(3) Rotate the new centroid "B_1" with an angle of "-ϕ"(clockwise direction).
(4) Then calculate the position vector of the point "B_1" with respect to the inertial frame.



$$\begin{bmatrix} y_{B_1} \\ z_{B_1} \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_{B_1} \\ z'_{B_1} \end{bmatrix}$$



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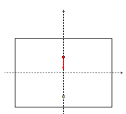
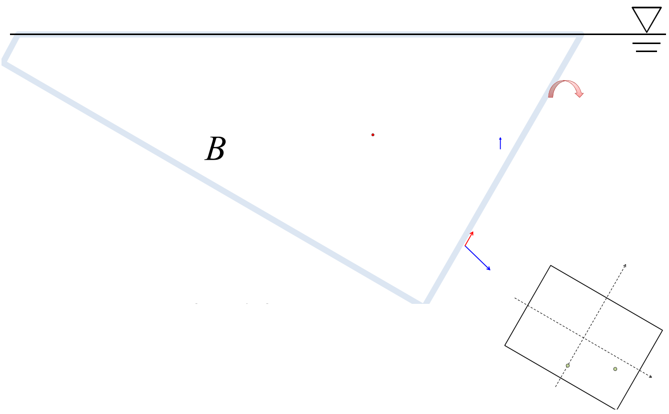
Stability of a Ship
- Stable Condition (1/3)



Stability of a Ship
- Stable Condition (2/3)



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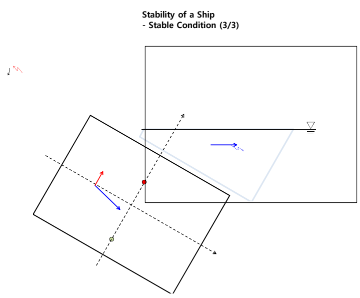
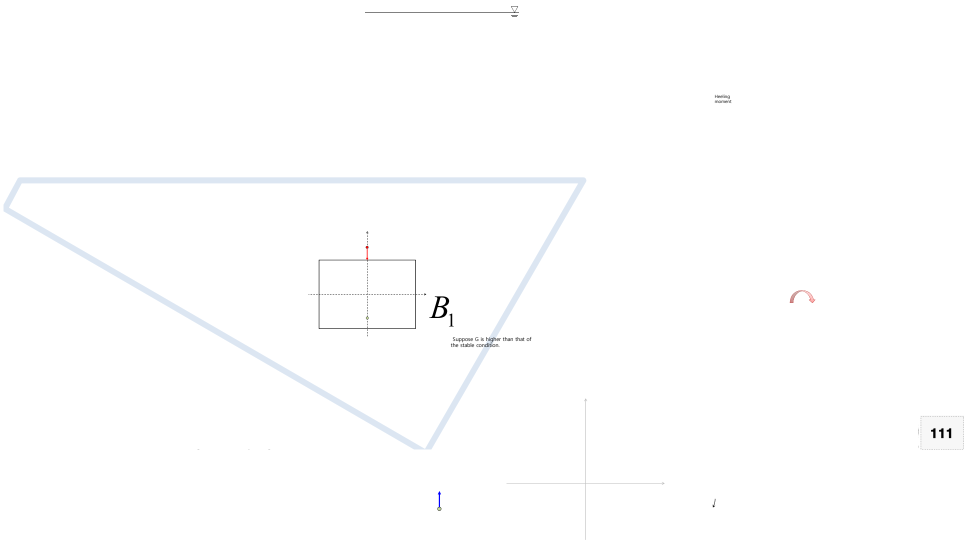
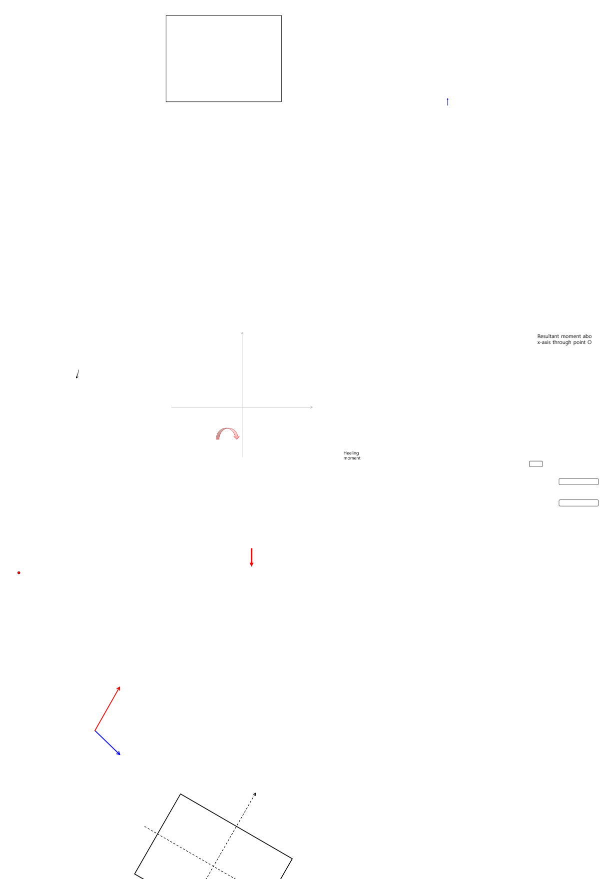
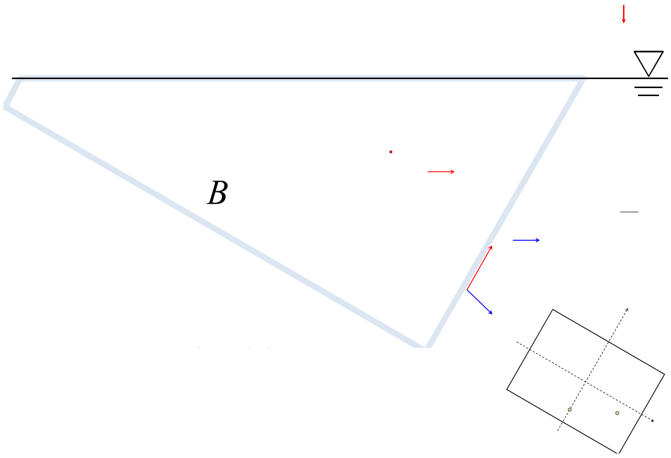


