Lecture Note of Naval Architectural Calculation

Ship Stability

Ch. 2 Review of Fluid Mechanics

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Contents

Ch. 2 Review of Fluid Mechanics

1. Hydromechanics and Hydrostatics

Introduction to Hydromechanics

- Today, the branch of physics, which encompasses the theories and laws of the behavior of water and other liquids, is known as .

- Hydromechanics itself is subdivided into three fields:

(1) , which deals with .
(2) , which studies .
(3) Hydraulics, dealing with the practical and engineering applications of hydrostatics and hydrodynamics.

Meaning of Hydrostatics

- What is Hydrostatics?

Hydrostatics (from Greek *hydro*, meaning , and *statics* meaning , or calm) describes the behavior of water in a state of rest.

This science also studies the forces that apply to immersed and floating bodies, and the forces exerted by a fluid.

Definition of Pressure

- **Pressure***

Let a small pressure-sensing device be suspended inside a fluid-filled vessel.

We define the pressure on the piston from the fluid as the force divided by area, and it has units Newton per square meter called 'Pascal'.

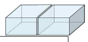
$$P = \frac{\Delta F}{\Delta A}$$

One newton per square meter is one Pascal.

We can find by experiment that at a given point in a fluid at rest, the pressure have the same value no matter how the pressure sensor is oriented.

Pressure is a , having no directional properties, and force is a vector quantity.

But is only the magnitude of the force.



Definition of Fluid

- **What is a Fluid?***

A fluid, in contrast to a solid, is a substance that , because it cannot withstand a shearing stress.

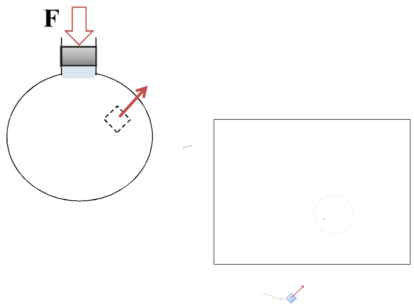
It can, however, exert a force in the direction perpendicular to its surface.

ΔF : Magnitude of perpendicular force between the two cubes
: Area of one face of one of the cubes

Pascal's Principle

- We will now consider a fluid element in static equilibrium in a closed container filled with a fluid which is either a gas or a liquid. The velocity of flow is everywhere zero.

- At first, we will neglect gravity. If a force F is applied on the cap of the container with an area A in this direction, then a pressure of F/A is applied.



Application of the Pascal's Principle

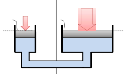
The idea of a Hydraulic jack

F_2

Pascal's Principle:

Consider a vessel with two pistons having area A_1 and area A_2. The vessel is filled with liquid everywhere.
Now a force F_1 on A_1 and a force F_2 on A_2 are applied. So the pressure on the left piston is F_1/A_1.
According to the Pascal's principle, everywhere in the fluid,
. The pressure on the right piston, F_2/A_2 must be the same as the pressure F_1/A_1, if the liquid is not moving. The effect of gravity does not change the situation very significantly.

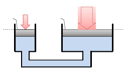
Pascal's Principle:
A change in the pressure applied an enclosed fluid is transmitted



Pascal's Principle:

(Pascal's Principle)

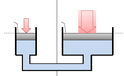
Example) If , then .



Example of Design of Hydraulic Jack (2/5)

F_2

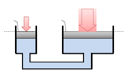
Pascal's Principle:



Displaced Volume:
(Incompressible fluid)

Pascal's Principle:

Conservation of Energy:

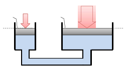


Displaced Volume:
(Incompressible fluid)

Example of Design of Hydraulic Jack (4/5)

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Pascal's Principle:



Displaced Volume:
(Incompressible fluid)

Conservation of Energy:

Example) If

→ (Pascal's Principle)

→ (Displaced Volume)

∴ Conservation of Energy is satisfied.

3. Hydrostatic Pressure

Hydrostatic Pressure (1/9)

- Hydrostatic Pressure

As every diver knows,
below the water.

As every mountaineer knows,
as one ascends into the atmosphere.

The pressure encountered by the diver and the mountaineer are usually called ,
because they are due to fluids that are static (at rest).