Stability of a Ship
- Neutral Condition (1/3)





Stability of a Ship
- Neutral Condition (3/3)



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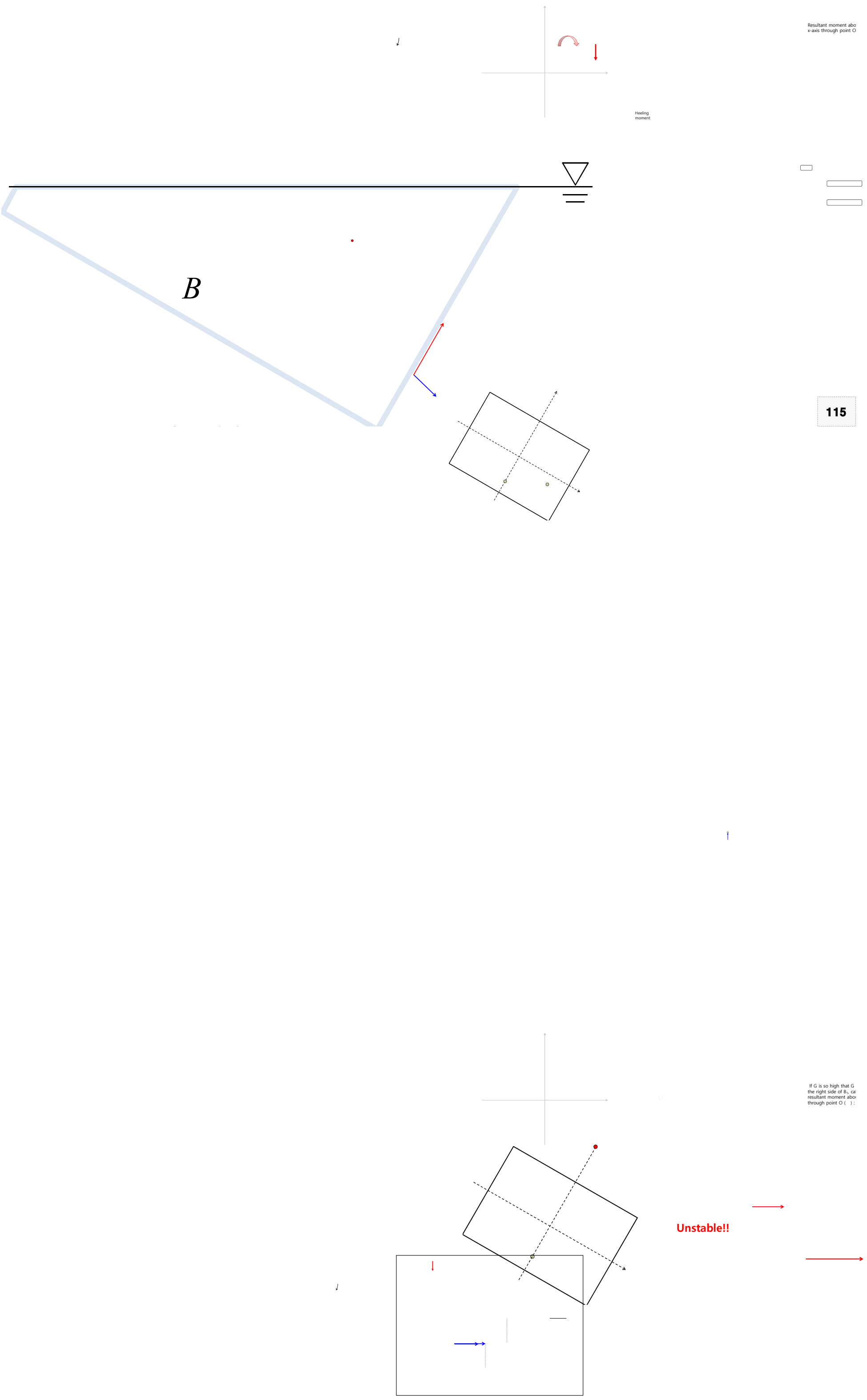
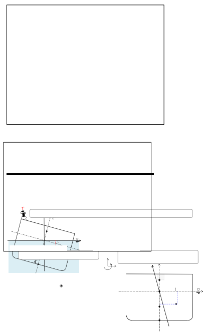
Stability of a Ship
- Unstable Condition (1/3)