Here we want to find an expression for hydrostatic pressure as a function of depth or altitude.

Hydrostatic Pressure (2/9)

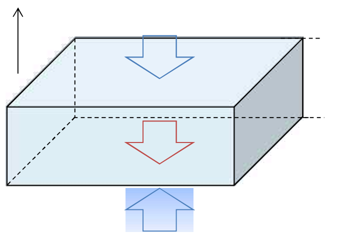
Now, gravity, of course, has an effect on the pressure in the fluid.

Hydrostatic pressure is due to fluids that are static (at rest).

Thus, there has to be
.

Consider a fluid element in the fluid itself and assume the upward vertical direction as the positive z-coordinate.

The mass of the fluid element is the volume times the density, and the volume is face area times delta z, and then times the density, which may be a function of z.



= Density of the fluid element
= Mass of the fluid element
= Horizontal base (or face) area
= Force that acts at the bottom surface (due to the water below the rectangular solid)
= Force that acts at the top surface (due to the water above the rectangular solid)
= Pressure at
= Pressure at

Hydrostatic Pressure (4/9)

Reference) Static Equilibrium

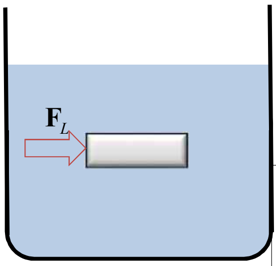
If a fluid is at rest in a container, all portions of the fluid must be in static equilibrium (at rest with respect to the observer).

Furthermore,

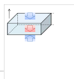
.

If this was not the case, a given portion of the fluid would not be in equilibrium.

For example, consider the small block of fluid. If the pressure were greater on the left side of the block than on the right, F_L would be greater than F_R, and the block would accelerate and thus would not be in equilibrium.



Newton's 2nd Law :



Hydrostatic Pressure (6/9)

Newton's 2nd Law :

$$\frac{P_{z+dz}-P_z}{dz}=-\rho(z)\cdot g$$

: Change of Hydrostatic Pressure
(Due to)

: Change in Hydrostatic Pressure
(Due to gravity)

Most liquids are practically incompressible. In other words, the density of the liquid cannot really change. And so therefore, we could always use the constant density, ρ, instead of the varying density ρ(z). We will assume from now on that fluids are completely incompressible. We can, then, do a very simple integration.

We have now dP in the L.H.S, which we can integrate from some value P_1 to P_2. And that equals now minus rho g dz in the R.H.S, integrated from z_1 to z_2.

Integrate from to .

Calculate the pressure difference between and .

Hydrostatic Pressure (8/9)

$$\int_{P_1}^{P_2} dP = -\int_{z_1}^{z_2} \rho(z) \cdot g \cdot dz$$

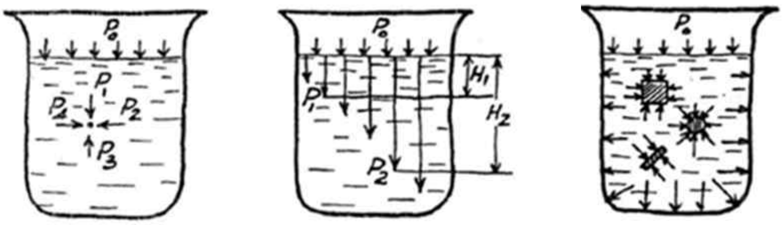
L.H.S:

R.H.S:

: Pascal's Principle (also Pascal's Law)

Hydrostatic Pressure (Incompressible fluid due to gravity)

Three Basic Characteristics of Pressure in a Body of Fluid

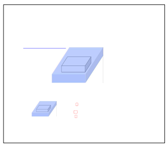


$P_1 = P_2 = P_3 = P_4$

Hydrostatic pressure at any point in a body of water is equal in all

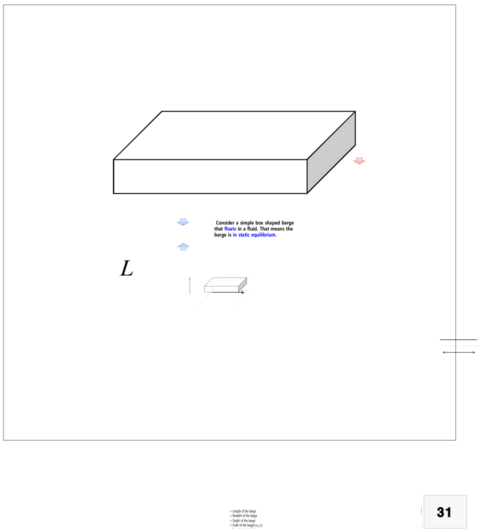
<Graphic presentation of the concept of hydrostatic pressure>

* Polevoy, S. L., Water Science and Engineering, Blackie Academic and Professional, pp.78, 1996



Archimedes' Principle and Buoyant Force (1/4)

- Static equilibrium of a **rigid body** in a fluid



Archimedes' Principle and Buoyant Force (3/4)

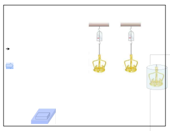
- **Buoyant force:** $F_B = F_1 - F_2$

Assumption: Buoyant force of air is neglected.

[Reference] Buoyant Force of Air

- **Static equilibrium of a barge**

$$F_B = (L \cdot B) \cdot (P_1 - P_2)$$



Archimedes' Principle and Buoyant Force
- Example: Archimedes and Crown Problem (2/2)

$$W_1 = V \rho_{crown} g$$

Archimedes lived in the third century B.C. Archimedes had been given the task to determine whether a crown was pure gold. He had the great vision to do the following: He takes the crown and he weighs it in a normal way. So the weight of the crown - we call it W_1 - is the volume of the crown times the density of which it is made. If it is gold, it should be 19.3 gram per centimeter cube (19.3 ton/m^3), and so volume of the crown x rho crown is the mass of the crown and multiplying mass by g is the weight of the crown. Cf. Silver: 10.49 ton/m^3

Now he takes the crown and he immerses it in the water. And he has a spring balance, and he weighs it again. And he finds that the weight is less and so now he has the weight immersed in the water.

So what he gets is the weight of the crown minus the buoyant force, which is the weight of the displaced water. And the weight of the displaced water is the volume of the crown times the density of water times g. And so V x rho w x g is 'weight loss'.

And he takes W_1 and divides by the weight loss and it gives him rho of the crown divided by rho of the water. And he knows rho of the water, so he can find rho of the crown. It's an amazing idea; he was a genius.

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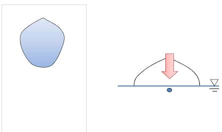
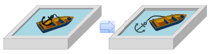




Archimedes' Principle and Buoyant Force
- Example: Floating Iceberg

Question)

What percentage of the volume of ice will be under the level of the water?

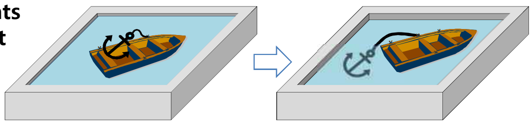
$$\rho_{ice} = 0.92\text{g/cm}^3, \rho_w = 1.0\text{g/cm}^3$$

Answer)

Archimedes' Principle and Buoyant Force
- Example: Waterline will change? (2/6)

Question)

A boat with an anchor on board floats in a swimming pool that is somewhat boat. Does the pool water level move up, move down, or remain the same if the anchor is



(a) Dropped into the water

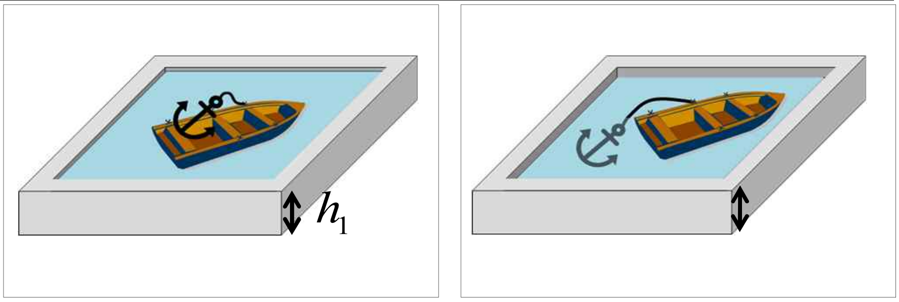
Answer)

The volume under the water level is composed of the water and the volume displaced by the boat and anchor. After the anchor is dropped into the water, the buoyant force exerted on the anchor cannot compensate the weight of the anchor.