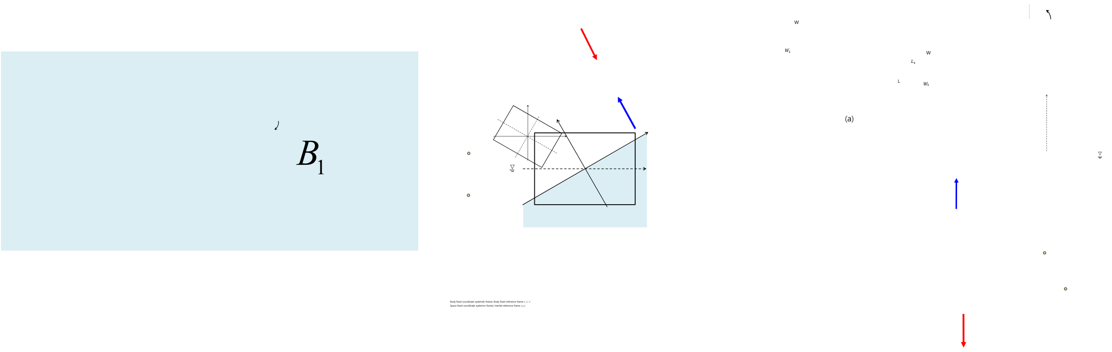
Heeling moment

Stability of a Ship
- Unstable Condition (2/3)



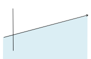


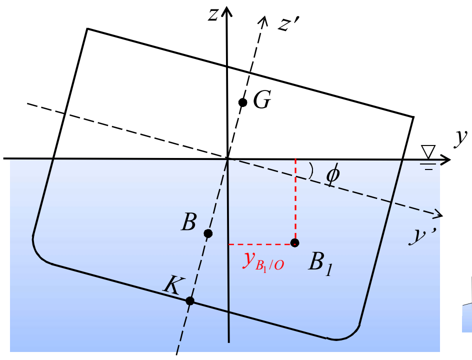
Orientation of a Ship with Respect to the Different Reference Frame



[Reference] Water Plane Fixed Reference Frame vs. Body Fixed Reference Frame

We can calculate the center of buoyancy with respect to the water plane fixed reference frame (inertial reference frame).





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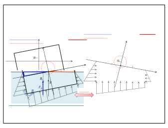
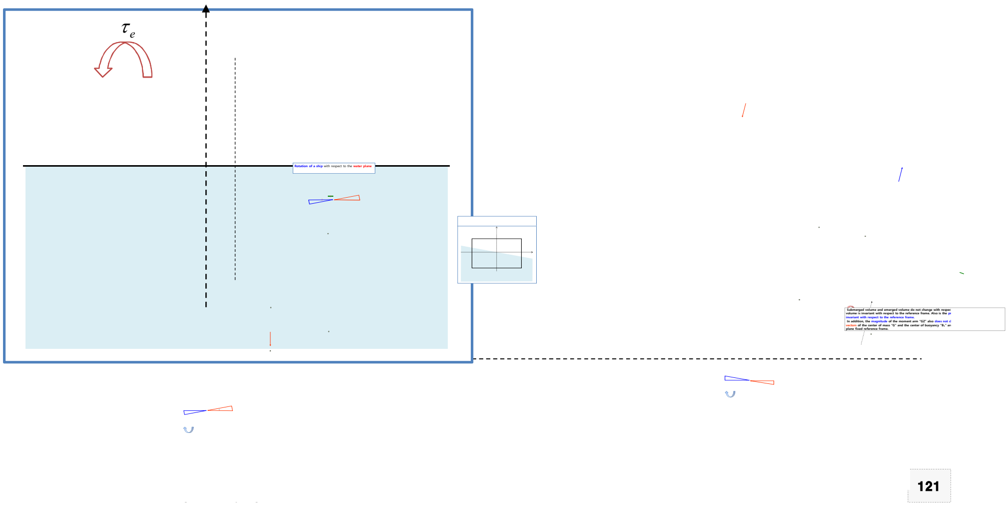


[Reference]

Orientation of a Ship with Respect to the Different Reference Frame

Inclination of a ship can be represented either with respect to the **water plane fixed frame("inertial reference frame")** or the **body fixed reference frame**.

Are these two phenomena with respect to the different reference frames the same?



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