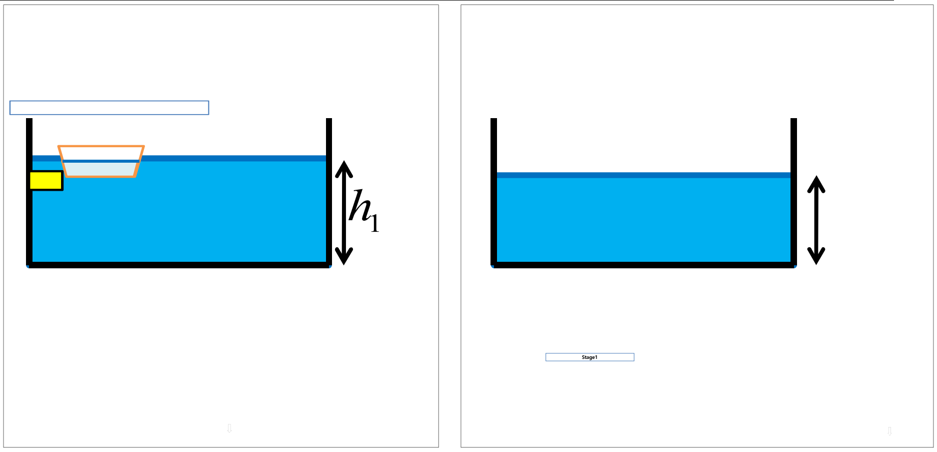
Thus the water level .

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.377, 2004

Archimedes' Principle and Buoyant Force
- Example: Waterline will change? (3/6)



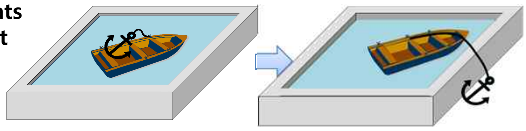
Example: Waterline will change? (4/6)
(a) Dropped into the water (1/2)



Archimedes' Principle and Buoyant Force
- Example: Waterline will change? (5/6)

Question)

A boat with an anchor on board floats in a swimming pool that is somewhat wider than the boat. Does the pool water level move up, move down, or remain the same if the anchor is



(b) Thrown onto the surrounding ground

Answer)

After the anchor is thrown onto the surrounding ground, the ground supports the weight of the anchor. So buoyant force exerted on the anchor is zero.

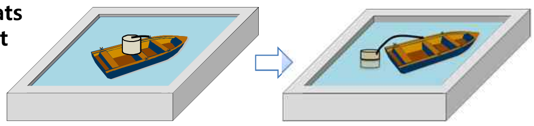
Thus the water level .

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.377, 2004

Archimedes' Principle and Buoyant Force
- Example: Waterline will change? (6/6)

Question)

A boat with an anchor on board floats in a swimming pool that is somewhat wider than the boat. Does the pool water level move up, move down, or remain the same if the anchor is



(c) If, instead, a cork is dropped from the boat into the water, where it floats, does the water level in the pool move upward, move downward, or remain the same?

Answer)

After the cork is dropped from the boat into the water, the cork floats in the water. So the buoyant force exerted on the cork has the same magnitude as that of the weight of the cork. Thus the volume displaced by the cork remains the same.

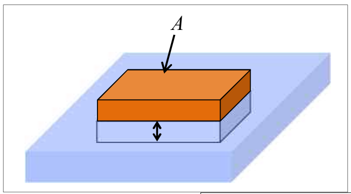
And the water level also .

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.377, 2004

Archimedes’ Principle and Buoyant Force
- Example: Floating Down the River (1/2)

Question)*

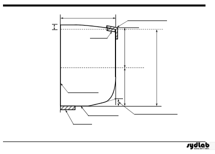
A raft is constructed of wood having a density of 600 kg/m³. Its water plane area is 5.7 m², and its volume is 0.60 m³. When the raft is placed in fresh water of density 1,000 kg/m³, as in the figure, to what depth does the raft sink in the water?



<A raft partially submerged in water>

Hint)

The magnitude of the upward buoyant force acting on the raft must equal the weight of the raft if the raft is to float. In addition, from Archimedes' Principle the magnitude of the buoyant force is equal to the weight of the displaced water.



Archimedes' Principle and Buoyant Force
- Example: 302,000DWT VLCC

Question)

A 302,000DWT VLCC has a mass of 41,000 metric tons when empty and it can carry up to 302,000 metric tons of oil when fully loaded. Assume that the shape of its hull is approximately that of a rectangular parallelepiped of 300m long, 60m wide, and 30m high.

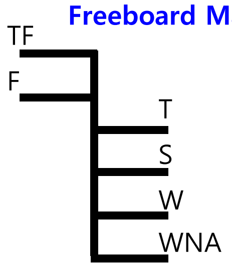
(a) What is the draft of the empty tanker, that is, how deep is the hull submerged in the water? Assume that the density of the sea water is $1.025 Mg/m^3$.

(b) What is the draft of the fully loaded tanker?

Archimedes' Principle and Buoyant Force - Freeboard (2/2)

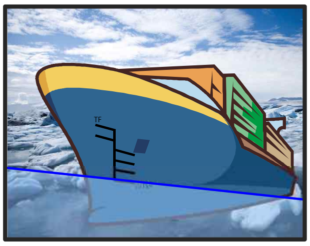
$$W = F_B$$
$$= (L \cdot B \cdot T) \cdot \rho_{sw} g$$

Freeboard Mark



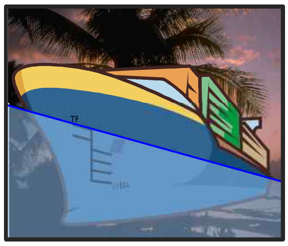
TF – Tropical Fresh Water
F – Fresh Water
T – Tropical Sea Water
S – Summer Sea Water
W – Winter Sea Water
WNA – Winter North Atlantic

The heaviest water is in the North Atlantic in winter time. Ships there displace much less water than in other areas of the world ocean.



The density of water in the world ocean is 1.026 g/cm^3. The density of water in the North Atlantic is 1.028 g/cm^3.

Tropical fresh water is lightest. It occurs in tropical rivers(Amazon, Congo, and others). Some of these rivers are navigable by ocean steamers.



The density of water in navigable tropical rivers is 0.997 g/cm^3.

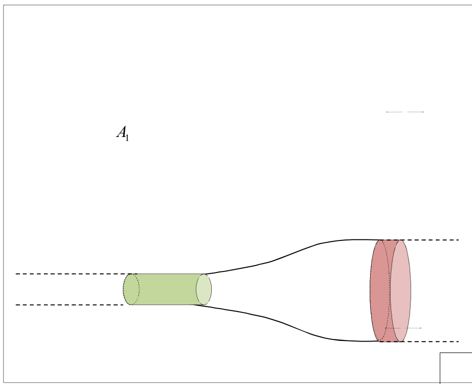
* Polevoy, S. L., Water Science and Engineering, Blackie Academic and Professional, p.93-97, 1996

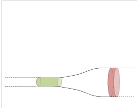
The Equation of Continuity* (1/3)

● **The Equation of Continuity**

The equation of continuity of flow is a mathematical expression of .

Here we wish to derive an expression that relates v and A for the steady flow of an ideal fluid through a tube with varying cross section.





The Equation of Continuity* (3/3)

- **The Equation of Continuity**

$$A_1 v_1 = A_2 v_2$$

This relation between speed and cross-sectional area is called the equation of continuity for the flow of an ideal fluid.

: Equation of Continuity
for the flow of an ideal fluid

The flow speed increases when we decrease the cross-sectional area through which the fluid flows.

Bernoulli's Equation (2/9)

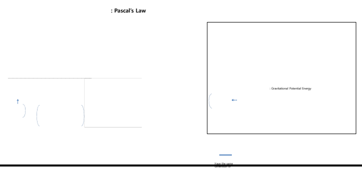
● **Bernoulli's Equation**

We can apply the principle of conservation of energy to the fluid.

Assumption: incompressible fluid

(1) If this fluid is completely static, it seems that it is not moving.

$$P_1 - P_2 = \rho g(z_2 - z_1) = \rho g h$$



Bernoulli's Equation (4/9)

- **Example: Eliminate 'z'**

If we take z to be a constant,
› that the fluid does not change
levation as it flows,

we assume that ,

$$A_1 v_1 = A_2 v_2$$

Bernoulli's Equation (6/9)
- Example: Siphon* (: Eliminate 'P') (1/3)

d

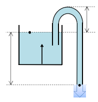


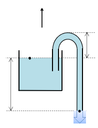
Figure on the right side shows a siphon, which is a device for removing liquid from a container, and a kind of tube.

A tube must initially be filled, but once this has been done, liquid will flow through the tube until the liquid surface in the container is the same level with the tube opening at z_1. The liquid has density ρ and negligible viscosity.

(a) With what speed does the liquid emerge from the tube at z_1?
(b) Theoretically, what is the greatest possible height d that a siphon can lift water?

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.383-384, 2004

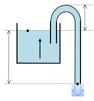
Bernoulli's Equation:



Bernoulli's Equation (6/9)
- Example: Siphon* (: Eliminate 'P') (3/3)

(b) Theoretically, what is the greatest possible height d that a siphon can lift water?

d



Barometric Pressure:

Therefore, This siphon would only work if is less than 10m.

→ ↑

→ ↓

↓

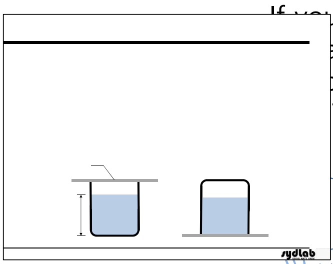
→ ↓

↑

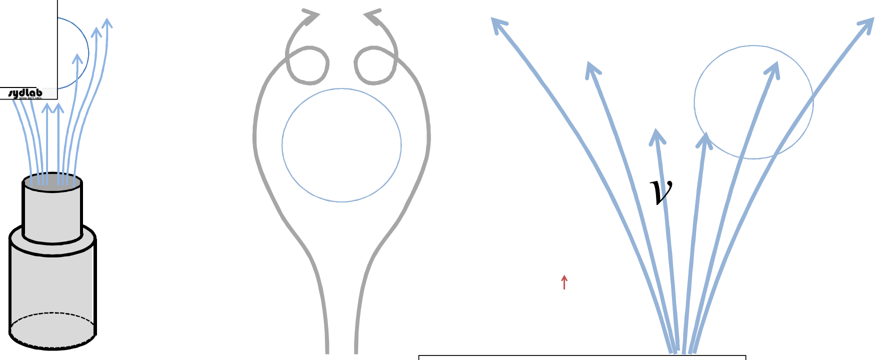
→ ↑

↑

Bernoulli's Equation (8/9)
- Example: Ping-Pong Ball in the Jet of Air*



If you place a ping-pong ball in the jet of air from a vacuum cleaner aimed vertically upward, the ping-pong ball will be held in stable ...brium with this jet. Explain this by means of Bernoulli's equation. ... The speed of air is maximum at the center of the jet.)

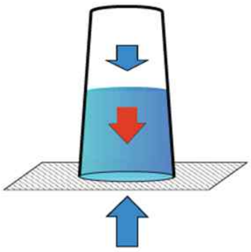


* Ohanian, H. C., Physics, 2nd Ed., W W. Norton & Company, p.354-355, pp.486, 1989

Bernoulli's Equation (9/9)
- Example: A Glass Filled with Water* (2/2)

☑ Any object in air is subject to pressure from air molecules colliding with it. At sea level, the mean air pressure is one "atmosphere" (=101,325 Pascals in standard metric units).

☑ The blue arrows indicate the forces due to air pressure above and below the water. The red arrow indicates the force of gravity. Together, the three forces balance out to cancel each other.



☑ The pressure of the outside air acts against the paper, and forces it against the glass, because it is stronger than the pressure of the water (weight of the water).